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THE INTERRELATION OF A Z(3) GAUGE THEORY ON FLAT LATTICES AND A SPIN-1 BEG MODEL

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1 Introduction

It is well known that symmetries play a key role in modern physics and have a direct relation to universality hypothesis. The power of universality ideas comes from the fact that they relate the critical properties of a gauge theory in $(d+1)$ space-time dimensions to those of a classical scalar field theory (or spin system) in d spatial dimensions and allow predictions of critical indices in theories with continuous phase transitions.

The lattice gauge theory with one-parameter representation of the action was introduced by Wilson [1]:

$$S = -\beta \sum_{p} \Re e U_p , \tag{1}$$

where U_p denotes the usual plaquette variable, the product of link gauge fields $U_{x,\mu}$ around a plaquette. The critical behavior of this model depends on the dimensionality and the gauge group and includes both first- and second-order phase transitions.

The enlarged lattice gauge actions involving new double plaquette interaction terms were proposed and studied in $3d$ and $4d$ by Edgar [2], Bhanot et al [3]. The $2d$ version of one of these lattice gauge models with Z(2) gauge symmetry formulated on the planar rectangular windows was investigated by Turban [4]. This model with pure gauge action has been reduced to the usual spin-$\frac{1}{2}$ Ising model on the square lattice and the point of a second-order phase transition was found.

In our paper we constructed the Z(3) gauge model on the flat triangular and square lattices with double plaquette (window) representation of the action. The choice of this mixed action allowed us to connect it with the Hamiltonian of the spin-1 BEG model [5] and receive the line of the second-order phase transition.

2 The BEG model

The BEG model plays an important role in the development of the theory of multicritical points. Its Hamiltonian is

$$-\beta H = \sum_{<i,j>} [J S_i S_j + K S_i^2 S_j^2] - \Delta \sum_i S_i^2 , \tag{2}$$

where $S_i = 0, \pm 1$ is the spin variable at the site i and $< i,j >$ designate the nearest-neighbor pairs of the lattice sites. This model exhibits a rich critical behavior including the first- as well as second-order phase transitions and reflects interesting phenomena connected with real physical systems such as the phase separation in ^{3}He $-$ ^{4}He mixture, multicomponent fluids, microemulsions and so on. Its phase diagrams have been investigated by means of mean-field [6], renormalization group [7] and other approximations. The exact results have been obtained on the honeycomb [8, 9] and on the Bethe [10, 11] lattices.

As to the exact result on the honeycomb lattice, it is limited to the certain subspace of the exchange-interaction constants

$$e^K \cosh J = 1\,. \tag{3}$$

This condition permits one to map the BEG model to the usual spin-$\frac{1}{2}$ Ising model and get the λ-line of the second-order phase transition.

In our previous paper [11] we solved the BEG model with condition (3) on the Bethe lattice and found the tricritical point of the second-order phase transition for the lattices with coordination number greater than 6.

Now it is interesting to apply the results obtained for the BEG model to the gauge theory, because of the deep interrelation between spin and gauge lattice models.

3 The Z(3) gauge model

The model is considered on the flat triangular and square lattices in terms of the bond variables U_b which take their values in the Z(3), the group of the third roots of unity. Let $U_{p_i} = \prod_{b\in\partial p} U_b$ denote the product of U_b's around an elementary plaquette i.

The action of the model is

$$S_{Gauge}(\beta_{2g}, \beta'_{2g}, \beta_g) = S_{pp} + S_p\,, \tag{4}$$

where

$$S_{pp} = -\sum_{<p_i p_j>} \left\{\beta_{2g}\left(\delta_{U_{p_i},1}\delta_{U_{p_j},1} + \delta_{U_{p_i},z}\delta_{U_{p_j},z}\right) + \beta'_{2g}\left(\delta_{U_{p_i},1}\delta_{U_{p_j},z} + \delta_{U_{p_i},z}\delta_{U_{p_j},1}\right)\right\},$$

$$S_p = \beta_g \sum_{p_i} \left(\delta_{U_{p_i},1} + \delta_{U_{p_i},z}\right).$$

The first summation is over all nearest-neighbor plaquettes and the second one is over all plaquettes of the lattice, $z = \exp(i\frac{2\pi}{3}) \in Z(3)$.

After introduction of spin variables S_i in the sites of the dual lattice such that

$$\begin{aligned} S_i &= \delta_{U_{p_i},1} - \delta_{U_{p_i},z}\,, \\ S_i^2 &= \delta_{U_{p_i},1} + \delta_{U_{p_i},z}\,, \end{aligned} \tag{5}$$

the action (4) becomes

$$S_{Spin}(\beta_{2g}, \beta'_{2g}, \beta_g) = -\sum_{<ij>} \left\{\frac{\beta_{2g} - \beta'_{2g}}{2} S_i S_j + \frac{\beta_{2g} + \beta'_{2g}}{2} S_i^2 S_j^2\right\} + \beta_g \sum_i S_i^2 \tag{6}$$

in which we recognize the Hamiltonian multiplied by $1/k_B T$ of the BEG model (2), where $J = \frac{1}{2}(\beta_{2g} - \beta'_{2g})$, $K = \frac{1}{2}(\beta_{2g} + \beta'_{2g})$ and $\Delta = \beta_g$.

The corresponding partition function of the model (4) on the triangular lattice is

$$Z^{Triangular}_{Gauge}(\beta_{2g}, \beta'_{2g}, \beta_g) = \sum_{\{U\}} \exp\left[-S_{Gauge}(\beta_{2g}, \beta'_{2g}, \beta_g)\right]\,, \tag{7}$$

where the sum is taken over all possible configurations of the gauge variables $\{U\}$. This partition function can be rewritten in terms of the spin variables S_i defined in the sites of the dual lattice (honeycomb):

$$Z^{Triangular}_{Gauge} = 3^{N/2} Z^{Honeycomb}_{Spin}\,, \tag{8}$$

where

$$Z^{Honeycomb}_{Spin} = \sum_{\{S\}} \exp\left[-S_{Spin}\right].$$

A factor $3^{N/2}$ has been included in the equation (8) to take into account the difference between the number of gauge $\{U\}$ and spin $\{S\}$ configurations, since for each spin configuration with N sites we have $3^{N/2}$ identical gauge ones.

The condition (3) in terms of the gauge coupling constants takes the following form

$$\exp(\beta_{2g}) + \exp(\beta'_{2g}) = 2\,. \tag{9}$$

Therefore, we can apply the exact solution obtained by Horiguchi and Wu [8, 9] to our gauge model and obtain the line of the second-order phase transition

$$\exp(\beta_g) = 2\left[\sqrt{3}\exp(\beta_{2g}) - (\sqrt{3}+1)\right]. \tag{10}$$

In the same way as for triangular lattice we can reduce the partition function of our model defined on the square lattice to the partition function of the BEG model on the dual lattice:

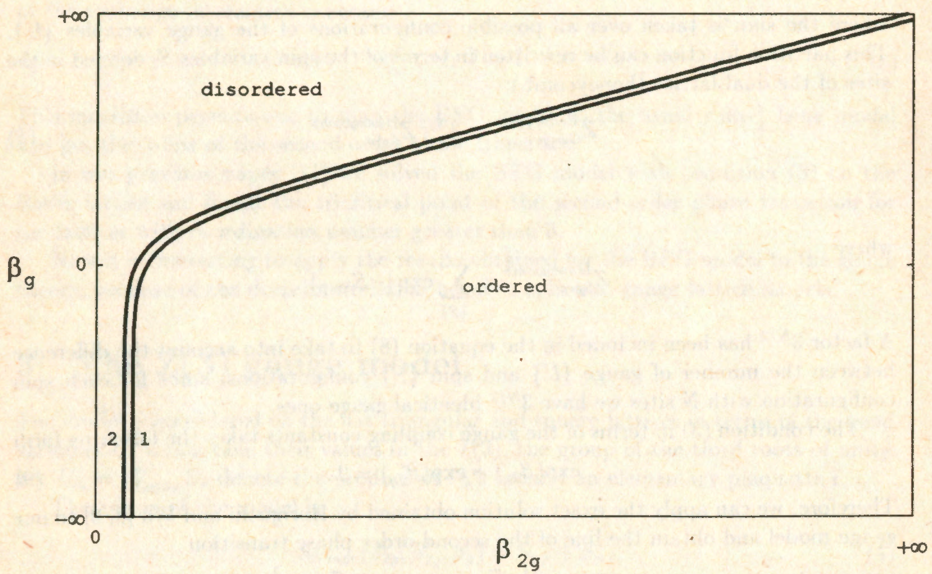
$$Z^{Square}_{Gauge}(\beta_{2g}, \beta'_{2g}, \beta_g) = 3^N Z^{Square}_{Spin}(\beta_{2g}, \beta'_{2g}, \beta_g)\,, \tag{11}$$

where the factor 3^N takes again into account the difference between the number of spin and gauge configurations.

For the BEG model formulated on the square lattice with condition (9) an approximate result has been obtained by Tang [12] using a free-fermion approximation technique [13]. Applying this solution to our model we get the critical line of the second-order phase transition

$$\exp(\beta_g) = 2(\sqrt{2}+1)\left[\exp(\beta_{2g}) - \sqrt{2}\right]. \tag{12}$$

The critical lines (10) and (12) of the second-order phase transition separate the (β_{2g}, β_g) plane into two regions as is shown in the figure. We have the phase transition from the ordered (Z(2) broken) phase which corresponds to confinement to the disordered one, since confinement in the gauge theory corresponds to ordering in the spin system. These transitions belong to the Ising type and hence they have the same critical exponents.



The plane of the coupling constants (β_g, β_{2g}) is separated by the lines of the second-order phase transition into two regions (ordered and disordered). The line 1 (2) corresponds to the triangular (square) lattice.

4 Conclusion

We constructed the Z(3) gauge lattice model with double plaquette (window) representation of the action and showed that this model is dual to the spin-1 BEG one. Using the exact solution of the BEG model on the honeycomb lattice and the solution obtained by the free-fermion approximation technique on the square lattice, we found the lines of the second-order phase transition.

The constructed action gives possibility to transform the Z(3) gauge theory defined on the generalized Bethe lattice of plaquettes [14, 15] to the BEG model on the usual Bethe lattice for which the exact solution was obtained [10] and receive exact expressions for the multicritical points and construct phase diagrams.

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