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INCLUSIVE RAPIDITY CORRELATIONS OF π^- MESONS IN pp-INTERACTIONS

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1. INTRODUCTION

It is known that at least an essential part of two-particle inclusive rapidity correlations are pseudocorrelations due to wrong normalization of the probability density functions and the multiplicity dependence of kinematic spectra [1]. Having approximations of semi-inclusive (for fixed multiplicity) one-particle rapidity spectra of investigated particles and their multiplicity distributions assuming the absence of any kind of correlations, one can obtain two- and many-particle rapidity spectra and look what part of the published experimentally investigated inclusive correlations the pseudocorrelations make up. One can also compare experimental multiplicity distributions in rapidity intervals and intervals separated by empty gaps as well as forward–backward, right–left correlations etc. with the calculated ones assuming the absence of semi-inclusive correlations.

Of course, this method coincides (for good approximations) with the commonly used procedure of studying the interference of identical particles (e.g., [2]) when real events are compared with false ones consisting of random combinations of the investigated particles taken from different events (*but with exactly the same multiplicity of these particles*). By the way, no "extra" correlations are usually observed in these precision measurements.

In comparison with all charged particles the investigation of negative ones in *pp* interactions is essentially more pure relative to "dynamic" rapidity correlations. The contribution of trivial correlations and background from: decays of resonances and long-lived particles; Dalitz pairs and γ-conversions; conservation laws of momentum (it can be compensated by neutral and positive particles as well) and charge (there is not only an even number of particles in the event); primary particles (leading particles always have an opposite rapidity sign in the c.m.s.) and their wrong mass identification (the ratio of π^+ mesons to protons is appr. 3:1 even at $E_{lab} = 400$ GeV) is much smaller here.

The spectra of negative particles at their fixed number are presented in some parts of papers [3–10] used here. The spectra of π^- mesons (the admixture of K^- mesons was statistically subtracted) for a fixed number of negative particles are given in the other papers and the spectra of π^- at a fixed number of π^- in the third ones. However, after normalization of the spectra these data do not differ within the errors: the admixture of K^- is small, and their spectra are similar to those of π^- since they are also "truly produced" particles. Therefore we are not going to distinguish between negative particles and π^- here.

2. APPROXIMATION OF SEMI-INCLUSIVE SPECTRA

Figure 1 presents the nomalized to unit one-particle semi-inclusive rapidity distributions of negative particles (π^- mesons) in pp interactions of various energies in events with a fixed multiplicity of negative particles n [4,6,9]

$$\rho'_n(y) \equiv \frac{1}{n\sigma_n}\frac{d\sigma_n}{dy} = \frac{\rho_n(y)}{n}; \qquad \int_{-\infty}^{\infty} \rho'_n(y)dy = 1. \tag{1}$$

The quantity $\rho'_n(y)dy$ is the probability that one π^-, randomly chosen from an event with n π^- mesons (e.g., by means of a random number choosing the π^- number), has rapidity $y \pm dy/2$.

The distributions are surely widened with increasing primary energy $\sqrt{s}$ (the mean energy of particles increases) and narrowed with increasing multiplicity (the energy per one particle decreases). The distributions with different $\sqrt{s}$ and n but equal width could be different in shape, but it is found to be the same — scaling of semi-inclusive spectra [11]. Therefore the two-parametric set of distributions for various $\sqrt{s}$ and n can be in principle described by the one-parametric function which parameter already depends on energy and multiplicity.

The spectra in Fig.1 are approximated by the function:

$$\rho'_n(y) = \frac{0.5}{\sqrt{2\pi Y}}\left(exp\frac{-(y-Y)^2}{2Y} + exp\frac{-(y+Y)^2}{2Y}\right) \tag{2}$$

which is two identical Gaussians with dispersions $\sigma^2 = Y$ shifted by $\pm Y$ from the c.m.s. The values of the parameter Y and χ^2 obtained by fitting these 12 spectra are presented in the figure. For all 48 spectra at 10 energies [3-10] $\Sigma\chi^2/\Sigma n.d.f. = 654/547$. To calculate χ^2, only the statistical errors of experimental points were used. This ratio equals 486/463 without 12 GeV/c points having very small statistical errors (175000 events, Fig.1).

The parameters Y obtained at fitting the spectra are shown in Fig.2 on the left versus $ln(\sqrt{s}/\sqrt{n})$. As seen, the groups of points corresponding to different multiplicities lie on one line for all energies, i.e. the shape of rapidity spectrum depends only on the ratio $\sqrt{s}/\sqrt{n}$ [11]. This experimental result was found to be intermediate between two extreme possible ones:

a) the multiplicity of π^- is proportional to the inelasticity coefficient for π^- $(\Sigma E_{\pi^-}/\sqrt{s})$, then Y would depend only on $\sqrt{s}$, i.e. the spectrum would be independent of multiplicity;

b) the multiplicity does not depend on the inelasticity coefficient, then Y would be dependent only on $\sqrt{s}/n$.

The curve in the figure is

$$Y = l - l^{0.67} + 0.34, \qquad \text{where} \qquad l = ln(\sqrt{s}/\sqrt{n}). \tag{3}$$

Figures 3 and 4 depict the spectra for other energies [3,4,8,9] in logarithmic scale. The curves are obtained by formulae (2) and (3).

On the right-hand side of Fig.2 one can see that the mean energy and mean transverse momentum of π^- mesons [3,6,8,10,12,13] also depend only on the ratio s/n. This allows us to assume that the total double-differential one-particle spectrum of π^- in semi-inclusive pp interactions also depends only on s/n. Such a behaviour of the spectra already gives some evidence for non-correlated, statistical character of π^- production for these energies.

3. APPROXIMATIONS OF MULTIPLICITY DISTRIBUTIONS

The multiplicity distributions of negative particles in pp interactions (P_n) from threshold energy (P_{lab} ~1.2 GeV/c) and at least up to ISR energies (2000 GeV/c) obey KNO scaling [14], more exactly its accurate realization [15,16], which, in contradiction to original asymptotic formulae [14], agrees with the condition $\sum_{n=0}^{\infty} P_n = 1$:

$$P_n = \int_n^{n+1} P(m)dm; \quad P(m) = \frac{1}{<m>}\Psi\left(\frac{m}{<m>}\right); \quad <m> \equiv \int_0^{\infty} mP(m)dm. \tag{4}$$

$\Psi(z)$ is an energy-independent universal function normalized by the conditions arising from (4):

$$\int_0^{\infty}\Psi(z)dz = \int_0^{\infty} z\Psi(z)dz = 1. \tag{5}$$

As a universal function Ψ, we used [15,16]

$$\Psi(z) = a(z+0.14)exp\left(-b(z+0.14)^2\right). \tag{6}$$

where a and b obtained from normalization conditions (5) are equal to 1.251 and 0.618, respectively.

The dependence on the primary energy of the parameter $<m>$, which is a scale (linear) characteristic of the quantity of produced π^-, from the threshold of π^- production and at least to 400 GeV/c is well described by the one-parametric function [16]

$$<m> = 0.81(\sqrt{s} - 2M_p)^{3/4}(\sqrt{s})^{-1/4}, \tag{7}$$

where M_p (proton mass) and $\sqrt{s}$ are expressed in GeV. In the Fermi thermodynamic model the multiplicity of π mesons in pp interactions at 10÷1000 GeV/c should be proportional to this expression [17].

The negative binomial distribution

$$P_n = \frac{(n+k-1)!}{n!(k-1)!}\left(\frac{\bar{n}}{\bar{n}+k}\right)^n\left(\frac{k}{\bar{n}+k}\right)^k \tag{8}$$

with parameters $\bar{n}$ and k presented by the authors was also used to describe the experimental data including only a "non-single-diffractive" (NSD) sub-ensemble of events.

4. INDEPENDENT PARTICLES PRODUCTION

The *probability density* that one π^-, randomly chosen from an event with n π^- mesons, has rapidity y_1 is equal to

$$\rho_n'(y_1) \equiv \frac{1}{n\sigma_n}\frac{d\sigma_n}{dy_1}; \qquad \int_{-\infty}^{\infty}\rho_n'(y_1)dy_1 = 1. \tag{9}$$

The *probability density* that 2 random π^-, successively chosen from an event with n π^- mesons ($n \geq 2$), have respectively y_1 and y_2 (the second π^- is chosen from the $n-1$ remaining ones) equals [1]:

$$\rho_n'(y_1, y_2) \equiv \frac{1}{n(n-1)\sigma_n}\frac{d^2\sigma_n}{dy_1 dy_2}; \qquad \int_{y_1,y_2}\rho_n'(y_1,y_2)dy_1dy_2 = 1. \tag{10}$$

The *probability density* that i random π^- mesons, successively chosen from an event with n π^- ($n \geq i$), have respectively y_1, $y_2 \ldots y_i$ (each following π^- meson is chosen from the lesser number of remaining ones) is written as

$$\rho_n'(y_1, y_2 \ldots y_i) \equiv \frac{(n-i)!}{n!\;\sigma_n}\frac{d^i\sigma_n}{dy_1dy_2\ldots dy_i}; \quad \int_y \rho_n'(y_1, y_2 \ldots y_i)dy_1dy_2\ldots dy_i = 1. \tag{11}$$

If the pairs of π^- mesons are uncorrelated, i.e. if the spectrum $\rho_n'(y_2)$ of other π^- in the sub-ensemble of events, where there is a π^- with y_1, is the same as in the total ensemble, then the compound probability density is equal to the product of elementary ones (e.g., [18]):

$$\rho_n'(y_1, y_2) = \rho_n'(y_1)\rho_n'(y_2). \tag{12}$$

The many-particle probability is factorized in the same way: if all π^- are produced independently, the compound probability is equal to the product of elementary ones:

$$\rho_n'(y_1, y_2 \ldots y_i) = \rho_n'(y_1)\rho_n'(y_2)\ldots\rho_n'(y_i). \tag{13}$$

The correlation function is the distinction of the compound probability from the product of elementary ones. The two-particle correlation function is:

$$C_n' \equiv \rho_n'(y_1, y_2) - \rho_n'(y_1)\rho_n'(y_2); \qquad \int_{y_1,y_2} C_n' dy_1 dy_2 = 0 \tag{14}$$

$$\text{or} \qquad R_n' \equiv \rho_n'(y_1, y_2)/\rho_n'(y_1)\rho_n'(y_2) - 1 \tag{15}$$

(try to use the most conventional designations). If π^- mesons are independent, the correlation function equals zero for all y_1, y_2.

It can be averaged over n (P_n is the probability of an event with n π^-):

$$C_{sh}' \equiv \sum_{n=2}^{\infty} P_n C_n' = \sum_{n=2}^{\infty} P_n\Big(\rho_n'(y_1, y_2) - \rho_n'(y_1)\rho_n'(y_2)\Big). \tag{16}$$

It can be also averaged with a weight, e.g. $n(n-1)/ < n(n-1) >$, i.e. proportionally to statistics (the number of π^- pairs) at a given n. By the way, in this case the lower limit of summation is automatically set to be 2.

Multiplicity n in these formulae concerns just the particles which correlations are being investigated. If an ensemble of events is selected from the total one according to some criteria (the presence of a strange particle, lack of diffraction) or the phase volume for π^- is somehow limited, then all quantities: n, P_n, σ_n, $\rho_n'(y_1)$, $\rho_n'(y_1, y_2)$ concern just this ensemble or volume.

These formulae correspond to the procedure of studying correlations when the two-particle spectrum is compared with that of pairs of particles where each particle is randomly chosen from different events (but with the same multiplicity of these particles).

5. TWO-PARTICLE CORRELATIONS

Other correlation functions are often used.

1. The semi-inclusive correlation function constructed from the one- and two-particle multiplicity densities in events with multiplicity n reads

$$\rho_n(y_1) \equiv \frac{1}{\sigma_n}\frac{d\sigma_n}{dy_1} = n\rho_n'(y_1); \qquad \int_{y_1}\rho_n(y_1)dy_1 = n; \tag{17}$$

$$\rho_n(y_1, y_2) \equiv \frac{1}{\sigma_n}\frac{d^2\sigma_n}{dy_1dy_2} = n(n-1)\rho_n'(y_1, y_2); \quad \int_{y_1,y_2}\rho_n(y_1, y_2)dy_1, dy_2 = n(n-1).$$

$$C_n \equiv \rho_n(y_1, y_2) - \rho_n(y_1)\rho_n(y_2)\;; \qquad \int_{y_1,y_2} C_n dy_1 dy_2 = -n. \tag{18}$$

Under no conditions the function C_n can be equal to 0 at all y_1 and y_2 since its integral is not 0. The first and second terms in (18) are normalized to a different number of π^- pairs. The product of one-particle distributions in this correlation function is the "model" of two-particle distribution with "cut off" correlations, but this model does not take into account that in a real event the second particle is chosen already from $n-1$ particle and not from n as the first one. Of course, when particles of different types are plotted on the y_1 and y_2 axes, such misunderstanding does not arise (e.g., we do not change the multiplicity of negative particles taking a positive particle out of the event).

The function C_n averaged over n is

$$C_{sh} \equiv \sum_{n=1}^{\infty} P_n C_n = \sum_{n=1}^{\infty} n P_n \Big((n-1)\rho_n'(y_1, y_2) - n\rho_n'(y_1)\rho_n'(y_2) \Big);$$

$$\int_y C_{sh} dy_1 dy_2 = - <n> \tag{19}$$

(making summation, from $n = 2$ the integral is equal to $P_1 - <n>$).

2. The inclusive correlation function constructed from the multiplicity densities averaged over all events is:

$$\rho(y_1) \equiv \frac{1}{\sigma_{in}} \frac{d\sigma}{dy_1} = \sum_{n=1}^{\infty} n P_n \rho_n'(y_1); \qquad \int_{-\infty}^{\infty} \rho(y_1) dy_1 = <n>;$$

$$\rho(y_1, y_2) \equiv \frac{1}{\sigma_{in}} \frac{d^2\sigma}{dy_1 dy_2} = \sum_{n=2}^{\infty} n(n-1) P_n \rho_n'(y_1, y_2); \tag{20}$$

$$\int_y \rho(y_1, y_2) dy_1 dy_2 = <n(n-1)>;$$

$$C \equiv \rho(y_1, y_2) - \rho(y_1)\rho(y_2); \qquad \int_{y_1, y_2} C dy_1 dy_2 = <n(n-1)> - <n>^2. \tag{21}$$

If the events with $n = 1$ are not used, the first integral in (20) equals $<n> - P_1$ and the integral in (21) $<n(n-1)> - (<n> - P_1)^2$.

The two-particle distribution and the product of one-particle ones have different normalizations here too. However, at uncorrelated particle production and the n independence of $\rho_n'(y)$ and $\rho_n'(y_1, y_2)$ the function C (in contrast to C_n) might be equal to 0 for the multiplicity distribution with $<n(n-1)> = <n>^2$ (e.g., Poisson). In this case the excess of π^- pairs in the product of one-particle spectra (as compared to two-particle ones with the same multiplicity) is precisely "compensated" by another incorrectness

different averaging of these spectra over the ensemble of events (see also sect. 8). The two-particle spectrum C contains a greater "percent" of events with larger multiplicity. The weight of events with multiplicity n in it is proportional to $n(n-1)/ <n(n-1)>$ and $n^2/ <n>^2$ in the product of one-particle spectra. For example, the first one does not contain events with $n = 1$ at all.

3. The function constructed from the normalized (on the average) multiplicity densities

$$\rho'(y_1) \equiv \frac{1}{<n>\, \sigma_{in}} \frac{d\sigma}{dy_1} = \frac{1}{<n>} \sum_{n=1}^{\infty} n P_n \rho_n'(y_1);$$

$$\rho'(y_1, y_2) \equiv \frac{1}{<n(n-1)>\, \sigma_{in}} \frac{d^2\sigma}{dy_1 dy_2} = \frac{<n(n-1)\rho_n'(y_1, y_2)>}{<n(n-1)>}; \tag{22}$$

$$\int_{y_1} \rho'(y_1) dy_1 = 1; \qquad \int_y \rho'(y_1, y_2) dy_1 dy_2 = 1;$$

$$C' \equiv \rho'(y_1, y_2) - \rho'(y_1)\rho'(y_2); \qquad \int_{y_1, y_2} C' dy_1 dy_2 = 0. \tag{23}$$

could be equal to 0 for all y_1 and y_2 in the case of uncorrelated particle production if the probability density functions did not depend on multiplicity. For the spectra depending on n this function despite its normalization (on the average) contains the same combinatorial pseudocorrelation as C.

Figure 5 presents these functions (more exactly, their sections at y_1=0) for 250 GeV/c [19]. The C and C' data are obtained from pp interactions, the points C_{sh} and C'_{sh} from $\pi^+ p$ and $K^+ p$, but, judging from the data of [19], they should be similar to pp. The curves in the figures are obtained assuming the absence of correlations (12) by approximations (2), (3) for $\rho_n'(y)$ and (4), (6), (7) for P_n. To obtain these correlation functions, the events with 1 π^- were also used in [19]. So, when summing up, beginning with $n = 1$ we get the following for the curves from (12) and (16), (19)-(23):

$$C = \sum_{n=1}^{\infty} n P_n \rho_n'(y_2) \left((n-1)\rho_n'(0) - \sum_{n=1}^{\infty} n P_n \rho_n'(0) \right); \tag{24}$$

$$C' = \frac{1}{<n(n-1)>} \sum_{n=1}^{\infty} n(n-1) P_n \rho_n'(0) \rho_n'(y_2) - $$
$$ - \frac{1}{<n>^2} \sum_{n=1}^{\infty} n P_n \rho_n'(0) \sum_{n=1}^{\infty} n P_n \rho_n'(y_2); \tag{25}$$

$$C_{sh} = - \sum_{n=1}^{\infty} n P_n \rho_n'(0) \rho_n'(y_2); \tag{26}$$

$$C'_{sh} = -P_1\rho'_1(0)\rho'_1(y_2). \quad (27)$$

The first term in C'_{sh} (16) at $n = 1$ in [19] is assumed to be equal to 0. The pseudocorrelation (curve) for C'_{sh} is obtained only due to the second term at $n = 1$ which is compensated by nothing. For positive particles it is out because their multiplicity in pp, K^+p and π^+p interactions is always larger than 1. A significant difference of the curve from the experimental points in Fig.5 is observed only for C'_{sh}. Perhaps, this is a true correlation (e.g., π^- interference) although it is not seen for positive particles either.

The pseudocorrelation in C_{sh} is obtained because of different normalization of the one- and two-particle spectra: the first term in (19) is always smaller than the second one.

The pseudocorrelation in C' is due to different averaging of these spectra over the ensemble of events: in the first term of C' (23) the weight of events with larger multiplicity ($\propto n(n-1)/ <n(n-1)>$) is bigger than in the second one ($\propto n^2/ <n>^2$). The width of the rapidity spectrum decreases with increasing multiplicity, so the first term in C' is a narrower and higher function than the second one, and their difference is seen in the figure.

The function C combines both previous pseudocorrelations.

The correlation function R at y_1=0 (the function C normalized to one-particle densities) for different primary energies is presented in Fig.6 [5,20–27]

$$R \equiv \sigma_{in}\frac{d^2\sigma/dy_1dy_2}{(d\sigma/dy_1)(d\sigma/dy_2)} - 1 = \frac{\sum_{n=1}^{\infty} n(n-1)P_n\rho'_n(y_1,y_2)}{\sum_{n=1}^{\infty} nP_n\rho'_n(y_1)\sum_{n=1}^{\infty} nP_n\rho'_n(y_2)} - 1. \quad (28)$$

Of course, the pseudocorrelations in R are the same as in C. The curves are also obtained assuming the independence of produced π^- (12):

$$R = \frac{\sum_{n=1}^{\infty} n(n-1)P_n\rho'_n(0)\rho'_n(y_2)}{\sum_{n=1}^{\infty} nP_n\rho'_n(0)\sum_{n=1}^{\infty} nP_n\rho'_n(y_2)} - 1. \quad (29)$$

6. MULTIPLICITY DISTRIBUTIONS IN RAPIDITY INTERVALS

The probability that one π^- randomly chosen from an event with N π^- falls within a given interval y is equal to (see (9)):

$$p = \int_{y_{min}}^{y_{max}} \rho'_N(y)dy \quad (30)$$

(p depends on N). If all π^- are independent (13), the probability for each next π^- chosen from the same event to fall within this interval is the same.

Then the probability that exactly n π^- from the event with N π^- fall within this interval is [18]:

$$P_{n|N} = \binom{N}{n} p^n(1-p)^{N-n} = \frac{N!}{n!(N-n)!}p^n(1-p)^{N-n} \quad (31)$$

(binomial distribution). The probability that exactly n π^- from a *random* event fall within this interval (averaging over N) is

$$P_n = \sum_N P_N P_{n|N} = \sum_{N=0}^{\infty} P_N\frac{N!}{n!(N-n)!}p^n(1-p)^{N-n}, \quad (32)$$

where P_N is the probability that the total multiplicity of π^- in an event is N.

Figure 7 presents the comparison of the multiplicity distributions of π^- inside different rapidity intervals from papers [28,29] with the points calculated by this formula (the points are connected by straight lines). The experimental data are taken from π^+p interactions, but, according to the author's statements, they coincide with pp in these rapidity intervals. Only "non-single-diffractive" (NSD) events selected from the total ensemble according to some criteria decreasing the number of events with small multiplicity were used in these papers. The results of fitting the total multiplicity distribution of π^- by negative binomial distribution (8) are only presented there. This distribution was used to obtain P_N in (32) with the parameters $\bar{n} = 3.47$ and $1/k = 0.013$ presented in [29]. It was also assumed that the spectrum of π^- in the events with multiplicity 1 and 2 (after excluding a part of them according to the NSD criteria) continued to be described by approximation (2), (3).

Figure 8 depicts the means and dispersions of the multiplicity distributions of π^- in pp interactions at 200 and 250 GeV/c [29,30] inside the given rapidity intervals, nonsymmetrical and symmetrical, relative to the c.m.s. Only NSD events were also used here, and the results are only given as the fit parameters of the distributions by the negative binomial one. The points in Fig.8 are obtained from these parameters by the formulae which are valid if the data are precisely described by this distribution [29,30]:

$$<n> \equiv \sum_n nP_n = \bar{n}; \quad D = \sqrt{<n^2> - <n>^2} = \sqrt{\bar{n} + \bar{n}^2/k}. \quad (33)$$

The curves in Fig.8 can be obtained directly from (32), but the calculations can be somewhat reduced. The mean and the mean square of the π^- multiplicity distribution within a given rapidity interval for the events with N π^-, i.e. the mean and the mean square of the binomial distribution (31), are equal to [18]:

$$<n>_N \equiv \sum_n nP_{n|N} = Np; \quad <n^2>_N \equiv \sum_n n^2P_{n|N} = Np(1-p+Np). \quad (34)$$

These quantities can be averaged over N in view of their linearity relative to $P_{n|N}$:

$$<n>\equiv \sum_n nP_n = \sum_n n \sum_N P_N P_{n|N} = \sum_N P_N \sum_n nP_{n|N} = \sum_N P_N <n>_N;$$
$$<n^2>\equiv \sum_n n^2 P_n = \sum_n \sum_N n^2 P_N P_{n|N} = \sum_N P_N <n^2>_N \qquad (35)$$

(D is obtained from these equalities according to (33)).

7. MULTIPLICITY DISTRIBUTION IN THE INTERVAL SEPARATED BY EMPTY GAPS

Figure 9 presents the characteristics of the multiplicity distributions for $\pi^+p \to N\pi^-$ 250 GeV/c in the rapidity interval $|y| < y_C$ in the sub-ensemble of events where none of π^- falls within the adjacent rapidity intervals $y_C < |y| < y_E$ [31]. Here the points are also calculated from the fit parameters according to (33). They are also rather well described by the curves, i.e. independent π^- emission; true, the calculation was made using the same approximations for $pp \to N\pi^-$.

The probability that one π^- randomly chosen from an event with N π^- falls within the interval $|y| < y_C$ (or $y_C < |y| < y_E$) is:

$$p_C = 2\int_0^{y_C} \rho'_N(y)dy; \qquad p_E = 2\int_{y_C}^{y_E} \rho'_N(y)dy. \qquad (36)$$

If all π^- are independent, the probability of an event with N π^- none of which falls within the forbidden intervals is equal to $P_N(1-p_E)^N$ where P_N is the total probability of the event with N π^-.

Let us consider only that the sub-ensemble of events where none of π^- falls within the interval $y_C < |y| < y_E$. The total multiplicity distribution of π^- mesons, N, in this sub-ensemble is:

$$P'_N = \frac{P_N(1-p_E)^N}{\sum_N P_N(1-p_E)^N}; \qquad \sum_N P'_N = 1. \qquad (37)$$

The probability to fall within the central interval for each of π^- from an event with N π^- entering into our sub-ensemble equals

$$p = p_C/(1-p_E) \qquad (38)$$

(p depends on N). The mean and the mean square of the number of π^- in the central interval for events with N π^- entering into our sub-ensemble are calculated in the same way as in (34) (binomial distribution). Averaging them over N as in (35), we get

$$<n>= \sum_N P'_N Np; \qquad <n^2>= \sum_N P'_N Np(1-p+Np). \qquad (39)$$

8. FORWARD–BACKWARD AND RIGHT–LEFT CORRELATIONS

Except "true" correlations and the multiplicity dependence of the spectra, the dependence of the mean multiplicity of π^- mesons, $<l(r)>$, in a rapidity interval L (left) on multiplicity r in a non-overlapping with it interval R (right) is determined by two other trivial reasons. Selecting events with large r, we select events with large total multiplicity N, and so we increase $<l(r)>$. On the other hand, selecting large r at fixed N, we decrease $<l(r)>$. So at a very narrow multiplicity distribution this "correlation" is negative and at a very wide one it is positive. In the case of the multiplicity distribution with $D^2 =<N>$ (e.g., Poisson), these contrary tendencies are precisely compensated as well as for the correlation function C which is just the characteristic of multiplicity correlations and not probability correlations.

Figure 10 gives the parameters a and b of a linear approximation

$$<l(r)>= a + br \qquad (40)$$

versus interval limits for pp NSD interactions at 250 GeV/c [32]. The left-hand side of the figure is for equal and symmetrical (relative to the c.m.s) intervals (forward–backward correlation, $n_B \equiv l$; $n_F \equiv r$) limited from above: $0 < |y| < y_{cut}$ and from below: $y_{cut} < |y| < \infty$ as a function of y_{cut}. The right-hand side is for the case when the left interval is $y_0 - 1 < y < y_0$ and the right one is $y_0 < y < y_0 + 1$ versus y_0 (right–left correlation, $n_L \equiv l$; $n_R \equiv r$).

If these correlations are precisely described by a linear fit (40), then the parameters are equal to [32]:

$$b = \frac{<lr> - <l><r>}{<r^2> - <r>^2}; \qquad a =<l> -b <r> \qquad (41)$$

since (P_r is the probability that exactly r π^- falls into R):

$$<l>= \sum_r P_r <l(r)>= \sum_r P_r(a+br) = a + b <r>; \qquad (42)$$

$$<lr>= \sum_r rP_r <l(r)>= \sum_r rP_r(a+br) = a <r> +b <r^2> . \qquad (43)$$

The numerator of the correlation parameter b in (41) is the integral from the correlation function C over y_1 and y_2 in rectangle $L \times R$.

The probability that one π^- randomly chosen from the event with N π^- falls within the interval R (L) is

$$p_R = \int\limits_R \rho'_N(y)dy; \qquad p_L = \int\limits_L \rho'_N(y)dy \tag{44}$$

(these probabilities depend on N). If π^- mesons are not correlated, the probability that exactly r π^- and exactly l π^- from an event with N π^- fall in R and L respectively is equal to (trinomial distribution [18]):

$$P_{r,l|N} = \frac{N!}{r!\,l!\,(N-r-l)!} p_R^r p_L^l (1 - p_R - p_L)^{N-r-l}. \tag{45}$$

The mean values of the first and second moments of this distribution are

$$<r>_N = Np_R; \quad <r^2>_N = Np_R(1 - p_R + Np_R); \quad <rl>_N = p_R p_L N(N-1).$$

They can be averaged over N as in (35) due to their linearity relative to $P_{r,l|N}$:

$$<r> = \sum_N P_N N p_R; \qquad <l> = \sum_N P_N N p_L;$$

$$<r^2> = \sum_N P_N N p_R(1 - p_R + Np_R); \; <rl> = \sum_N P_N p_R p_L N(N-1). \tag{46}$$

The curves corresponding to independent π^- emission in Fig.10 are obtained by formulae (41), (46), i.e. assuming a precise linear dependence (40). It is also assumed that P_N is described by negative binomial distribution (8) with the parameters obtained from a and b for unlimited symmetric intervals $0 < |y| < \infty$ [32]:

$$\bar{n} = 2a/(1-b) = 3.63; \qquad 1/k = b/a = 0.011. \tag{47}$$

The second equality is valid only in the case of lack of correlations.

For forward–backward correlations for unlimited intervals the formulae are essentially simplified, the probabilities $p_R = p_L = 0.5$ do not depend on multiplicity (need for spectra approximations disappears), and from (41), (46) it follows [33]:

$$a = <N>^2/(D^2 + <N>); \qquad b = (D^2 - <N>)/(D^2 + <N>), \tag{48}$$

where $<N>$ and D are the mean and the dispersion of the total multiplicity distribution. It has been shown [33] that these correlations for a variety of

reactions agree with the assumption of independent particle production (see also [34]).

It is clear that for independent π^- emission in pp interactions their forward–backward correlations for total intervals are the same as up–down correlations (perpendicular to the reaction axis) or in any other direction in the c.m.s. For all charged particles it is of course not so: both leading protons can have a transverse momentum up, but they are both practically unable to emit forward in the c.m.s.

9. CONCLUSIONS

The main features of multiple π^- production in pp interactions at E_{lab} at least to 400 GeV are rather well described within the framework of independent π^- emission without any additional assumptions of correlations, clusters, clans, jets and so on. To describe the rapidity two- and many-particle correlations considered here, it is enough to know the multiplicity distributions of negative particles (practically π^- mesons) and their one-particle spectra for each multiplicity. Numerous models, which describe badly correlations (pseudocorrelations) in the cited papers, evidently describe merely badly even multiplicity distributions and one-particle spectra.

Simplicity of the description obtained here is surely connected with purity of the used material: π^- production in pp interactions. Of course, it is impossible to describe the mixture of all charged particles so simply even if due to a great number of trivial correlations enumerated in the introduction.

The researchers of correlations for some reason do not often use the simple and reliable method allowing one to check whether a given correlation exists, i.e. whether two- or many-particle probability differs from the product of one-particle ones. In other words, whether these correlations differ in a real event and in a false one constructed from the investigated particles randomly chosen from different events *but with the same multiplicity of these particles.*

Of course, correlations which are due to processes with very small cross sections, e.g. π^- interference, are not seen within the errors against the background of the general picture considered here. But it is possible that an excess of the positions of the points over the curve for most experiments in Fig.6 at $y_2 = 0$ and for C'_{sh} in Fig.5 arises from this phenomenon. The origin of π^- correlations versus the rapidity difference $y_1 - y_2$ from papers [1] and [35] is possibly due to it as well.

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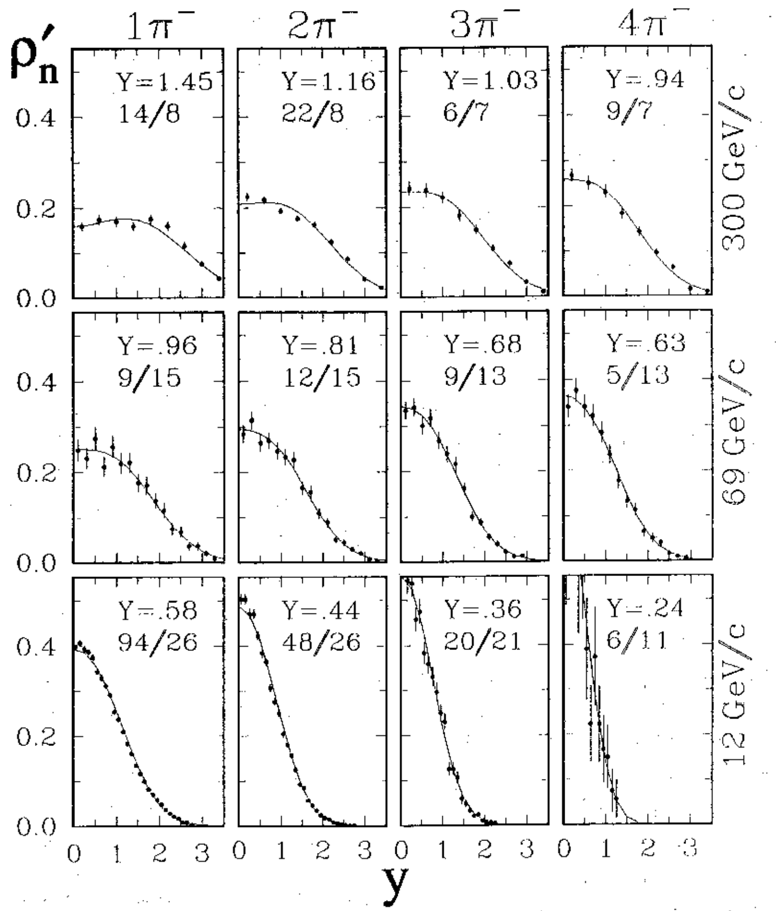

Fig.1. $\rho'_n(y) = (1/n\sigma_n)(d\sigma_n/dy)$. Normalized to 1 one-particle semi-inclusive rapidity spectra of π^- mesons in pp interactions at 12, 69 and 300 GeV/c for π^- multiplicity: 1, 2, 3 and 4. Curves — fit (2).

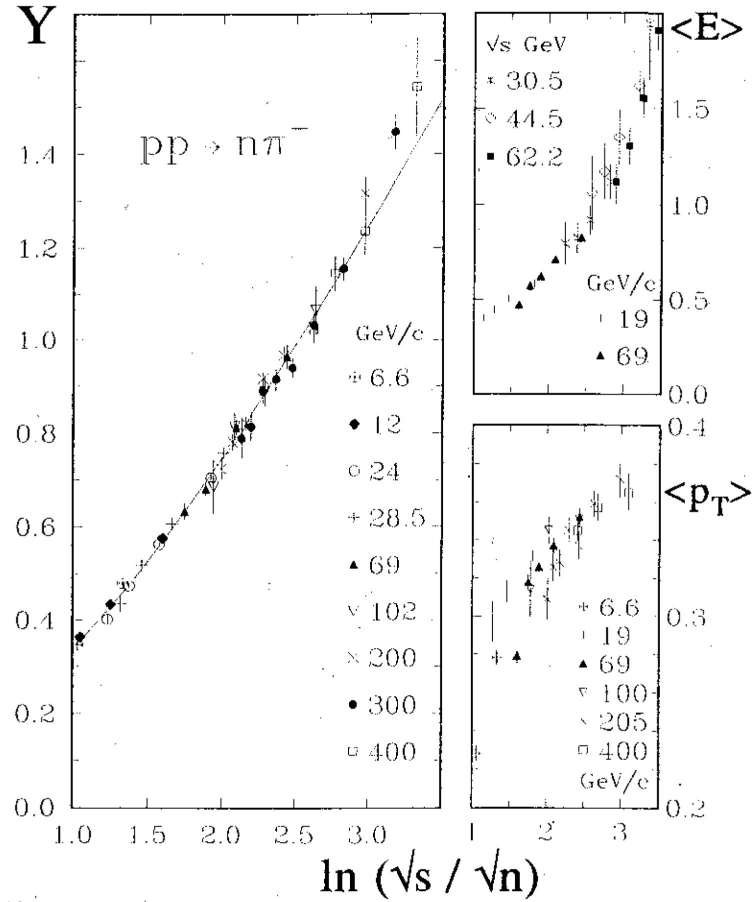

Fig.2. Parameter Y determining the shape of rapidity spectrum (2), mean energy and mean transverse momentum of π^- mesons in semi-inclusive events as a function of $\sqrt{s}/\sqrt{n}$. Curve — fit (3).

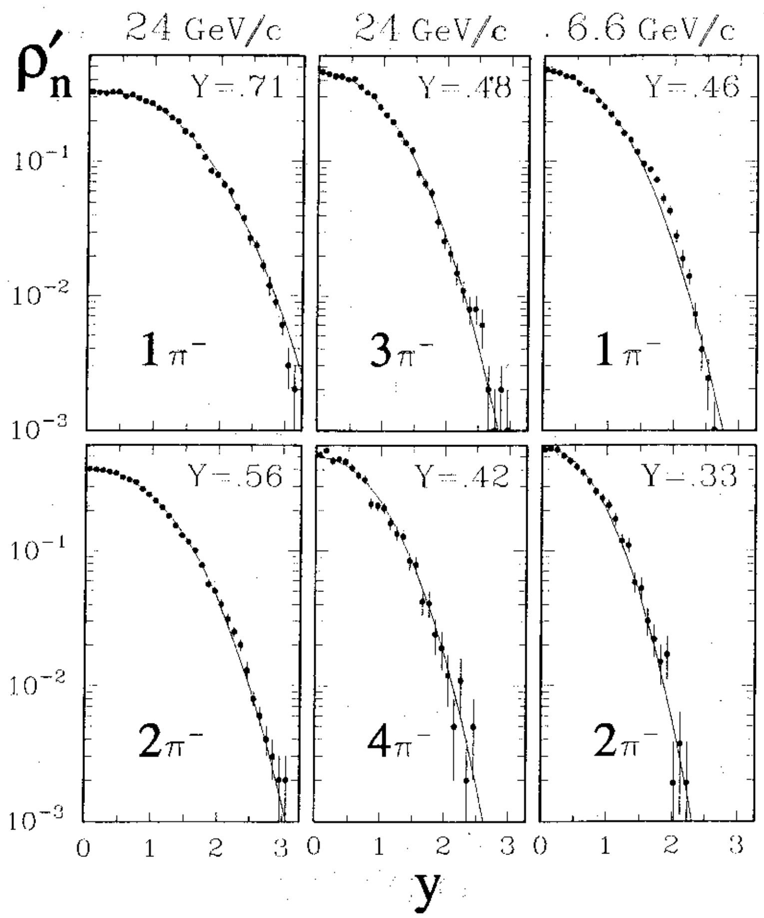

Fig.3. $\rho'_n(y) = (1/n\sigma_n)(d\sigma_n/dy)$. Normalized to 1 one-particle semi-inclusive rapidity spectra of π^- mesons in pp interactions at 6.6 GeV/c and 24 GeV/c for various π^- multiplicity. Curves — (2), (3).

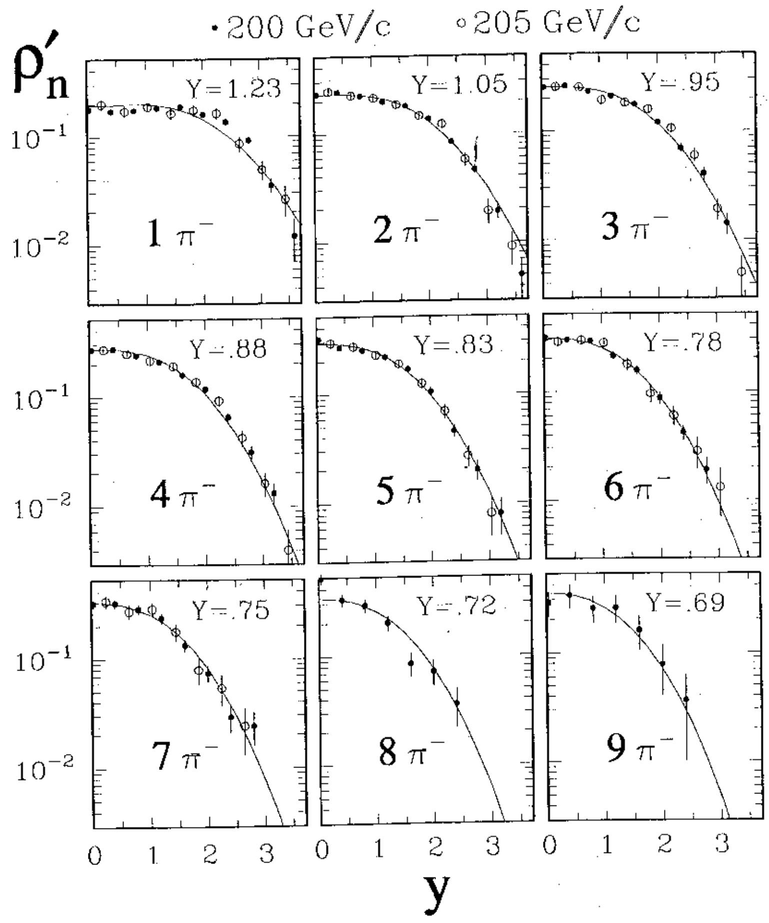

Fig.4. $\rho'_n(y) = (1/n\sigma_n)(d\sigma_n/dy)$. Normalized to 1 one-particle semi-inclusive rapidity spectra of π^- mesons in pp interactions at 200 and 205 GeV/c for π^- multiplicity: 1, 2..., 9. Curves — (2), (3).

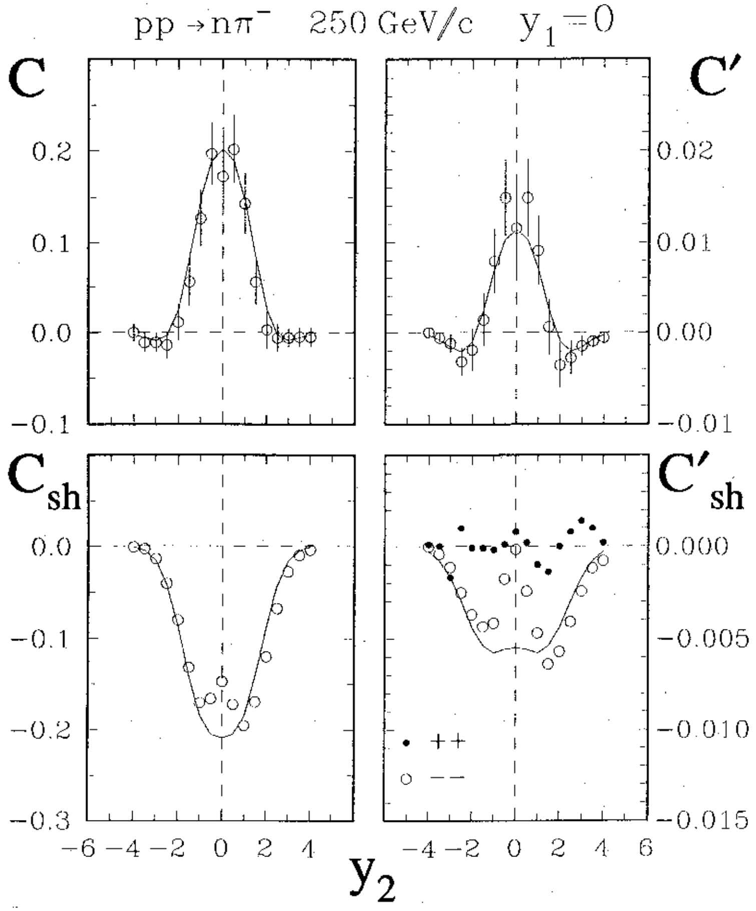

Fig.5. Two-particle rapidity correlations of π^- mesons ($y_1 = 0$) at 250 GeV/c [19] for different definitions of the correlation function: (16), (19), (21), (23). Curves ((24)–(27)) correspond to independent π^- emission (12).

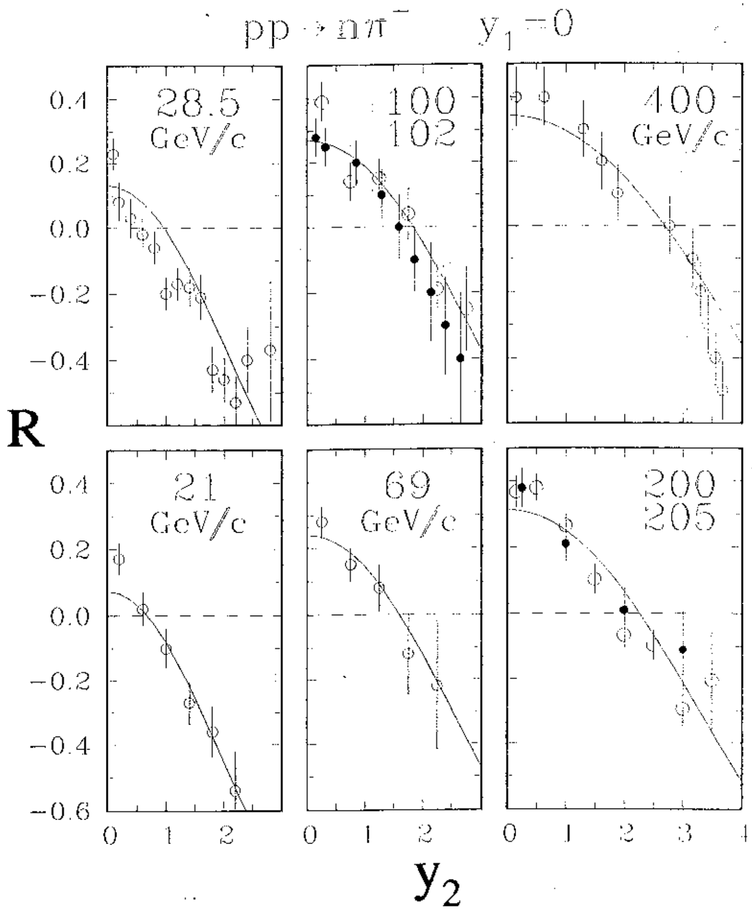

Fig.6. $R = \sigma_{in}\rho(y_1; y_2)/\rho(y_1)\rho(y_2) - 1$ two-particle rapidity correlations of π^- mesons (28) at $y_1 = 0$ in pp interactions for $21 \div 400$ GeV/c [5,20–27]. Curves (29) correspond to independent π^- emission (12).

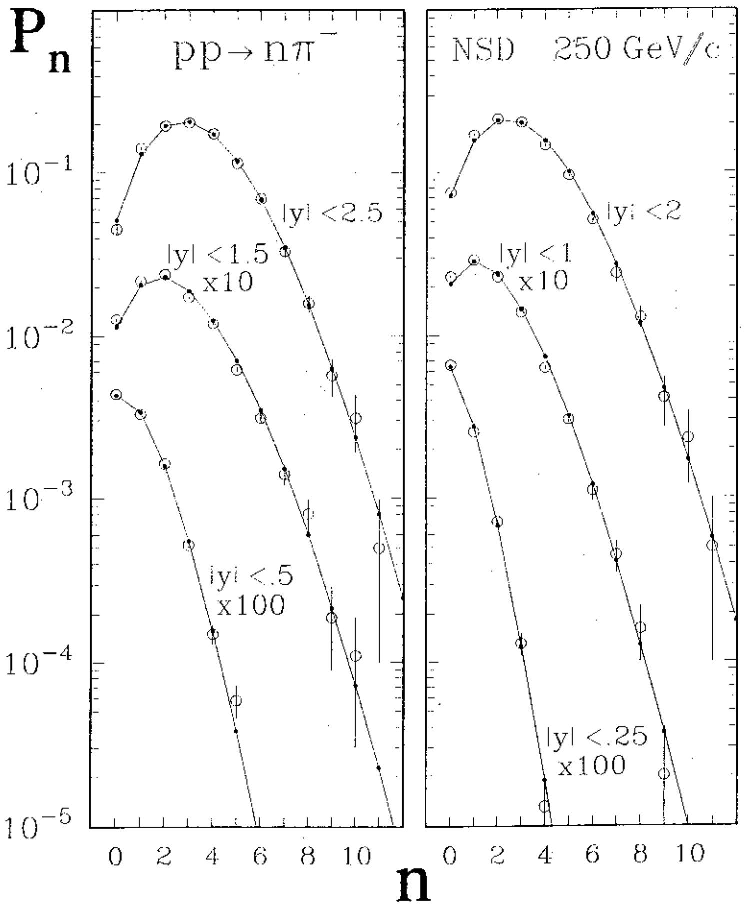

Fig.7. Multiplicity distributions of π^- mesons inside different rapidity intervals [28,29]. The segments of the straight lines connect the points (32) corresponding to independent π^- production (13).

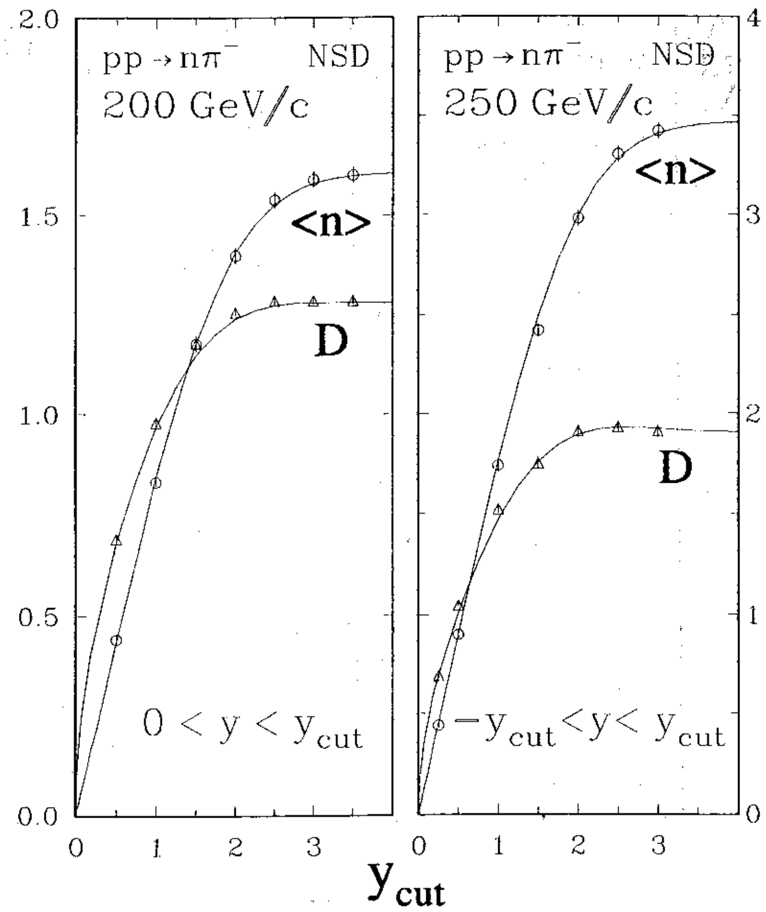

Fig.8. Means and dispersions of the π^- multiplicity distributions inside different rapidity intervals in *pp* interactions at 200 and 250 GeV/c [29,30]. Curves (35) — independent π^- production (13).

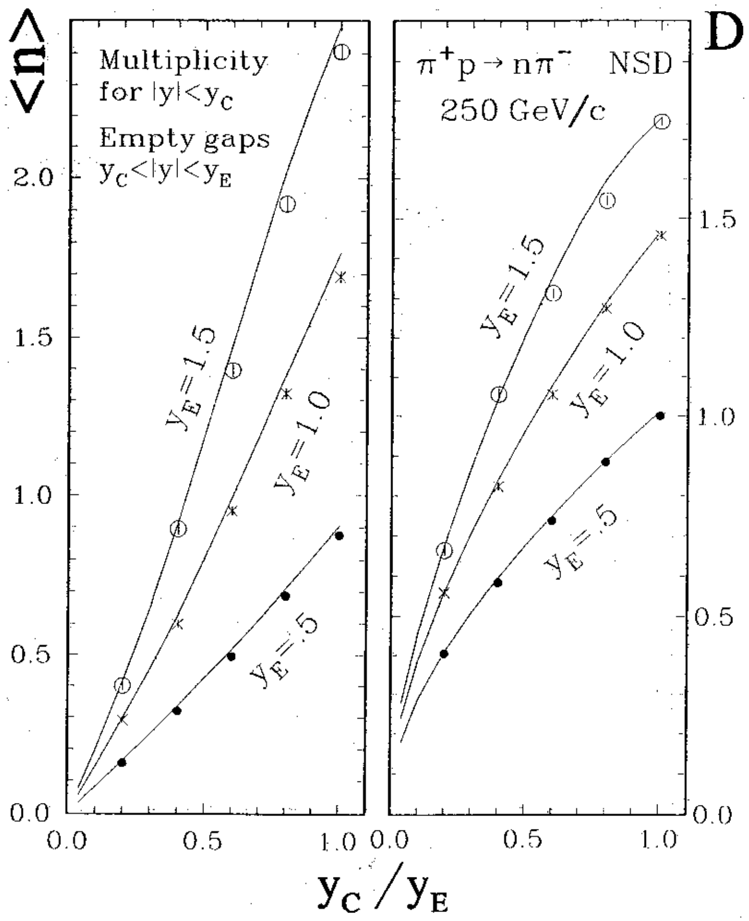

Fig.9. Means and dispersions of the π^- multiplicity distributions in the central interval $|y| < y_C$ in the sub-ensemble of events where none of π^- falls within the adjacent intervals $y_C < |y| < y_E$ versus the ratio y_C/y_E [31]. Curves (39) — independent π^- production (13).

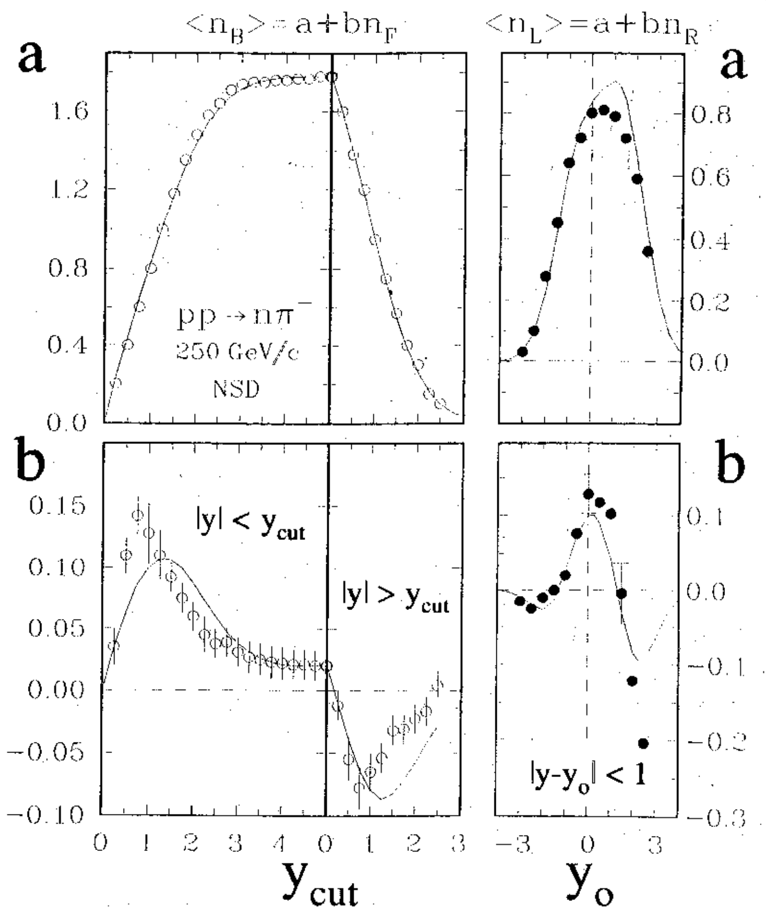

Fig.10. Values of the parameters a and b [32] for forward backward multiplicity correlations relative to the c.m.s.: $<n_B>= a + bn_F$ in the central ($|y| < y_{cut}$) and peripheral ($|y| > y_{cut}$) rapidity region (left). For right left multiplicity correlations relative to y_0: $<n_L>= a + bn_R$ (right). Curves (41), (46) – independent π^- production (13).

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