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RADIATIVE AND HADRONIC DECAYS OF HEAVY VECTOR MESONS*

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I. Introduction

The study of radiative and hadronic decays of the vector mesons containing a single heavy quark (b or c) is of great interest now both from the experimental and theoretical point of view. The recent measurements of the $D^* \to D\gamma$ and $D^* \to D\pi$ branching ratios have been done by CLEO and ARGUS -collaborations [1, 2]. These decays have been considered in the constituent quark model [3, 4] and in the light-front quark model [5]. A model independent approach based on the incorporation of chiral $SU(3)_L \times SU(3)_R$ symmetry and heavy quark symmetry for such decays has been developed in [6, 7]. There are two sets of adjustable parameters in this approach: two effective coupling constants coming from the effective Lagrangian which incorporates chiral and heavy quark symmetries, and the constituent heavy quark masses (M_c and M_b). The available experimental data allow one just to find the range for that parameters with quite large uncertainties and to make predictions for total and partial widths of charm and bottom vector mesons.

The combination of the heavy quark and chiral symmetries with the vector meson dominance hypothesis was used in [8]. It was shown that using experimental data on the $D \to \pi l\nu$, $D \to K l\nu$ and $D \to K^* l\nu$ transitions as an input gives the predictions for the radiative branching ratios $D^* \to D\gamma$, the hadronic rate $D^* \to D\pi$ as well as for the full D^* width.

In this paper, we shall analyze the radiative and hadronic decays of vector heavy mesons within the relativistic quark model with confined light quarks [9, 10]. This model is specified by the effective Lagrangian describing the point-like interaction of hadron with quarks, and the compositeness condition which allows one to determine the effective hadron-quark coupling constant. The behavior of light quarks is defined by the non-pole propagator with shape being defined by fitting well-known experimental data of meson interactions at low energies. This model has been extended to the heavy quark physics [11] by assuming that a heavy quark inside quark loops is described by the usual pole propagator. It was shown that the heavy quark symmetry is reproduced in the limit of large mass, and the Isgur-Wise function was calculated both for heavy mesons and baryons [11, 12].

II. Model

A heuristic bozonization procedure starting from the QCD generation functional is the basis of meson-quark interaction Lagrangian in QCM. Since our present objective is the application of this Lagrangian to actual physical problems, we refer the interested reader to Ref. [9]. There one can find how our Lagrangian may be reached within this type of approach.

The explicit form of the strong interaction Lagrangian which is needed in this paper is

$$\mathcal{L}_H = \frac{g_\pi}{\sqrt{2}}\{\pi^0[\bar{u}i\gamma^5 u - \bar{d}i\gamma^5 d] + \pi^+(\bar{u}i\gamma^5 d) + \pi^-(\bar{d}i\gamma^5 u)\} \tag{1}$$
$$+\left\{g_D\left[D^+(\bar{c}i\gamma^5 d)+D^0(\bar{c}i\gamma^5 u)\right]+g_{D^*}\left[D^{*+}_\mu(\bar{c}\gamma^\mu d)+D^{*0}_\mu(\bar{c}\gamma^\mu u)\right]+\text{h.c.}\right\}$$
$$+\left\{g_B\left[B^+(\bar{u}i\gamma^5 b)+B^0(\bar{d}i\gamma^5 b)\right]+g_{B^*}\left[B^{*+}_\mu(\bar{u}\gamma^\mu b)+B^{*0}_\mu(\bar{d}\gamma^\mu b)\right]+\text{h.c.}\right\}.$$

In fact, meson masses are the input to our model. Their values are given from Review of Particle Properties [13].

In QCM we assume that mesons are bound states of $\bar{q}q$, which is expressed in the *compositeness condition* that the renormalization constant for meson H with mass M_H is equal to zero:

$$Z_H = 1 - h_H \tilde{\Pi}'_H(M_H^2) = 0 \tag{2}$$

where $h_H = N_c g_H^2/(2\pi)^2$ with $N_c = 3$ is the effective coupling constant and $\tilde{\Pi}'_H$ is the derivative of the (renormalized) meson mass operator. It is important to remark that (i) our interaction Lagrangian together with the compositeness condition (2) has been shown [9] to be equivalent to the one obtained by the QCD bozonization based on the ideas of the Nambu-Jona Lasinio model [14], and (ii) the compositeness condition (2) allows one to determine the coupling constant h_H (or g_H) as a function of the physical meson mass.

Mesonic interactions in the QCM are defined by diagrams involving closed quark loops which can be obtained from the S-matrix.

$$S = \int d\sigma_{\rm vac} T \exp\{i \int dx \mathcal{L}_H(x)\}. \tag{3}$$

Here, the T-product is the ordinary Wick time ordering for the mesonic and quark fields. The quark propagator in the presence of the gluonic vacuum background $B_{\rm vac}$ has the following form:

$$S(x_1, x_2|B_{\rm vac}) =< 0|T(q(x_1)q(x_2))|0 >= i(\hat{\partial} + \hat{B}_{\rm vac})^{-1}\delta(x_1 - x_2). \tag{4}$$

We then assume [9] that an average over $B_{\rm vac}$ of the quark loops generated by the S-matrix provides the quark confinement and makes the theory ultraviolet-finite. It is to be remarked that there exist the vacuum gluon configurations with constant strengths (see, e.g. [15, 16]) which provide the quark confinement in this sense. This averaging for the light quarks takes the following form:

$$\int d\sigma_{\rm vac} {\rm tr}[H(x_1)S(x_1, x_2|B_{\rm vac})...H(x_n)S(x_n, x_1|B_{\rm vac})]. \tag{5}$$

where $\sigma_{\rm vac}$ is a set of variables characterizing $B_{\rm vac}$. Our confinement ansatz [9] for light quarks is to replace this equation by

$$\int d\sigma_v {\rm tr}[H(x_1)S_v(x_1 - x_2)...H(x_n)S_v(x_n - x_1)], \tag{6}$$

Here,

$$S_v(x_1 - x_2) = \int \frac{d^4p}{(2\pi)^4 i} e^{-ip(x_1-x_2)} \frac{1}{v\Lambda - \not p} \tag{7}$$

is the propagator of a light quark with a smeared constituent mass $m_q = v\Lambda$ where v is a complex parameter and the scale parameter Λ characterizes the size of the confinement region. The complex measure $d\sigma_{\rm vac}$ is defined to provide the absence of singularities corresponding to the physical quark production in eq.(8):

$$\int \frac{d\sigma_v}{v - \not p} = G(\not p) = a(-p^2) + \not p b(-p^2) \tag{8}$$

with the functions a and b being defined by

$$a(-p^2) = \int \frac{v d\sigma_v}{v^2 - p^2} \qquad b(-p^2) = \int \frac{d\sigma_v}{v^2 - p^2}. \tag{9}$$

The function $G(z)$ called the confinement function is an entire analytical function which decreases faster than any degree of z in an Euclidean direction $z^2 \to -\infty$; $G(z)$ is a universal function, i.e., is independent of color and flavor. In other words, the function $G(z)$ is common for all quark diagrams defining the hadron interaction at low energies. We have used a simple choice of the confinement functions [9]

$$a(u) = a_0 \exp(-u^2 - a_1 u) \qquad b(u) = b_0 \exp(-u^2 + b_1 u). \tag{10}$$

The parameters a_i, b_i and Λ have been determined from the best model description of data in low energy processes. The following values were found [9]: $a_0 = b_0 = 2$, $a_1 = 1$, $b_1 = 0.4$, and $\Lambda = 460$ MeV. The calculations of numerous low-energy effects of the meson-meson and meson-baryon interactions performed in the QCM have shown that the model allows one to describe quite well physical quantities [9]-[12].

It is clear that additional physical ideas are needed for applying the QCM to heavy quark physics. It is known that heavy quarks weakly interact with vacuum gluon fields, e.g. instantons [17]. Therefore, we have adopted [11] the following idea for describing the processes with heavy and light quarks. The interaction of light quarks is completely defined by the confinement mechanism whereas a heavy quarks is considered as an ordinary Dirac particle with a large mass. Hence, in expression

$$\int d\sigma_v \mathrm{tr}[H(x_1)S_v(x_1 - x_2)...H(x_n)S^{\mathrm{heavy}}(x_n - x_1)]. \tag{11}$$

propagator of a heavy quark is used as

$$S^{\mathrm{heavy}}(x) = \int \frac{d^4p}{(2\pi)^4 i} e^{-ipx} \frac{1}{M_Q - \not p}. \tag{12}$$

where M_Q is the constituent mass of a heavy quark.

Here, we just give the results for the coupling constants g_H which are determined from the compositeness condition (2).

$$g_H = \frac{2\pi}{\sqrt{3}} \frac{1}{\sqrt{\Pi'_H(M_H^2)}} \tag{13}$$

We can write $\Pi_H(M_H^2) = \Lambda^2 I_H(M_H^2/\Lambda^2)$ and $\Pi'_H(M_H^2) = I'_H(\frac{M_H^2}{\Lambda^2})$. Hereafter, we assume for simplicity that all momenta and masses in the structural integrals are given in units of Λ. The mass operator of heavy mesons is defined by the diagram on Fig.1. We have for pseudoscalar mesons :

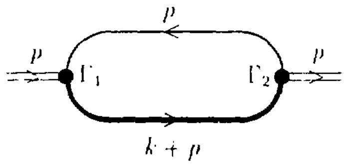

Figure 1: The mass operator of heavy meson.

$$I_{HP}(M_H^2) = -\int \frac{d^4k}{4\pi^2 i}\int d\sigma_v \mathrm{tr}\left\{ i\gamma^5 \frac{1}{v-\not k} i\gamma^5 \frac{1}{M_Q-\not k-\not p}\right\} \tag{14}$$

and for vector mesons :

$$\begin{aligned} I^{\mu\nu}_{HV}(M_H^2) &= -\int \frac{d^4k}{4\pi^2 i}\int d\sigma_v \mathrm{tr}\left\{ \gamma^\mu \frac{1}{v-\not k} \gamma^\nu \frac{1}{M_Q-\not k-\not p}\right\} \\ &= -g^{\mu\nu} I_{HV}(M_H^2) + p^\mu p^\nu I^*_{HV}(M_H^2). \end{aligned} \tag{15}$$

The integrals are calculated by using the standard Feyman's α-parametrization and the prescription (9) to integrate over $d\sigma_v$. We have

$$I_{HP}(M_H^2) = B_1 + \int_0^1 d\alpha\alpha \left[\frac{1}{(1-\alpha)^2} M_Q^2 - M_H^2\right] \times \tag{16}$$

$$\times\left\{ M_Q a\Big(\Delta(\alpha)\Big) + \Big((1-\frac{\alpha}{2})M_H^2 - M_Q^2\Big) b\Big(\Delta(\alpha)\Big)\right\}$$

$$I_{HV}(M_H^2) = B_1 + \int_0^1 d\alpha\alpha \left[\frac{1}{(1-\alpha)^2} M_Q^2 - M_H^2\right] \times \tag{17}$$

$$\times\left\{ M_Q a\Big(\Delta(\alpha)\Big) + \left[(1-\frac{1}{6}\alpha^2)M_H^2 - \frac{2-\alpha}{2-2\alpha} M_Q^2\right] b\Big(\Delta(\alpha)\Big)\right\}$$

The notation are

$$\Delta(\alpha)=\frac{\alpha}{1-\alpha}M_Q^2-\alpha M_H^2,\quad A_n=\int\limits_0^\infty du a(u)u^n,\quad B_n=\int\limits_0^\infty du b(u)u^n. \quad (18)$$

Finally, we get

$$g_{HP}=\frac{2\pi}{\sqrt{3}}\frac{1}{\sqrt{I'_{HP}(M_H^2)}},\qquad g_{HV}=\frac{2\pi}{\sqrt{3}}\frac{1}{\sqrt{I'_{HV}(M_H^2)}}, \quad (19)$$

$$I'_{HP}(M_H^2)=\int\limits_0^1 d\alpha\alpha\Bigg\{\left[\frac{2\alpha^2-5\alpha+4}{2(1-\alpha)^2}M_Q^2-(2-\alpha)M_H^2\right]b\Big(\Delta(\alpha)\Big)$$
$$+\alpha\left[\frac{1}{(1-\alpha)^2}M_Q^2-M_H^2\right]\left[M_Q^2-(1-\frac{\alpha}{2})M_H^2\right]b'\Big(\Delta(\alpha)\Big)$$
$$-M_Q a\Big(\Delta(\alpha)\Big)-M_Q a'\Big(\Delta(\alpha)\Big)\Bigg\}, \quad (20)$$

$$I'_{HV}(M_H^2)=\int\limits_0^1 d\alpha\alpha\Bigg\{\left[\frac{2\alpha^2-9\alpha+12}{6(1-\alpha)^2}M_Q^2-(2-\frac{\alpha^2}{3})M_H^2\right]b\Big(\Delta(\alpha)\Big)$$
$$+\alpha\left[\frac{1}{(1-\alpha)^2}M_Q^2-M_H^2\right]\left[\frac{2-\alpha}{2-2\alpha}M_Q^2-(1-\frac{\alpha^2}{6})M_H^2\right]b'\Big(\Delta(\alpha)\Big)$$
$$-M_Q a\Big(\Delta(\alpha)\Big)-M_Q a'\Big(\Delta(\alpha)\Big)\Bigg\}. \quad (21)$$

The details of calculation of coupling constants for light pseudoscalar mesons with mass m_{PL} may be found in Refs.[9, 10].

$$g_{LP}=\frac{2\pi}{\sqrt{3}}\sqrt{\frac{2}{R_{PP}(m_{PL}^2)}},\quad R_{PP}(x)=B_0+\frac{x}{4}\int\limits_0^1 du b(-\frac{ux}{4})\frac{(1-u/2)}{\sqrt{1-u}}. \quad (22)$$

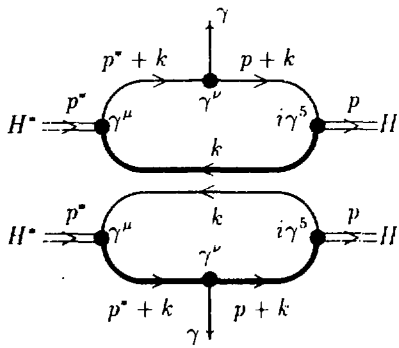

Figure 2: The diagram of radiative decay.

III. Radiative and hadronic decays of heavy vector mesons

The invariant matrix elements corresponding to the diagram Fig.2 are defined by

$$M^{\mu\nu}(p^*,p) = e_Q M_h^{\mu\nu}(p^*,p) + e_q M_l^{\mu\nu}(p^*,p) \tag{23}$$

with $M_h^{\mu\nu}(p^*,p)$ and $M_l^{\mu\nu}(p^*,p)$ being the hadronic matrix elements of heavy and light quark currents, respectively. e_Q and e_q are the charges of heavy and light quarks. p^* and p are the four-momenta of heavy vector and pseudoscalar mesons ($q = p^* - p$). We have

$$M_h^{\mu\nu}(p^*,p) = g_H g_{H^*}\Lambda \frac{3}{(2\pi)^2}\int \frac{d^4k}{4\pi^2 i}\int d\sigma_v \times \tag{24}$$

$$\times \mathrm{tr}\left\{\gamma^\nu \frac{1}{M_Q - \not k - \not p^*}\gamma^\mu \frac{1}{v - \not k} i\gamma^5 \frac{1}{M_Q - \not k - \not p}\right\} = e^{\mu\nu\rho\epsilon} p_\rho p^*_\epsilon I_h$$

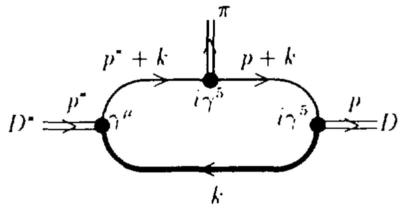

Figure 3: The diagram of hadronic decay.

$$M_l^{\mu\nu}(p^*,p) = g_H g_{H^*} \Lambda \frac{3}{(2\pi)^2} \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v \times \tag{25}$$

$$\times \mathrm{tr}\left\{ i\gamma^5 \frac{1}{M_Q - \not k}\gamma^\mu \frac{1}{v - \not p^* - \not k}\gamma^\nu \frac{1}{v - \not p - \not k}\right\} = \epsilon^{\mu\nu\rho\epsilon} p_\rho p^*_\epsilon I_l$$

with the integrals I's being defined by

$$\begin{vmatrix} I_h \\ I_l \end{vmatrix} = g_H g_H \ \frac{3}{(2\pi)^2}\frac{1}{\Lambda}\int_0^1 d\alpha_1 \int_0^1 d\alpha_2 \begin{vmatrix} \alpha_1 \\ (1-\alpha_1) \end{vmatrix} \zeta(\alpha_1,\alpha_2), \tag{26}$$

Here

$$\zeta(\alpha_1,\alpha_2) = a\Big(\Delta(\alpha_1,\alpha_2)\Big) + M_Q \frac{\alpha_1}{1-\alpha_1} b\Big(\Delta(\alpha_1,\alpha_2)\Big). \tag{27}$$

$$\Delta(\alpha_1,\alpha_2) = \frac{\alpha_1}{1-\alpha_1} M_Q^2 - \alpha_1 M_H^2 - \alpha_1\alpha_2 (M_{H^*}^2 - M_H^2). \tag{28}$$

The invariant matrix elements of hadronic decays are defined by the diagram in Fig.3.:

$$M_{\pi^+}^{\mu}(p^*,p) = g_D g_{D^*} \Lambda \frac{3}{(2\pi)^2} \int \frac{d^4k}{4\pi^2 i} \int d\sigma_v \times$$

$$\times \mathrm{tr}\left\{ i\gamma^5 \frac{1}{v - \not k - \not p^*}\gamma^\mu \frac{1}{M_Q - \not k} i\gamma^5 \frac{1}{v - \not k - \not p}\right\} = f_{(p)} p^\mu + f_{(p^*)} p^{*\mu}. \tag{29}$$

$$M_{\pi^0}^{\mu}(p^*,p) = \frac{1}{\sqrt{2}} M_{\pi^+}^{\mu}(p^*,p).$$

The width of π meson radiation includes only $f_{(p)}$ which is written down

$$f_{(p)} = g_H g_{H^*} \frac{3}{(2\pi)^2}\left[\int_0^1\!\!\int_0^1 d\alpha_1 d\alpha_2 \left\{ M_Q a\Big(\Theta(\alpha_1,\alpha_2)\Big) - M_Q^2 b\Big(\Theta(\alpha_1,\alpha_2)\Big) \right.\right.$$

$$\left. + (1-\alpha_1)\Big(M_H^2 + \alpha_2(M_{H^*}^2 - M_H^2)\Big) b\Big(\Theta(\alpha_1,\alpha_2)\Big)\right\}$$

$$\left. + \int_0^\infty du \int_0^1 d\alpha\, b\Big(u - \alpha(1-\alpha)m_\pi^2\Big)\right] \qquad (30)$$

where

$$\Theta(\alpha_1,\alpha_2) = \Delta(\alpha_1,\alpha_2) - (1-\alpha_1)\alpha_2(1-\alpha_2)m_\pi^2. \qquad (31)$$

with $\Delta(\alpha_1,\alpha_2)$ being defined by (28).

The widths of radiative and hadronic decays is defined in a standard manner

$$\Gamma(H^* \to H\gamma) = \frac{\alpha}{3}k^3\left[e_Q I_h + e_q I_l\right]^2, \qquad \Gamma(D^* \to D\pi) = \frac{g_\pi^2}{6\pi}\frac{p^3}{(2M_{D^*})^2} f_{(p)}^2. \qquad (32)$$

Where $p = \lambda^{\frac{1}{2}}(M_{D^*}^2, M_D^2, m_\pi^2)/2M_{D^*}$ and $k = \lambda^{\frac{1}{2}}(M_{H^*}^2, M_H^2, 0)/2M_{H^*}$ is the total momentum in the rest frame of decay particle.

IV. Numerical results and discussions

As it follows from (20),(21),(26),(30) the invariant amplitudes of radiative decays depend on the only adjustable parameter-heavy quark mass M_Q which may be varied in a resonable range. We shall define the value of charm quark mass by using the available experimental data taking into account that $\mathrm{Br}(D^{*0} \to D^0\gamma) + \mathrm{Br}(D^{*0} \to D^0\pi^0) = 1$, and $\mathrm{Br}(D^{*+} \to D^+\gamma) + \mathrm{Br}(D^{*+} \to D^+\pi^0) + \mathrm{Br}(D^{*+} \to D^0\pi^+) = 1$. The dependence of the branching ratios of decays D^{*0} and D^{*+} and the full decay width $\Gamma(D^{*+})$ on the c-quark mass are plotted in Fig.4 to Fig.7. One can see from Fig.7 that the experimental bound on the full decay width $\Gamma(D^{*+}) < 131$ KeV imposes the constraint on the upper bound of the mass of c-quark $M_c < 1.73$ GeV. The obtained branching ratios of D^{*0}-meson decays (see, Fig.4) are in accordance with available experimental data from CLEO if the value of M_c varies approximately in the interval 1.3 GeV $< M_c <$ 1.65 GeV. This interval includes all known estimations for the value of c-quark mass. We are not able to fix the value of c quark mass more precisely since the experimental data have very large uncertainties. Actually, one has to remark that the branching ratios of D^{*+}-meson decays in Fig.5 and Fig.6 depend very weakly on the value of c-quark mass and for all M_c agree with experimental data from CLEO and ARGUS.

It is interesting to see the suppression of the decay mode $D^{*+} \to D^+ + \gamma$ in comparison with the decay mode $D^{*0} \to D^0 + \gamma$. The ratio of their decay widths is written as

$$\frac{\Gamma(D^{*+} \to D^+\gamma)}{\Gamma(D^{*0} \to D^0\gamma)} \sim \frac{1}{4}\frac{(I_l - 2 \cdot I_h)^2}{(I_l + I_h)^2}. \tag{33}$$

Since the integrals I_l and I_h are positive this ratio is mainly defined by the difference $I_l - 2 \cdot I_h$ which as it follows from Fig.9 may be equal to zero for $M_c \approx 1.39$ GeV.

Using the allowed interval for the value of c-quark mass, we may be able to make predictions for the partial and full decay widths of heavy vector mesons. The magnitudes of radiative decay widths of D^*-meson for some values of M_c are shown in Table 1. In addition, we have calculated the decay widths of B^*-meson (see, Table 1 and Fig.8) although their experimental values are not yet measured.

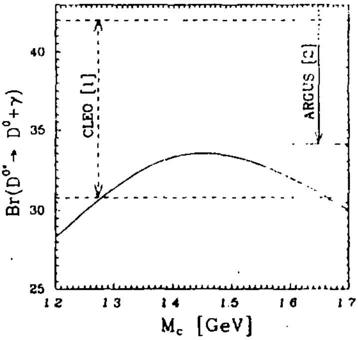

Figure 4: The dependence of branching ratio for the decay $D^{*0} \to D^0 + \gamma$ on M_c.

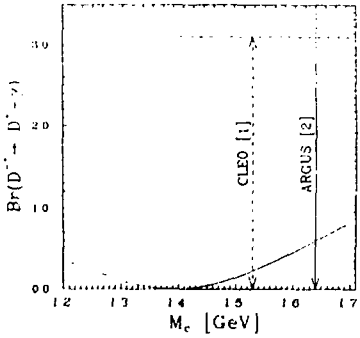

Figure 5: The dependence of branching ratio for the decay $D^{*+} \to D^+ + \gamma$ on M_c.

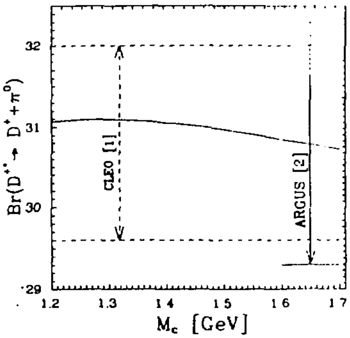

Figure 6: The dependence of branching ratio for the decay $D^{*+} \to D^+ + \pi^0$ on M_c.

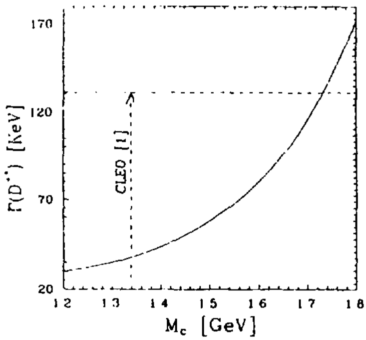

Figure 7: The dependence of total width $\Gamma(D^{*+})$ on M_c.

We compare our results with those obtained in other approaches [7],[8] (see. Table 2 and 3). One has to remark that the uncertainties of our results are only related to the adjustment of M_c to the experimental data with available errors. All other model parameters (confinement functions, etc.) are fixed by the low-energy physics with quite good accuracy. This means that our predictions may be considered as more reliable than those obtained in other approaches which usually have more adjustable parameters.

It is interesting to see the heavy quark limit in the invariant amplitudes when $M_Q \to \infty$ with $E = M_H - M_Q = \mathrm{const}$. Then we can get in the leading order over $1/M_Q$

$$g_{HP} = g_{HV} \to g_H^{\mathrm{asym}} = \frac{2\pi}{\sqrt{3}}\sqrt{\frac{M_Q}{\Lambda}\frac{1}{C_1(E)}}. \tag{34}$$

$$I_h \to I_h^{\mathrm{asym}} = \frac{1}{M_Q}, \qquad I_l \to I_l^{\mathrm{asym}} = \frac{1}{\Lambda}\frac{C_0(E)}{C_1(E)}. \tag{35}$$

where

$$C_n(E) = \int_0^\infty dt t^n \Big[a\Big(\Delta(t,E)\Big) + tb\Big(\Delta(t,E)\Big)\Big] \tag{36}$$

and

$$\Delta(t,E) = t^2 - 2Et. \tag{37}$$

We will neglect the pion mass under the calculation of asymptotics of hadronic decay amplitudes (22) and (30). Then one can obtain

$$g_{LP} \to g_L^{\mathrm{asym}} = 2\pi\sqrt{\frac{2}{3B_0}}. \tag{38}$$

Table 1: The widths of radiative and hadronic decays of D^*, D_s^* and B^* mesons

M_c	GeV	1.3	1.5	1.7
$\Gamma(D^{*0} \to D^0\gamma)$	KeV	7.57	13.73	23.55
$\Gamma(D^{*0} \to D^0\pi^0)$	KeV	16.57	27.49	51.89
$\Gamma(D^{*0})$	KeV	21.14	41.23	78.44
$\Gamma(D^{*+} \to D^+\gamma)$	KeV	0.03	0.08	0.95
$\Gamma(D^{*+} \to D^+\pi^0)$	KeV	10.88	17.96	35.78
$\Gamma(D^{*+} \to D^0\pi^+)$	KeV	21.08	39.98	79.74
$\Gamma(D^{*+})$	KeV	35.01	58.03	116.48
$\Gamma(D_s^* \to D_s\gamma)$	KeV	0.082	0.005	0.411
M_b	GeV	4.5	4.7	5
$\Gamma(B^{*0} \to B^0\gamma)$	KeV	$2.3 \cdot 10^{-2}$	$4.4 \cdot 10^{-2}$	$1.31 \cdot 10^{-1}$
$\Gamma(B^{*+} \to B^+\gamma)$	KeV	$4.2 \cdot 10^{-2}$	$1.04 \cdot 10^{-1}$	$4.01 \cdot 10^{-1}$

Table 2: The comparison of our results with those obtained in [7] and [8]

References		Our	Cho et al. [7]	Colangelo et al. [8]
M_c	GeV	1.7	1.7	
$\Gamma(D^{*0} \to D^0\gamma)$	KeV	23.55	8.8 ± 17.1	
$\Gamma(D^{*0} \to D^0\pi^0)$	KeV	51.89	41.8 ± 59.4	
$\Gamma(D^{*0})$	KeV	78.44	50.6 ± 61.9	36.7 ± 9.7
$\Gamma(D^{*+} \to D^+\gamma)$	KeV	0.95	8.3 ± 8.1	
$\Gamma(D^{*+} \to D^+\pi^0)$	KeV	35.78	27.7 ± 39.4	
$\Gamma(D^{*+} \to D^0\pi^+)$	KeV	79.74	61 ± 86.8	
$\Gamma(D^{*+})$	KeV	116.48	97 ± 95.6	46.1 ± 14.2
M_b	GeV	5	5	
$\Gamma(B^{*0} \to B^0\gamma)$	KeV	0.131	0.127 ± 0.203	0.075 ± 0.027
$\Gamma(B^{*+} \to B^+\gamma)$	KeV	0.401	0.66 ± 0.931	0.22 ± 0.09

Table 3: Branching ratios of radiative and hadronic decays

Branching ratio %	Our $M_c = 1.44 \pm 0.19$	Colahgelo et al. [8]	Experiment CLEO	Experiment ARGUS
$Br(D^{*0} \to D^0\gamma)$	31.76 ± 1.74	43.6 ± 17.8	$36.4 \pm 2.3 \pm 3.3$	$40.4 \pm 3.5 \pm 2.8$
$Br(D^{*0} \to D^0\pi^0)$	68.23 ± 1.74	56.4 ± 27.1	$63.6 \pm 2.3 \pm 3.3$	$59.6 \pm 3.5 \pm 2.8$
$Br(D^{*+} \to D^+\gamma)$	0.412 ± 0.412	1.1 ± 0.9	$1.1 \pm 1.4 \pm 1.6$	$0.0 \pm 2.8 \pm 1.6$
$Br(D^{*+} \to D^+\pi^0)$	30.89 ± 0.18	31.2 ± 17.4	$30.8 \pm 0.4 \pm 0.8$	$31.2 \pm 1.1 \pm 0.8$
$Br(D^{*+} \to D^0\pi^+)$	68.71 ± 0.25	67.7 ± 34.2	$68.1 \pm 1.0 \pm 1.3$	$68.1 \pm 2.4 \pm 1.3$

Table 4: The comparison of exact and asymptotic results for $M_c = 1.7$ GeV and $M_b = 5$ GeV

		Exact	Asymptotic
$\Gamma(D^{*0} \to D^0\gamma)$	KeV	23.55	24.80
$\Gamma(D^{*0} \to D^0\pi^0)$	KeV	54.89	27.44
$\Gamma(D^{*0})$	KeV	78.44	52.24
$\Gamma(D^{*+} \to D^+\gamma)$	KeV	0.95	0.95
$\Gamma(D^{*+} \to D^+\pi^0)$	KeV	35.78	17.82
$\Gamma(D^{*+} \to D^0\pi^+)$	KeV	79.74	39.74
$\Gamma(D^{*+})$	KeV	116.48	58.52
$\Gamma(B^{*0} \to B^0\gamma)$	KeV	0.131	0.134
$\Gamma(B^{*+} \to B^+\gamma)$	KeV	0.401	0.407

$$f_{(P)} \to f_{(p)}^{\mathbf{asym}} = \frac{M_Q}{\Lambda} \frac{C_0(E) + B_0 - 2\int\limits_0^\infty dt \left[(E-t) b\Big(\Delta(t, E) \Big) \right]}{C_1(E)}. \tag{39}$$

If we are assuming that $E = 0$, then

$$g_{HP} = g_{HV} \to g_H^{\mathbf{asym}} = \frac{2\pi}{\sqrt{3}} \sqrt{\frac{M_Q}{\Lambda} \frac{2}{A_0 + B_{\frac{1}{2}}}}, \quad I_h \to I_h^{\mathrm{asym}} = \frac{1}{M_Q}, \tag{40}$$

$$I_l \to I_l^{\mathbf{asym}} = \frac{1}{\Lambda} \frac{A_{-\frac{1}{2}} + B_0}{A_0 + B_{\frac{1}{2}}}, \quad f_{(p)} \to f_{(p)}^{\mathbf{asym}} = \frac{M_Q}{\Lambda} \frac{A_{-\frac{1}{2}} + B_0}{A_0 + B_{\frac{1}{2}}}. \tag{41}$$

where A_n and B_n are defined by (18). It is readily seen that the asymptotical formulas reproduce the well-known classical result

$$\Gamma(H^* \to H\gamma) = \frac{4\alpha}{3} k^3 \left[\mu_q + \mu_Q \right]^2. \tag{42}$$

where $\mu_q = e_q/(2m_q)$ and $\mu_Q = e_Q/(2M_Q)$ are the magnetons of light and heavy quarks, respectively.

Even though there is no the constituent mass of light quark in our approach according to the assumption on its confinement, we may be able to introduce the dimension parameter m_q which may be considered as an analogy of this value

$$m_q = \Lambda \frac{A_0 + B_{\frac{1}{2}}}{A_{-\frac{1}{2}} + B_0}. \tag{43}$$

One can see that the value m_q is defined by the model parameters only $A_0 = 1.09$, $A_{-\frac{1}{2}} = 2.73$, $B_0 = 2.25$, $B_{\frac{1}{2}} = 1.67$, and is equal to ≈ 255 MeV.

One has to remark that in the heavy quark limit the width of π-meson emission is defined by the magnetons of heavy and light quark too:

$$\Gamma(D^* \to D\pi^+) = \frac{g_\pi^2}{6\pi} \frac{p^3}{(2M_{D^*})^2} \left(\frac{e_Q}{e_q} \right)^2 \left[\frac{\mu_q}{\mu_Q} \right]^2. \tag{44}$$

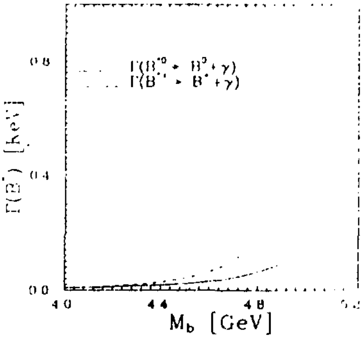

Figure 8: The dependence of $B^{*0}\to B^0+\gamma$ and $B^{*+}\to B^+ + \gamma$ decay widths on M_b.

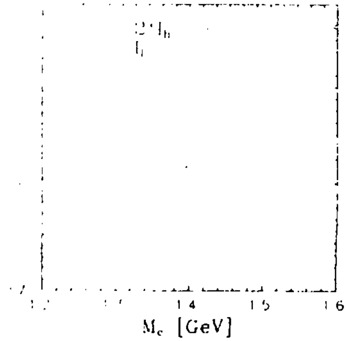

Figure 9: The dependence of structural integrals $2 \cdot I_h$ and I_l on M_c.

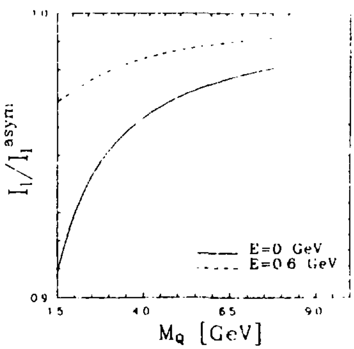

Figure 10: The comparison of exact and asymptotic results for the structural integral I_l.

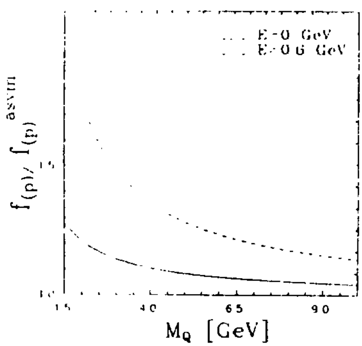

Figure 11: The comparison of exact and asymptotic results for the structural integral $f_{(p)}$.

We plot the dependencies of the ratios I_l/I_l^{asym} and $f_{(p)}/f_{(p)}^{asym}$ on the heavy quark mass in Fig.10 and Fig.11 to demonstrate how the results of precise calculation reach their asymptotics. The decay widths and branching ratios obtained by both the exact and asymptotic calculations are given in Table 4. One can see that the heavy quark limit for radiative decays of D and B mesons reproduce the results of exact calculations with quite good accuracy. The uncertainties in the structural integrals are of $I_h^{asym} \approx 7\% - 15\%$ and $I_l^{asym} \approx 1\% - 10\%$ depending on the different values of E for D-mesons, and less than 1% for B-mesons. More dramatic situation takes place for the processes with emission of π-meson. Here, the deviation of the asymptotic results from the exact calculations is of order $\approx 30\%$ through $\approx 200\%$ depending on the different values of the parameter E for D-meson. This means that the heavy quark limit cannot give the reliable results for the processes with emission of π-meson.

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