объединенный
институт
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исследований
дубна

E2-94-339

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NJL MODEL WITH NONPERTURBATIVE DEPENDENCE ON GLUON CONDENSATE

Submitted to «Modern Physics Letters A»

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1994

The NJL model with quark and gluon condensates has been investigated in papers [1-3]. The G^2-approximation by the gluon condensate (GC) has shown that taking account of GC saves the general structure of the NJL model. It leads to the redefinition of divergent integrals and to some changes of basic parameters of the model. Really, all divergent integrals acquire additional terms proportional to G^2. The cut-off parameter Λ is slightly decreasing and the constant of four-quark interaction is increasing. At the finite temperatures, GC offers the stabilizing action on the behaviour of constituent quark masses, quark condensate and different physical quantities of mesons, prevents from restoring the spontaneously broken chiral symmetry. Such an influence of GC reminds the influence of strong external magnetic fields, which in contrast with the electric fields also prevent from restoring the chiral symmetry (see [4-6]).

In this short note it will be shown that taking account of GC in a more complete functional approximation within the NJL model gives results, close to those, which have been obtained in the G^2-approximation earlier [1-3]. The Schwinger method will be used in such a form as it has been described in papers [5,7-9].

The Lagrangian of the NJL model with external gluon SU(3) color fields has the form [1-3,5]

$$L = \bar{q}(i\hat{D} - m^0)q + \kappa[(\bar{q}q)^2 + (\bar{q}i\gamma_5\vec{\tau}q)^2] - \frac{1}{4}G^a_{\mu\nu}G^{\mu\nu}_a, \tag{1}$$

where $\bar{q}, q$ are the quark fields, m^0 is the current quark mass,

$$G^{\mu\nu}_a = \partial^\mu G^\nu_a - \partial^\nu G^\mu_a - g f_{abc} G^\mu_b G^\nu_c, \tag{2}$$

$$D^\mu = \partial^\mu + ig\mathbf{G}^\mu, \quad \mathbf{G}^\mu = \frac{1}{2}\lambda_a G^\mu_a, \tag{3}$$

$\vec{\tau}$ denotes the Pauli matrices, λ_a are the Gell-Mann matrices. Here we consider only the classical parts of gluon fields because the quantum parts of these fields have already been used when obtaining effective four-quark interactions.

The detailed derivation of the gap equation for constituent quark mass in constant external gluon fields can be found in papers [5,9]. Here, we give the final form of this equation derived by the above mentioned authors for the case of constant fields considered in the flux-tube model

$$m = m^0 + 8m\kappa I_1(m, \Lambda) + \kappa F(m, G), \tag{4}$$

where

$$I_1(m, \Lambda) = -i\frac{N_c}{(2\pi)^4}\int^\Lambda \frac{d^4k}{m^2 - k^2} = \frac{3}{(4\pi)^2}\left[\Lambda^2 - m^2 \ln\frac{\Lambda^2}{m^2}\right], \tag{5}$$

is the quadratically divergent integral describing the quark condensate with the help of the cut-off parameter Λ, m denotes the constituent light quark mass and N_c is the number of colors. The function $F(m, G)$ has the following form:

$$F(m, G) = \frac{m}{2\pi^2}\left\{\int_0^\infty \frac{ds}{s^2} e^{-sm^2}\left[\left(\frac{Es}{3}\right)\cot\left(\frac{Es}{3}\right) - 1\right] + \right.$$

$$+2\int_0^\infty \frac{ds}{s^2} e^{-sm^2} \left[\left(\frac{Es}{6}\right) \cot\left(\frac{Es}{6}\right) - 1 \right] \Big\} = \tag{6}$$

$$= -i\frac{m}{\pi^2}\frac{E}{3}\left[\ln(\Gamma(z)\Gamma(2z)) + (3 - 2\ln 2)z + (1 - 3z)\ln z - \ln(\sqrt{2}\pi)\right],$$

where $z = i3m^2/2E$. The quantity E is expressed through the chargeless gauge fields G_3^μ and G_8^ν, which are connected with a color isotopic charge Q_3 and of color hypercharge G_8, respectively. So far as the external field is homogeneous, it can be expressed in terms of these fields only.

Let us now suppose that the quantity E is expressed via the gluon condensate $E^2 = cG^2$ and we find this connection by comparing the G^2 term, following from (4), with the corresponding expression derived from perturbation theory. For this aim one can use the G^2 approximation for the gap equation, obtained in papers [1,3]

$$m = m^0 + 8m\kappa I_1(m,\Lambda) + \kappa\frac{G^2}{6m}, \tag{7}$$

where

$$G^2 = \frac{g^2}{4\pi^2}\langle G^a_{\mu\nu} G^{\mu\nu}_a\rangle = (330\ \mathrm{MeV})^4. \tag{8}$$

Here we have used the value of GC taken from the article [10]. As a result, we get the equation

$$E^2 = -6\pi^2 G^2 \quad \text{or} \quad E = i\pi\sqrt{6G^2} \tag{9}$$

From here one can see that GC plays the role of a magnetic field in the case, where we consider the NJL model in strong electromagnetic fields [5,6]. When the strong electric field destroys the quark condensate, the magnetic field, as well as GC, intensifies the bond of $\bar{q}q$ pairs in the quark condensate. In particular, by studying the NJL model at finite temperatures one can notice that GC plays the stabilizing role at increasing temperatures (see [2]).

Now let us fix the parameters of the NJL model with nonperturbative dependence on GC and compare the obtained results with the G^2 approximation used in papers [1-3].

By using the value (8) for GC and $m = 300$ MeV for the mass of constituent u-quark, we obtain for the function $F(m,G)$ the following estimate:

$$F(m,G) = 0.17\ m\sqrt{G^2} = 0.0055\ \mathrm{GeV}^3. \tag{10}$$

This quantity is close to the value of the last term of equation (7) derived in the G^2 approximation

$$\frac{G^2}{6m} = 0.0066\ \mathrm{GeV}^3. \tag{11}$$

Therefore, the parameters of the NJL model, evaluated in the G^2 approximation for GC, change a little in a more complete nonperturbative approximation for GC. [1]

Really, we find the cut-off parameter Λ from the condition that the total quark condensate should be equal to its standard value

$$\langle\bar{q}q\rangle_0^{tot} = -4mI_1(m,\Lambda) - \frac{F(m,G)}{2} = (-250\ \mathrm{MeV})^3. \tag{12}$$

Then, we have

$$4mI_1(m,\Lambda) = \frac{3m}{(2\pi)^2}\left[\Lambda^2 - m^2\ln\left(\frac{\Lambda}{m}\right)^2\right] = (250\ \mathrm{MeV})^3 - \frac{F(m,G)}{2} =$$

$$= 15.6\times 10^6\ \mathrm{MeV}^3 - 2.8\times 10^6\ \mathrm{MeV}^3 = 12.8\times 10^6\ \mathrm{MeV}^3. \tag{13}$$

From here we obtain the value $\Lambda = 870$ MeV. From formula (13) one can see that the gluon corrections to the value of the total quark condensate amount to 18%.

For the coupling constant κ we get

$$\kappa^{-1} = \left(\frac{m_\pi F_\pi}{m}\right)^2 - \frac{2\langle\bar{q}q\rangle_0^{tot}}{m} = 0.124\ \mathrm{GeV}^2, \quad \kappa = 8.06\ \mathrm{GeV}^{-2} \tag{14}$$

and for the current quark mass m^0,

$$m^0 = \frac{m_\pi^2 F_\pi^2 \kappa}{m} = 4.6\ \mathrm{MeV}. \tag{15}$$

It is easily seen that these values are close to those which have been obtained in the G^2 approximation (see [1] and the footnote). This fact shows, that for the value of GC (8), used here, the G^2 approximation should give us completely reasonable results.

We would like to emphasize that the estimates given here have only the qualitative character since the function (6) has been obtained in [5,6] for a very special case of constant gluon fields G_3^μ and G_8^μ. However, we can see, that the form of this function keeps its form for both electromagnetic and chromodynamical fields. That is why one can hope, that also for the gluon condensates the form of this function changes only a little and, therefore, the calculations performed in our paper are completely legal. Of course, the problem to obtain more accurate form of the function $F(m,G)$ directly for GC remains an open and very interesting task.

An interesting question is connected with investigations of the temperature dependence of the function $F(m,G)$ and its influence on temperature behaviour of quark masses, quark condensates and different meson characteristics in the NJL model, particularly, in the neighbourhood of critical temperature. These problems will be topics of our further investigations.

MKV would like to thank the JSPS Program of Japan for their financial support.

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[1] Let us note, that in article [1] other values have been used for $G^2 = (410\ \mathrm{MeV})^4$ and $m_u = 330$ MeV. For $G^2 = (330\ \mathrm{MeV})^4$ and $m_u = 300$ MeV we obtain $\Lambda = 820$ MeV, $\kappa = 9.5\ \mathrm{GeV}^{-2}$ and $m^0 = 5$ MeV.

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Received by Publishing Department
on August 19, 1994.