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THE INTERACTION OF MAGNETIZATIONS WITH AN EXTERNAL ELECTROMAGNETIC FIELD AND A TIME-DEPENDENT MAGNETIC AHARONOV-BOHM EFFECT

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1. Introduction

Usually, investigating the topological effects in quantum mechanics, one studies the scattering of charged particles on the space regions where electromagnetic strengths, not potentials, are equal to zero. There are known the magnetic Aharonov-Bohm (AB) effect when the particles are scattered on a static magnetic vector potential (VP) / 1 / and the electric AB effect when the phase of particle wave function is changed by the electric scalar potential / 2 /.

Here we propose to change the aforementioned phase by the time-dependent magnetic VP. The plan of our exposition is as follows. In § 2 we study the properties of the magnetic fields originatig from different choices of magnetizations. The physical meaning of the scalar functions entering into the Neumann-Helmholtz parametrization of the current density is clarified in §3. It turns out that the selectivity of the interaction to the time dependence of an external electromagnetic field arises for a specific choice of these functions. This can be used for the storage and ciphering of information. The time-dependent AB effect is studied in § 4. For this we construct the charge-particle configuration having the property that the magnetic field strength differs from zero in the space region inaccessible for the incoming charged particles while the time-dependent magnetic VP differs from zero everywhere. The static electric field differs from zero in the finite space region adjoining the charge-current configuration. Although charged particles may scatter on this electric field, the latter contributes only to the static background. It is the time variation of the magnetic VP that changes the phase of the wave function and the observed interference picture. This may be used as a new channel for the information transfer. As far as we know, this is the first example of the time-dependent AB effect. In the physical literature (see, e.g., / 3 /) a quite different physical effect is known under the same title, when one studies the time evolution of the wavepacket in the field of the usual static magnetic string.

2. Magnetization and toroidization

Consider the circular current lying in the Z=0 plane (the upper part of fig.1):

$$\vec{J} = \vec{n}_\varphi \cdot I\,\delta(\rho-d)\,\delta(z) = \frac{1}{d}\cdot I\,\vec{n}_\varphi\cdot\delta(r-d)\cdot\delta\left(\theta-\frac{\pi}{2}\right). \tag{2.1}$$

As div J = 0, the equivalent magnetization / 4,5 / can be used instead of J:

$$\vec{J} = \mathrm{rot}\,\vec{M}_1, \tag{2.2}$$

$$\vec{M}_1 = I\,\vec{n}_z\,\delta(z)\cdot\Theta(d-\rho) = -I\,\vec{n}_\theta\,\frac{1}{d}\,\Theta(d-r)\,\delta\left(\theta-\frac{\pi}{2}\right), \tag{2.3}$$

$\Theta(x)$ is the step function.

This relation is a mathematical expression of the Ampere hypothesis according to which the closed circular current is equivalent to the magnetized sheet. The magnetic field can be calculated either from (2.1) or (2.3). For example, the magnetic vector potential is given by

$$\vec{A} = \frac{1}{c}\,I\int\frac{1}{|\vec{r}-\vec{r}\,'|}\,\vec{n}_\varphi\,\delta(\rho'-d)\,\delta(z')\,dV' =$$

$$= -\frac{1}{c}\int\frac{\vec{r}-\vec{r}\,'}{|\vec{r}-\vec{r}\,'|^3}\times\vec{M}_1(\vec{r}\,')\,dV'. \tag{2.4}$$

For an infinitely small d, the current J in Eq.(2.1) is not well defined (the vector $\vec{n}_\varphi$ loses its sense at the origin). On the other hand, the magnetization M_1 in Eq.(2.3) is well defined. In the limit $d\to 0$ Eqs. (2.1)-(2.3) mean that the circular current of an infinitely

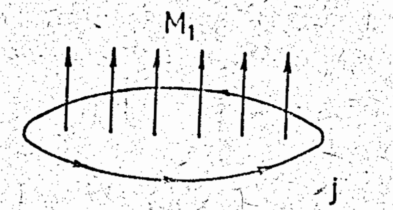

Fig.1 : Two magnetizations M_1 and M_2 corresponding to the same current J

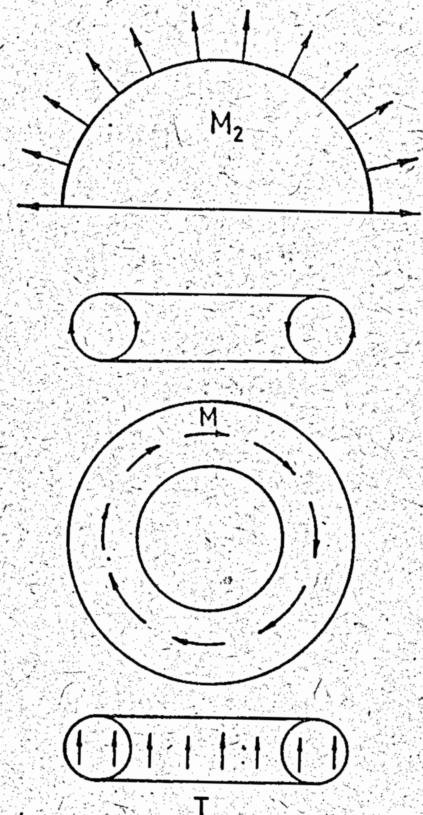

Fig.2 : The poloidal current J flowing on the torus surface is equivalent to the magnetization M which in turn is equivalent to the toroidization T

small radius is equivalent to the magnetic dipole. More complex case is the poloidal current flowing on the torus surface $(\rho-d)^2+z^2=R^2$ (see fig.2). To parametrize J it is convenient to introduce the coordinates $\tilde{R}, \psi$

$$x=(d+\tilde{R}\cos\psi)\cos\varphi, \quad y=(d+\tilde{R}\cos\psi)\sin\varphi, \quad z=\tilde{R}\sin\psi.$$

In these coordinates,

$$\vec{J} = \vec{n}_\psi \frac{\delta(\tilde{R}-R)}{d+R\cos\psi}\,\frac{j_0}{R^2 d}. \tag{2.5}$$

Here $\vec{n}_\psi$ is the unit vector tangential to the torus surface:

$$\vec{n}_\psi = \vec{n}_z\cos\psi - \vec{n}_\rho\sin\psi.$$

It lies in the $\varphi=\text{const}$ plane and defines the direction of J. The factor R^2 in the denominator of J is introduced for convenience and may be absorbed into J. As div J = 0, the current J may be expressed through the magnetization: J = rot M . It turns out that M is enclosed inside the torus T and has only the φ component:

$$\vec{M} = -\frac{1}{R^2 d}\, j_0\, \vec{n}_\varphi\, \frac{1}{d+\tilde{R}\cos\psi}\,\Theta(R-\tilde{R}). \tag{2.6}$$

As div M = 0, it can be presented as M = rot T, where

$$\vec{T} = \vec{n}_z \cdot T \cdot j_0 / Rd. \tag{2.7}$$

Here

$$T = \ln\frac{d-\sqrt{R^2-z^2}}{d+\sqrt{R^2-z^2}} \tag{2.8}$$

inside the torus hole ($-R\le z\le R,\ 0\le\rho\le d-\sqrt{R^2-z^2}$) and

$$T = \ln\frac{\rho}{d+\sqrt{R^2-z^2}} \tag{2.9}$$

inside the torus itself ($-R\le z\le R,\ d-\sqrt{R^2-z^2}\le\rho\le d+\sqrt{R^2-z^2}$).
In other space regions T = 0. In analogy with the magnetization M, the

distribution T may be called the toroidization. It follows from Eqs.(2.5)-(2.9) that

$$J = \text{rot rot } T, \tag{2.10}$$

$$\vec{A} = \frac{4\pi}{c}\vec{T}(\vec{r}) + \frac{1}{c}\,\text{grad}\int\frac{1}{|\vec{r}-\vec{r}'|}\,\text{div}\,\vec{T}(\vec{r}')\,dV'. \tag{2.11}$$

The magnetic field strength differs from zero only inside the torus :

$$H_\varphi = -\frac{4\pi}{c}\,\frac{j_0}{dR^2}\,\frac{1}{\rho}.$$

We consider now the case when the torus dimensions d,R tend to zero. Since R is always less than d we let to tend R to zero first and d later. In the limit $R\to 0$ the current J (see fig.2) becomes ill-defined. On the other hand, M and T remain well-defined:

$$\vec{M} \to -\vec{n}_\varphi\,\frac{\pi}{d^2}\,j_0\,\delta(\rho-d)\,\delta(z),$$

$$\vec{T} \to -\vec{n}_z\,\frac{\pi j_0}{d^2}\,\Theta(d-\rho)\,\delta(z) \tag{2.12}$$

for $R\to 0$.

When d goes to zero, the magnetization M becomes ill-defined, but the vector T is still well-defined:

$$\vec{T} \to -\vec{n}_z\,\frac{\pi}{2\rho}\,j_0\,\delta(z)\,\delta(\rho) = -\vec{n}_z\,j_0\,\pi^2\,\delta(\vec{r}). \tag{2.13}$$

The VP corresponding to this toroidization is given by

$$A_x = -\frac{3\pi^2 j_0 x z}{c r^5},\quad A_y = -\frac{3\pi^2 j_0 y z}{c r^5},\quad A_z = \pi^2 j_0\frac{r^2-3z^2}{c r^5} - \frac{8\pi^3}{3c}\,j_0\,\delta(\vec{r}).$$

We consider now the sequence of toroidal solenoids each turn of which is again a toroidal solenoid. For the case shown in fig.3

$$\vec{J} = (\text{rot})^3\,\vec{T}(\vec{r}),$$

$$\vec{A} = \frac{4\pi}{c}\,\text{rot}\,\vec{T}(\vec{r}). \tag{2.14}$$

We see that for this current both the vector potential and magnetic field differ from zero in those space regions where $\vec{T}\neq 0$.

When the space region in which $T \neq 0$ is contracted to a point, the vector potential and magnetic field differ from zero in that point only. Now we clarify how the treated current distributions interact with an external electromagnetic field. Let J be of the form

$$\vec{J} = (\mathrm{rot})^n \vec{T},$$

where T is either confined to the finite region of space or decreases sufficiently fast at infinity. Then the interaction energy is proportional to

$$U \sim \int \vec{A}_{ext} (\mathrm{rot})^n \vec{T} \, dV = \int \vec{T} (\mathrm{rot})^{n-1} H_{ext} \, dV.$$

The final answer is different for n even and odd. If $n = 2k+1$, then

$$U \sim (-1)^k \int \vec{T} \cdot \left(\frac{1}{c}\frac{\partial}{\partial t}\right)^{2k} \vec{H}_{ext} \, dV.$$

For $n = 2k+2$ one has

$$U \sim (-1)^k \int \vec{T} \cdot \left(\frac{1}{c}\frac{\partial}{\partial t}\right)^{2k+1} \vec{E}_{ext} \, dV.$$

Thus, we obtain the sequence of current configurations which interact with the time-dependent magnetic or electric field. For example, the current loop shown in fig.1 interacts with external magnetic field. The poloidal current presented in fig.2 interacts with the first derivative of the electric field. The current configuration of fig.3 interacts with the second derivative of the magnetic field, etc.

3. Magnetizations and Helmholtz-Neumann theorem

According to the Helmholtz-Neumann theorem, an arbitrary vector function can be represented in the form

$$\vec{j} = \mathrm{grad}\,\Psi_1 + \mathrm{rot}\,(\vec{r}\,\Psi_2) + \mathrm{rot}\,\mathrm{rot}\,(\vec{r}\,\Psi_3). \tag{3.1}$$

Comparing this with Eqs. (2.1),(2.2) we get

$$\Psi_1 = \Psi_3 = 0, \quad \Psi_2 = \frac{1}{2} I \,\delta(r-d)\, \tilde{\Theta}\left(\frac{\pi}{2}-\theta\right). \tag{3.2}$$

The corresponding magnetization is given by

$$\vec{M}_2 = \vec{n}_r \cdot \delta(r-d)\, \tilde{\Theta}\left(\frac{\pi}{2}-\theta\right). \tag{3.3}$$

This magnetization covers the upper semi-sphere of the radius d and is directed along its radius. It is certainly different from the magnetization (2.3). This means that the current representation (3.1) although being correct does not describe the sequence of toroidal distributions discussed in the previous section. The magnetizations M_1 and M_2 are connected by the gradient transformation

$$\vec{M}_2 = \vec{M}_1 + \mathrm{grad}\,\chi, \quad \chi = -\tilde{\Theta}(d-r)\,\Theta\left(\frac{\pi}{2}-\theta\right), \tag{3.4}$$

i.e., the function χ differs from zero inside the upper semi-sphere. This equation means that the magnetizations M and M despite their different functional forms lead to the same observable effects. The reason for the appearance of different magnetizations is that the equation rot M = J does not fix M uniquely. We note in passing that magnetic strength H satisfies almost the same equation rot H = $\frac{4\pi}{c}$ J but with the additional condition div H = 0. These two equations are sufficient for fixing H. In general, the condition div M = 0 is not imposed on M. As we have said, the requirement of M disappearance in the nearest vicinity of J does not fix M unambiguously.

Now we are able to clarify the physical meaning of the functions Ψ defining the current density J. For this we consider the interaction of the current J with an external electromagnetic field defined by the scalar potential φ_{ext} and vector potential $\vec{A}_{ext}$

$$\mathcal{U} \sim \int \vec{A}_{ext} \cdot \vec{J}\, dV.$$

Substituting here J, integrating by parts and assuming that J_{ext} does not overlap with the space region S where J ≠ 0 we get:

$$\mathcal{U} = \mathcal{U}_e + \mathcal{U}_d + \mathcal{U}_t,$$
$$\mathcal{U}_e = -\frac{1}{c}\int \dot{\varphi}_{ext} \cdot \psi_1 dV,$$
$$\mathcal{U}_d = \int \vec{r}\, \vec{H}_{ext} \cdot \psi_2 dV,$$
$$\mathcal{U}_t = \int \vec{r} \cdot \mathrm{rot}\, \vec{H}_{ext}\, \psi_3 dV. \qquad (3.5)$$

Here H = rot A is the magnetic strength of the external field and the dot above the letter means the time derivative. Let the dimensions of S be small as compared with the distance from the sources of the external field. Then, the external fields varying rather slowly over S can be approximated by their values taken at some point $\vec{r}_0$ lying inside S. Thus,

$$\mathcal{U}_e^{(1)} = -\frac{1}{c}\dot{\varphi}_{ext}(0) \cdot \int \psi_1 dV,$$
$$\mathcal{U}_d^{(1)} = \vec{H}_{ext}(0)\int \vec{r}\,\psi_2 dV, \quad \mathcal{U}_t^{(1)} = \mathrm{rot}\,\vec{H}_{ext}(0) \cdot \int \vec{r}\,\psi_3 dV. \qquad (3.6)$$

Here

$$\varphi_{ext}(0) \equiv \varphi_{ext}(\vec{r}_0), \quad \vec{H}_{ext}(0) \equiv H_{ext}(\vec{r}_0).$$

It follows from this that

$$\vec{\mu}_d = \int \vec{r}\,\psi_2 dV \quad \text{and} \quad \vec{\mu}_t = \int \vec{r}\,\psi_3 dV$$

are the magnetic dipole and toroidal / 6 / moments,resp. The next terms in the development of $\mathcal{U}_d$ are

$$\mathcal{U}_d^{(2)} = \frac{\partial H_i(0)}{\partial x_k}\mu_{ik}, \qquad \mu_{ik} = \int (x_i x_k - \tfrac{1}{3}\delta_{ik} r^2)\psi_2 dV,$$

$$\mathcal{U}_d^{(3)} = \frac{1}{2}\frac{\partial^2 H_i(0)}{\partial x_k \partial x_j}\mu_{ijk} + \frac{1}{10c^2}\frac{\partial^2 \vec{H}(0)}{\partial t^2}\vec{\mu}_d^{(2)},$$
$$\mu_{ijk} = \int [x_i x_k x_j - \tfrac{1}{5}(\delta_{ij}x_k + \delta_{ik}x_j + \delta_{kj}x_i) r^2]\psi_2 dV,$$
$$\vec{\mu}_d^{(2)} = \int \vec{r}\, r^2 \psi_2 dV. \qquad (3.7)$$

Obviously, μ_{ij} and μ_{ijk} coincide with the quadrupole and octupole magnetic moments, resp. Thus, the function ψ_2 describes the set of magnetic moments of different multipolarities. Similarly, one obtains the next terms in the expansion of $\mathcal{U}_t$

$$\mathcal{U}_t^{(2)} = \frac{\partial}{\partial x_k}[\mathrm{rot}\,\vec{H}(0)]_i\, t_{ik},$$
$$\mathcal{U}_t^{(3)} = \frac{1}{2}\frac{\partial^2}{\partial x_k \partial x_j}[\mathrm{rot}\,\vec{H}(0)]_i\, t_{ikj} + \frac{1}{10}\left(\frac{1}{c}\frac{\partial}{\partial t}\right)^3 \vec{E}\,\vec{\mu}_t^{(2)},$$
$$t_{ik} = \int (x_i x_k - \tfrac{1}{3}\delta_{ik} r^2)\psi_3 dV,$$
$$t_{ikj} = \int [x_i x_k x_j - \tfrac{1}{5}(\delta_{ik}x_j + \delta_{ij}x_k + \delta_{kj}x_i) r^2]\psi_3 dV,$$
$$\vec{\mu}_t^{(2)} = \int \vec{r}\, r^2 \psi_3 dV. \qquad (3.8)$$

This means that the function ψ_3 describes the toroidal moments of higher multipolarities / 6 /. Their physical realization via the toroidal solenoids enclosed into each other has been given in ref / 7 /.

In what follows we consider the stationary current distribution for which $\psi_1 = 0$ (as $\mathrm{div}\,\vec{J} = \Delta\psi_1 = -\partial\rho/\partial t$). Let ψ_2 be of the form

$$\psi_2 = \Delta \psi_2^{(1)}. \qquad (3.9)$$

Then,

$$U_d = \frac{1}{c^2}\frac{\partial^2 \vec{H}_{ext}(0)}{\partial t^2}\int \vec{r}\,\Psi_2^{(1)}\,dV. \qquad (3.10)$$

It follows from this that such a current configuration does not interact either with the stationary or with the linearly growing with time external magnetic field. It interacts with the magnetic field which grows not slower than t^2. Further, if Ψ_2 is presented in the form

$$\Psi_2 = (\Delta)^n\,\Psi_2^{(n)}, \qquad (3.11)$$

then

$$U_d = \left(\frac{1}{c}\frac{\partial}{\partial t}\right)^{2n}\vec{H}_{ext}(0)\int \vec{r}\,\Psi_2^{(n)}\,dV. \qquad (3.12)$$

Such a current distribution interacts with the magnetic field which grows not slower than t^{2n}. Now we turn to the toroidal moments. Taking into account the Maxwell equation and the fact that j_{ext} is not overlapping with S, we rewrite U_t as

$$U_t = \frac{1}{c}\int \frac{\partial \vec{E}_{ext}}{\partial t}\,\vec{r}\,\Psi_3\,dV. \qquad (3.13)$$

Now let Ψ_3 be of the form

$$\Psi_3 = \Delta^n\,\Psi_3^{(n)} \qquad (3.14)$$

Then

$$U_t = \left(-\frac{1}{c}\frac{\partial}{\partial t}\right)^{2n+1}\vec{E}_{ext}(0)\int \vec{r}\,\Psi_3^{(n)}\,dV. \qquad (3.15)$$

This means that this current configuration interacts with the electric field which grows not slower than t^{2n+1}. Obviously, such a selectivity of interaction can be used for the storage and ciphering of information. One precaution is needed, however. When presenting Ψ_2 or Ψ_3 in the form (3.11) (or (3.14)) we have implicitly

assumed that $\Psi_2^{(n)}$ or $\Psi_3^{(n)}$ are confined to the finite space region or that they decrease sufficiently fast for large distances. This is needed for the disappearance of surface integrals arising when the transition from (3.5) to (3.10),(3.12) or (3.15) is performed. In fact, every function Ψ can be represented in the form $\Psi = \Delta f$ where

$$f = -\frac{1}{4\pi}\int \frac{1}{|\vec{r}-\vec{r}\,'|}\,\Psi(\vec{r}\,')\,dV',$$

but there is no guarantee that f decreases sufficiently fast. As a result, Eqs.(3.10),(3.12) and (3.15) are valid for the very specific current configurations. We elucidate now which magnetic field corresponds to the choice of Ψ functions in the form (3.11) and (3.14). The convenient parametrization of VP corresponding to the stationary current density has been found in ref. /8/ (see Eqs.(3.10) and (3.13) therein). Substituting the current parametrization (3.1) into it, we get outside the space region S to which the current density is confined

$$\vec{A} = \frac{4\pi}{c}\sum \frac{1}{2l+1}\,r^{-l-1}(\vec{r}\times\vec{\nabla})Y_l^m \int r^l Y_l^{m*}\Psi_2\,dV + \frac{4\pi}{c}\nabla\sum\frac{l}{2l+1}\,r^{-l-1}Y_l^m\int r^l Y_l^{m*}\Psi_3\,dV. \qquad (3.16)$$

The magnetic field H disappears if

$$\int r^l Y_l^{m*}\Psi_2\,dV = 0. \qquad (3.17)$$

This relation is automatically satisfied if Ψ_2 has the form (3.11). The condition for the disappearance of A is

$$\int r^l Y_l^{m*}\Psi_3\,dV = 0.$$

Obviously, this is satisfied if Ψ_3 has the form (3.14). Thus, the simultaneous fulfilment of Eqs. (3.11) and (3.14) leads to the disappearance of the VP and magnetic field outside the space region S to which the current configuration J is confined. This fact has been interpreted in / 6 / (see Appendix C therein) as a "loss of toroid moments" in the multipole expansion. Our viewpoint differs from that. The representation (3.16) of VP, which is valid only outside S, disappears for the specific current distributions defined by Eqs.(3.11) and (3.14). This does not mean that VP vanishes everywhere. Inside S one should use either general formula

$$\vec{A}(\vec{r}) = \frac{1}{c}\int \frac{1}{|\vec{r}-\vec{r}\,'|}\vec{J}(\vec{r}\,')\,dV'$$

(as was done in § 2) or its development over the vector spherical harmonics. The latter certainly differs from (3.16) inside S. It follows from this that no experiment performed outside S (including Aharonov-Bohm like) can give information on the current distribution inside S. To this end, we have a wonderful electromagnetic object with a number of interesting properties. It does not act on the test charge or magnetic needle. On the other hand, it interacts with a time-dependent external electromagnetic field. The difficult question on the equality of action and counter-action lies beyond the scope of present consideration. The question arises on the practical realizations of this object. One of them (see § 2) is the family of toroidal solenoids built into each other (each turn of such a solenoid is again toroidal solenoid). The ambiguity in the magnetizations choice (see § 3) implies that this realization is not unique.

4. Nonstationary current configurations and time-dependent magnetic Aharonov-Bohm effect

Consider at first the charge-current configuration enclosed into the finite multiconnected space region S and periodically changing with

time. If S does not include the origin, then the conditions for the disappearance of electromagnetic field strengths outside S are / 8 / :

$$\int j_l Y_l^{m*}\Psi_2\,dV = \int n_l Y_l^{m*}\Psi_2\,dV = 0,$$
$$\int j_l Y_l^{m*}\Psi_3\,dV = \int n_l Y_l^{m*}\Psi_3\,dV = 0. \tag{4.1}$$

Here $j_l \equiv j_l(kr)$ and $n_l \equiv n_l(kr)$ are the spherical Bessel and Neumann functions, k = ω/ c. These conditions are obviously satisfied if

$$\Psi_2 = (\Delta + k^2)^n\,\Psi_2^{(n)}, \quad \Psi_3 = (\Delta + k^2)^n\,\Psi_3^{(n)}. \tag{4.2}$$

Outside S there are electromagnetic potentials Φ and $\vec{A}$ periodically changing with time

$$\Phi = -\frac{1}{c}4\pi k^2 \sum h_l Y_l^m \int j_l Y_l^{m*}\Psi_1\,dV\cdot\exp(-i\omega t),$$
$$\vec{A} = \frac{1}{c}4\pi i k\nabla \sum h_l Y_l^m \int j_l Y_l^{m*}\Psi_1\,dV\cdot\exp(-i\omega t). \tag{4.3}$$

Here $h_l \equiv h_l(kr)$ is the spherical Hankel function.
The criterion for the VP observability is the nonvanishing of the integral $\oint A_l\,dl$ taken along the closed path lying outside S and encircling the magnetic flux. Regretfully, we have not found the concrete realization of the function Ψ_1 meeting these requirements.
Instead, we consider now the poloidal current on the torus surface ($(\rho - d)^2 + z^2 = R^2$) which grows linearly with time:

$$\vec{j} = \vec{j}_0\, t, \quad \vec{j}_0 = \vec{n}_\psi \frac{\delta(\tilde{R}-R)}{d+R\cos\psi}\frac{j_0}{R^2}. \tag{4.4}$$

It turns out / 9 / that outside the torus only the electric

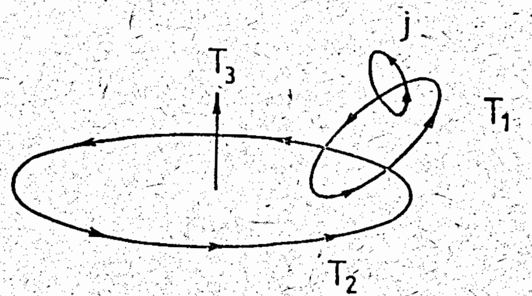

Fig.3 : The family of toroidal solenoids each turn of which is again toroidal solenoid (only the particular turns are shown)

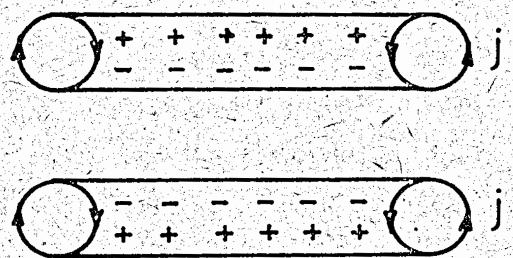

Fig. 4 : The poloidal current J linearly growing with time is equivalent to the doubly charged layer (the upper part of fig.). The lower part of this fig. illustrates that the electric field of the current may be compensated by that of the doubly charged layer

strength E differs from zero. For simplicity we consider the infinitely thin torus. The following representation for the VP is valid / 10,11 / :

$$A_x = \frac{\Phi_0 t}{4\pi}\frac{\partial^2 \alpha}{\partial x \partial z}, \quad A_y = \frac{\Phi_0 t}{4\pi}\frac{\partial^2 \alpha}{\partial y \partial z},$$

$$A_z = -\frac{\Phi_0 t}{4\pi}\left(\frac{\partial^2 \alpha}{\partial x^2} + \frac{\partial^2 \alpha}{\partial y^2}\right). \tag{4.5}$$

Here

$$\Phi_0 = -\frac{4\pi^2}{d}\frac{\dot{d}_0}{c}, \quad \alpha = \int \frac{dx'dy'}{|\vec{r}-\vec{r}'|}.$$

The integration in α is performed over the circle $z=0, \rho \le d$ coinciding with the hole of the infinitely thin torus.It was shown in / 11 / that VP has nowhere singularities except for the line $z=0, \rho=d$ into which torus T degenerates itself. Outside this line the electromagnetic strengths are given by

$$\vec{H}=0, \quad E_x = -\frac{\Phi_0}{4\pi c}\frac{\partial^2 \alpha}{\partial x \partial z}, \quad E_y = -\frac{\Phi_0}{4\pi c}\frac{\partial^2 \alpha}{\partial y \partial z},$$

$$E_z = \frac{\Phi_0}{4\pi c}\left(\frac{\partial^2 \alpha}{\partial x^2} + \frac{\partial^2 \alpha}{\partial y^2}\right). \tag{4.6}$$

On the other hand, the electric field produced by two oppositely charged layers $(\rho \le d,\ z = \pm \varepsilon)$ is given by

$$E_x = \frac{2e\varepsilon}{\pi d^2}\frac{\partial^2 \alpha}{\partial x \partial z}, \quad E_y = \frac{2e\varepsilon}{\pi d^2}\frac{\partial^2 \alpha}{\partial y \partial z},$$

$$E_z = \frac{2e\varepsilon}{\pi d^2}\frac{\partial^2 \alpha}{\partial z^2} = \frac{2e\varepsilon}{\pi d^2}\left(-\frac{\partial^2 \alpha}{\partial x^2} - \frac{\partial^2 \alpha}{\partial y^2}\right) - \frac{8e\varepsilon}{d^2}\delta(z)\,\Theta(d-\rho). \tag{4.7}$$

We see that E_z has a singularity on the circle $z=0, \rho \le d$

From comparison of Eqs. (4.6) and (4.7) it follows that the electric field of the linearly growing poloidal current coincides with that of the double layer everywhere except for the position of the layer itself (the upper part of fig.4). Thus, the electric field of the current (4.4) can be compensated by that of the double layer everywhere except for the layer position itself (the lower part of fig.4). Consider the scattering of charged particles on such a charge-current configuration (to prevent the particle penetration into the solenoid interior, it may be made impenetrable). Outside it the magnetic field H = 0 everywhere, the electric field is also everywhere zero except for the singularity at the torus hole. The VP differs from zero everywhere and linearly grows with time. The integral $\oint A_\ell \, d\ell$ taken along the closed path passing through the torus hole also grows linearly with time. This means that the time-dependent interference picture will be observed at the screen installed behind the solenoid. Obviously, the charged particles will scatter on the double layer within the torus hole, but this scattering being time-independent contributes only to the static background. As far as we know, this is the first example of the time-dependent Aharonov-Bohm effect originating from the time dependence of the magnetic VP. Although the toroidal solenoid with a linearly growing current and the double charged layer produce the same electric field in the space surrounding them, they in fact represent quite different systems. The following example illustrates this. Consider an arbitrary closed curve C at each point of which we install (perpendicular to this curve) an infinitely thin toroidal solenoid (TS) with a current linearly growing with time. The whole set of these solenoids forms a toroidal-like surface S. The magnetic strength is everywhere zero except on the surface S. The electric strength and time-dependent magnetic VP will be different from zero only inside the tube T, surrounded by the surface S. It seems at first that this contradicts

the vanishing of VP outside S (the VP should be everywhere continuous). The reason is the same as the discontinuity of the usual electric scalar potential on the surface of double charged layer: it turns out that the surface S is an example of the double current layer. This construction realizes a pure current capacitor (the static electric field, produced by the time-dependent current, is confined to the interior of the tube T). If the set of the charged layers (instead of TS) is installed on the same curve C, the electric strength will vanish inside the tube T. However, the nontrivial electric induction will be different from zero there / 12 /.

The possibility to simulate the charge distribution by the time-dependent currents has been pointed out earlier in refs./ 13 / where it is referred to as "current electrostatics". The present investigation may be viewed as a concrete realization of these ideas and their application to the time-dependent Aharonov-Bohm effect. Excellent measurements of the electric fields produced by the time-dependent currents have been described in book /14/.

5. Discussion

As we have learned from a previous section it is possible to find current configurations producing a static electric field E inside the tube T. As E is due to the currents, so div E = 0, and it can be represented in the form E = rot A_ℓ. The integral $\int E \, dS$ taken over the tube cross-section differs from zero. Then, the Stokes theorem $\int \vec{E} \, d\vec{S} = \int \vec{A}_\ell \, d\vec{s}$ (the linear integral is taken along the contour embracing the tube T) tells us that $\vec{A}_\ell$ differs from zero outside T. Or, in other words, there is a nontrivial electric VP outside T. This was admitted earlier in ref./12/. The drawback of these considerations is that we have not taken into account the singular fields in the infinitely thin layer on the surface of T (where the currents flow). It may happen that they exactly compensate the flux of E inside T. Then the total electric strength flux will be zero and there will be no need to introduce the electric VP. The electric field is given by $\vec{E}=-\frac{1}{c}\frac{\partial\vec{A}}{\partial t}$, where A is the VP of the closed chain of TS, installed along the closed curve C perpendicular to it. Thus, we should evaluate the integral

$$\vec{A}(\vec{r}) = \int \vec{A}_{TS}(\vec{r},\vec{r}_0(s))ds. \tag{5.1}$$

Here $\vec{A}_{TS}(\vec{r},\vec{r}_0)$ is the VP of the particular infinitely thin TS with its center at the point $\vec{r}_0$. The integraton in (5.1) is performed along the curve C defined as $\vec{r}=\vec{r}_0(s)$. For the treated case the time-dependent VP is given by $\vec{A}_{TS}=\vec{A}_{TS}^{(0)}t$, where $\vec{A}_{TS}^{(0)}$ is the VP of TS with a static current / 9 /. However, we unable to evaluate the integral (5.1) along an arbitrary closed curve. Instead, we integrate along the infinite straight line parallel to TS' symmetry axis. In the special gauge the VP of the TS with its axis parallel to the Z one is given by /10,15/ $\vec{A}_{TS}^{(0)}=$ $=-g\cdot T\cdot\vec{n}_z$, where T is given by Eqs.(2.7) and (2.8) in which one should use $z-z_0$ instead of Z (z_0 is the position of TS' center); $g=\Phi[2\pi(d-\sqrt{d^2-R^2})]^{-1}$ and Φ is the magnetic flux inside TS. Now we integrate this VP along the Z axis:

$$A_z(\rho) = \int (A_{TS}^{(0)})_z\, dz_0.$$

It turns out that $A_z=\Phi$ for $\rho<d-R$,

$$A_z = \Phi - 2g\xi\ln\rho + 2g\int_0^{\xi} dz\,\ln(d-\sqrt{R^2-z^2})$$

for $d-R<\rho<d$ $(\xi=[R^2-(\rho-d)^2]^{1/2})$,

$$A_z = -2g\xi\ln\rho + 2g\int_0^{\xi} dz\,\ln(d+\sqrt{R^2-z^2})$$

for $d<\rho<d+R$ and $A_z=0$ for $\rho>d+R$.

The flux of $\vec{A}$ is obtained by the integration over the cylinder cross-section

$$\int A_z\,\rho\, d\rho\, d\varphi = 2\pi^2 g d R^2.$$

Thus, we have proved that for the treated current configuration (TS continuously distributed over the cylinder C surface) the VP equals zero outside C, but its flux over the cross-section of C differs from zero. As div $\vec{A}$ = 0, we may put $\vec{A}=\mathrm{rot}\,\vec{B}$. Using the Stokes theorem one sees that there is the nontrivial vector function $\vec{B}$ outside C although A = 0 there.
The main problem is that B does not enter into the Schroedinger or Dirac equation. Nevertheless, such a current configuration interacts with an external electromagnetic field (see sections 2 and 3) and, particularly, with that of the incoming charged particle.

6. Conclusion

We mention the main results obtained:

1. We have investigated how the choice of magnetization inside a sample affects its interaction with an external electromagnetic field. Strong selectivity to the time dependence of the external field arises for particular choices of magnetization. This can be used for the storage and ciphering of the information.
2. We have constructed the nonstatic charge -current configuration outside which the magnetic field equals zero everywhere, while the static electric field is confined to the finite region of space. The nontrivial time-dependent magnetic vector potential differs from zero everywhere. This configuration can be used for the performance of time-dependent Aharonov-Bohm-like experiment. The arising time-dependent interference picture may be viewed as a new channel for the information transfer.

3. There are given examples of nontrivial current configurations for which not only the magnetic field H and the vector potential A have meaning but also the vector B the curl of which is just the vector potential.

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Взаимодействие намагниченностей с внешним электромагнитным полем и зависящий от времени эффект Ааронова — Бома

Выясняется, как выбор распределения намагниченности образца влияет на его взаимодействие с внешним электромагнитным полем. При специальном выборе намагниченности возникает сильная избирательность к временной зависимости внешнего поля. Это может быть использовано для хранения и шифровки информации. Предлагается зависящий от времени эффект типа Ааронова — Бома, в котором изменение фазы волновой функции связано с временной зависимостью векторного магнитного потенциала. При этом меняющаяся во времени интерференционная картина может быть использована для передачи информации.

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The Interaction of Magnetizations with an External Electromagnetic Field and a Time-Dependent Magnetic Aharonov-Bohm Effect

We investigate how the choice of the magnetization distribution inside the sample affects its interaction with the external electromagnetic field. The strong selectivity to the time dependence of the external electromagnetic field arises for the particular magnetizations. This can be used for the storage and ciphering of information. We propose a time-dependent Aharonov-Bohm-like experiment in which the phase of the wave function is changed by the time-dependent vector magnetic potential. The arising time-dependent interference picture may be viewed as a new channel for the information transfer.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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