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FORM FACTOR OF THE PROCESS $\gamma^*\gamma^* \to \pi^0$ FOR SMALL VIRTUALITY OF ONE OF THE PHOTONS AND QCD SUM RULES (II): SUM RULE

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1 Introductory remarks

In this paper we continue our study of the form factor $F_{\gamma^*\gamma^*\to\pi^0}(q_1^2, q_2^2)$ for the process $\gamma^*\gamma^* \to \pi^0$ at small virtualities of one of the photons $|q_1^2| \ll |q_2^2| \geq 1$ GeV^2 within the QCD sum rule approach [1]. In our first paper [2] (hereafter referred to as (I)), we formulated a QCD sum rule approach to the problem and analyzed the structure of the operator product expansion for the relevant three-point correlation function (I.2.1)

$$\mathcal{F}_{\alpha\mu\nu}(q_1, q_2) = i \int e^{-iq_1x - iq_2y} \langle 0|T\left\{J_\mu(x)\, J_\nu(y)\, j_\alpha^5(0)\right\}|0\rangle d^4x\, d^4y\,, \tag{1.1}$$

with a particular emphasis on the infrared (mass) singularities specific for the $q_1^2 \to 0$ limit. It was shown, that all the non-analytic dependence on q_1^2, the virtuality of the softer photon, which arises in the formal $q_1^2 \to 0$ limit of the standard OPE for $\mathcal{F}_{\alpha\mu\nu}(q_1, q_2)$, comes from the integration region where the intervals $x^2, (x-y)^2$ are large. Furthermore, it was established that these singularities are associated with the two lowest-twist composite operators of quark and gluonic fields. For the reader's convenience, we present here once more the figure from (I) illustrating the schematic structure of the modified OPE for the three-point function (1.1). It gives a graphical representation of the theoretical part of the relevant QCD sum rule (see Fig. 1).

The first row in Fig.1 corresponds to the usual operator expansion for the three-point correlation function. It was constructed in (I) using the standard approach [3, 4] valid in the kinematics when all the momentum invariants are large. In the $q_1^2 \to 0$ limit, this OPE is singular. In particular, the condensate terms explode like inverse powers of q_1^2. The perturbative term (the triangle loop) is formally finite in the small-q_1^2 limit, but it contains non-analytic contributions like $q_1^2 \ln(-q_1^2)$ and $q_1^4 \ln(-q_1^2)$ (see (I.4.1)). Just like in the pion form factor case [5], because of these mass singularities, one should perform an additional factorization of short and long distance contributions (some details of the method are described in Sec.2 of (I); see also refs.[6] - [10]).

Schematically, the appropriate factorization procedure adds some extra terms into the original OPE. These terms are shown in the next three rows of Fig.1. The total contribution of terms staying inside the same bracket defines the so-called SD(I)-regime for every graph of the first row. In this regime, all three currents are separated by short distances (cf. [5]), *i.e.*, all the intervals x^2, y^2 and $(x-y)^2$ are small. When q_1^2 goes to zero, they have the same singular behavior as the corresponding terms of the first row. Thus, in the OPE constructed for the essentially non-symmetric, small-q_1^2 kinematics (see Sec.2 of (I)), all the non-analytic terms mentioned above can be removed, and the final expression is regular in the $q_1^2 \to 0$ limit. The remaining terms in Fig.1 represent the SD(II)-regime, corresponding to a situation when the electromagnetic current $J_\mu(x)$ is separated by long distances from the two other currents, *i.e.*, the interval y^2 is small, while x^2 and $(x-y)^2$ are large.

The short-distance contributions are factorized into coefficient functions (CF), which, in our case (we do not consider radiative corrections), are given by a propagator or a product of propagators. In its turn, the long-distance contribution is represented by a two-point correlator (see [7, 8, 5]) of the electromagnetic current $J_\mu(x)$ and a composite operator of quark and gluonic fields denoted by $\otimes$ in Fig.1.

We emphasize that it is the twist rather than dimension of the composite operators which determines the power behaviour of the contribution associated with these two-point correlators ("bilocals"; see Sec. 4.2 of (I)). This implies that, for a fixed twist, one should include the composite operators with an arbitrary number n of derivatives inside them. In principle, multiplying the original correlator by the delta-function $\delta(q_1 y)$, one can eliminate all the terms containing derivatives. This trick was proposed in ref. [9] to avoid summation over n in a similar situation. However, an introduction of such a singular factor looks rather risky and we do not use it. Instead, we developed in (I) a method which enables us to sum up the whole $(q_1 y)^n$-series.

By definition, the long-distance factor (the bilocal object) accumulates nonperturbative information. One cannot calculate it in a straightforward perturbation theory. The idea is to incorporate again the QCD sum rule method. The starting point is a dispersion relation for the two-point correlator (with subtractions, in general). As the next step, one should construct the OPE for this correlator in the region of large space-like q_1^2 and then analyze the resulting ("auxiliary") QCD sum rule to determine the parameters of the relevant model spectral density. In our case, they include the moments $\langle x^n\rangle_\rho$ of the ρ^0-meson wave functions $\varphi_\rho(x)$. To obtain the correlator at small q_1^2-values, one should just use this model spectral density in the original dispersion relation (this procedure was used earlier in refs. [5, 9, 10]).

The paper is organized as follows. In Sec.2, we derive a spectral expansion for the bilocal correlators related to different CFs. We show that the leading terms correspond to the ρ-meson wave functions of twist 2 and twist 3. The next-to-leading term due to the operators having twist 5 is calculated as well. In this section, we also discuss the contribution of higher resonances (the continuum). In Sec.3, we consider a specific type of power corrections, so-called contact terms, in a situation when they appear in the leading-twist bilocals with an arbitrary number n of derivatives. In Sec.4, we analyze the final sum rule corresponding to small-q_1^2 kinematics. We argue that, to a good accuracy, one can use the asymptotic expressions for the ρ-meson wave functions which appear in the bilocals. Finally, we obtain a QCD sum rule estimate for the $\gamma^*\gamma^* \to \pi^0$ form factor and demonstrate that our results are in good agreement with existing experimental data [12].

2 Bilocal correlators

2.1 Bilocals related to single-propagator coefficient function

In the simplest case (see Fig.1d,i), the coefficient function of the SD(II) regime is given by a single quark propagator $S(y) = \hat{y}/2\pi^2(y^2 - i0)^2$, and the corresponding contribution into the three-point amplitude $\mathcal{F}_{\alpha\mu\nu}$ can be written as

$$\mathcal{F}^{SD(II)}_{\alpha\mu\nu} = -\frac{e^2}{3\sqrt{2}} \int d^4y\, e^{-iq_2 y}\, \frac{y^\beta}{2\pi^2 y^4} \sum_{n=0}^{\infty} \frac{1}{n!}\, y^{\mu_1} \ldots y^{\mu_n}$$
$$\times \Bigg[\Bigg\{ -S_{\nu\beta\alpha\sigma} \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overleftarrow{\partial}_{\mu_1} \ldots \overleftarrow{\partial}_{\mu_n})\gamma_\sigma\gamma_5 u(0)\}|0\rangle$$

$$+i\epsilon_{\nu\beta\alpha\sigma} \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overleftarrow{\partial}_{\mu_1} \ldots \overleftarrow{\partial}_{\mu_n})\gamma_\sigma u(0)\}|0\rangle \Bigg\} \tag{2.2}$$
$$+\Bigg\{ S_{\nu\beta\alpha\sigma} \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overrightarrow{\partial}_{\mu_1} \ldots \overrightarrow{\partial}_{\mu_n})\gamma_\sigma\gamma_5 u(0)\}|0\rangle$$
$$+i\epsilon_{\nu\beta\alpha\sigma} \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overrightarrow{\partial}_{\mu_1} \ldots \overrightarrow{\partial}_{\mu_n})\gamma_\sigma u(0)\}|0\rangle \Bigg\}\Bigg],$$

where $S_{\nu\beta\alpha\sigma} \equiv (g_{\nu\beta}\, g_{\alpha\sigma} - g_{\nu\alpha}\, g_{\beta\sigma} + g_{\nu\sigma}\, g_{\alpha\beta})$.

We will consider in detail only the bilocal correlators with the right-sided derivatives

$$R^5_n(q_1, y) = \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(y\overrightarrow{\partial})^n \gamma_\sigma\gamma_5 u(0)\}|0\rangle,$$
$$R_n(q_1, y) = \int d^4x\, e^{-iq_1 x}\, \langle 0|T\{J_\mu(x)\, \bar{u}(0)(y\overrightarrow{\partial})^n \gamma_\sigma u(0)\}|0\rangle. \tag{2.3}$$

The bilocals $L^5_n(q_1, y)$ and $L_n(q_1, y)$ with the left-sided derivatives can be treated in the same way.

As stressed before, these correlators are responsible for the long distance ($\sim 1/|q_1|$) contributions and, for this reason, they are not directly calculable in perturbation theory. However, we may write down a dispersion relation for them, assuming the standard spectral density ansatz: "lowest resonance" + "continuum contribution", with the continuum starting at some effective threshold s_0. Quantum numbers of the electromagnetic current $J_\mu(x)$, which appears in all the bilocals, dictate that the lowest resonance is represented by the ρ^0-meson:

$$R_n(q_1, y) = \frac{1}{\pi} \int_0^\infty ds \frac{\delta R_n(s)}{s - q_1^2} + (\text{ subtractions })$$
$$= \text{"}\rho^o\text{-meson contribution"} + \frac{1}{\pi} \int_{s_0}^\infty ds \frac{\delta R_n^{pt}(s, y)}{s - q_1^2} + (\text{ subtractions }) \tag{2.4}$$

where $\delta R_n(s, y)$ is the discontinuity: $\delta R_n(s, y) \equiv (R_n(s + i0, y) - R_n(s - i0, y))/2i$.

Taking the ρ^o-meson term in the completeness condition for a set of physical hadronic states

$$\delta^{(+)}(p^2 - m_\rho^2)\, \frac{d^4p}{(2\pi)^3} \sum_{\lambda=-1}^{1} |\rho^o_\lambda; \vec{p}\rangle\, \langle\rho^o_\lambda; \vec{p}\,| \subset \hat{\mathbf{I}}, \tag{2.5}$$

where λ is the helicity of the ρ^o, one can extract the ρ^0 contribution.

As a result, we obtain a set of matrix elements which can be parameterized as follows:

$$\langle 0|\bar{\psi}(0)\gamma_\sigma\psi(y)|\rho^o_{\lambda=0}; \vec{p}\rangle = ip_\sigma\, f^V_\rho\, \phi^V_\rho(yp, \mu^2) + \cdots \tag{2.6}$$
$$\langle 0|\bar{\psi}(0)\gamma_\sigma\psi(y)|\rho^o_{|\lambda|=1}; \vec{p}\rangle = \varepsilon^\perp_\sigma\, f^V_\rho m_\rho\, \phi^V_{\rho\perp}(yp, \mu^2) + ia_{V1}\, p_\sigma\, f^V_\rho m_\rho(\varepsilon^\perp y)\, \phi^{V1}_{\rho\perp}(yp, \mu^2) + \cdots \tag{2.7}$$
$$\langle 0|\bar{\psi}(0)\gamma_\sigma\gamma_5\psi(y)|\rho^o_{|\lambda|=1}; \vec{p}\rangle = \epsilon_{\sigma\alpha\beta\rho}\, \varepsilon^\perp_\alpha\, p_\beta\, y_\rho\, f^A_\rho\, \phi^A_\rho(yp, \mu^2) + \cdots\,. \tag{2.8}$$

Here the dots stand for the higher twist ($t \geq 5$) contributions, ε_σ is the polarization vector of the ρ^o-meson, and the helicity components have an evident interpretation in terms of the longitudinal and transverse polarization: $\rho^o_{\lambda=0} \equiv \rho^o_L$, $\rho^o_{|\lambda|=1} \equiv \rho^o_\perp$.

In a standard way, the functions $\phi_\rho(yp,\mu^2)$ can be related to the usual wave functions $\varphi_\rho(x,\mu^2)$ describing the light-cone momentum distribution inside the ρ:

$$\phi_\rho^{V,V1,A}(yp,\mu^2) = \int_0^1 dx e^{-i(yp)x} \varphi_\rho^{V,V1,A}(x,\mu^2), \tag{2.9}$$

with μ^2 being the renormalization parameter for the relevant composite operators. The constant f_ρ^V fixing the normalization of the simplest wave function is known from previous QCD sum rule studies $f_\rho^V \simeq 200\,\text{MeV}$ [1, 15], and the constants f_ρ^A and a_{V1} in eqs. (2.7), (2.8) can be fixed by equations of motion (cf. [13, 14]), which form an infinite set of relations connecting the moments of different wave functions (see Appendix A).

For our purposes, it is more convenient to use matrix elements for an arbitrary polarization of the ρ^o-meson:

$$\begin{aligned}\langle 0|\bar\psi(0)\gamma_\sigma\psi(y)|\rho^o_\lambda;\vec{p}\rangle &= \varepsilon_\sigma^{(\lambda)} f_\rho^V m_\rho \left[\phi_{\rho\perp}^V(yp,\mu^2) + C_{V5}\, y^2\phi_{\rho\perp}^{V5}(yp,\mu^2) + \cdots\right] + \\ &+ i a_{V1}\, p_\sigma\, f_\rho^V m_\rho(\varepsilon^{(\lambda)}y)\left[\phi_{\rho\perp}^{V1}(yp,\mu^2) + C_{V15}\, y^2\phi_{\rho\perp}^{V15}(yp,\mu^2) + \cdots\right] + \\ &+ f_\rho^{V25}\left(y_\sigma(\varepsilon^{(\lambda)}y) - \varepsilon_\sigma^{(\lambda)}y^2/4\right)\phi_{\rho\perp}^{V25}(yp,\mu^2) + \cdots \end{aligned} \tag{2.10}$$

$$\langle 0|\bar\psi(0)\gamma_\sigma\gamma_5\psi(y)|\rho^o_\lambda;\vec{p}\rangle = \epsilon_{\sigma\alpha\beta\rho}\,\varepsilon_\alpha^{(\lambda)}\, p_\beta\, y_\rho\, f_\rho^A\left[\phi_\rho^A(yp,\mu^2) + C_{A5}\, y^2\phi_\rho^{A5}(yp,\mu^2) + \cdots\right] \tag{2.11}$$

Since the C-parity of the ρ^o-meson is negative, its wave functions have the following properties (here and below $\bar x \equiv 1-x$):

$$\begin{aligned}&\varphi_{\rho\perp}^{V,V5,V25,A,A5}(x) = \varphi_{\rho\perp}^{V,V5,V25,A,A5}(\bar x) \quad , \quad \varphi_{\rho\perp}^{V1,V15}(x) = -\varphi_{\rho\perp}^{V1,V15}(\bar x), \\ &\int_0^1 dx\, \varphi_{\rho\perp}^{V,V5,V25,A,A5}(x) = 1 \quad , \quad \int_0^1 dx\, x\varphi_{\rho\perp}^{V1,V15}(x) = 1.\end{aligned} \tag{2.12}$$

In the relations above, we have explicitly displayed wave functions up to twist 5, which we will need in the following. Note, that for a longitudinally polarized ρ^o-meson

$$\varepsilon_\sigma^{\lambda=0} \simeq i\, p_\sigma/m_\rho + \mathcal{O}(m_\rho/p_z)$$

as $p_z \to \infty$, and the leading-twist part in eq.(2.10) coincides with the well known definition (2.6).

Let us consider first the leading-twist ρ^o-meson contribution in eq. (2.4). Applying (2.5), (2.10) and (2.11), we obtain:

$$\left(\sum_{n=0}^\infty \frac{1}{n!}R_n(q_1,y)\right)_{\substack{\rho^o\text{-meson}\\ \text{contribution}}} = \frac{(-i)(f_\rho^V m_\rho)^2}{m_\rho^2 - q_1^2}\times$$
$$\times\left\{\varepsilon_\mu\varepsilon_\sigma^* \int_0^1 dx\, e^{-i(yq_1)\bar x}\,\varphi_{\rho\perp}^V(x) - i a_{V1}\, q_{1\sigma}\varepsilon_\mu\,(\varepsilon^* y)\int_0^1 dx\, e^{-i(yq_1)\bar x}\,\varphi_{\rho\perp}^{V1}(x)\right\} \tag{2.13}$$

$$\left(\sum_{n=0}^\infty \frac{1}{n!}R_n^5(q_1,y)\right)_{\substack{\rho^o\text{-meson}\\ \text{contribution}}} = \frac{(-i)(f_\rho^A f_\rho^V m_\rho)}{m_\rho^2 - q_1^2}\times$$
$$\times\left\{\varepsilon_\mu\varepsilon_\delta^*\,\epsilon_{\sigma\delta\beta\rho}\, q_{1\beta}y_\rho \int_0^1 dx\, e^{-i(yq_1)\bar x}\,\varphi_{\rho\perp}^A(x)\right\}. \tag{2.14}$$

Here we have used the shorthand notation:

$$\varepsilon_\mu\varepsilon_\sigma^* \equiv \sum_{\lambda=0,\pm1}\varepsilon_\mu^\lambda\varepsilon_\sigma^{\lambda *} = -g_{\mu\sigma} + \frac{q_{1\mu}q_{1\sigma}}{m_\rho^2}. \tag{2.15}$$

Substituting eqs. (2.13), (2.14) into eq. (2.2) and extracting the proper tensor structure we get:

$$F^{\text{bilocal},CF(I)}_{1,\rho-\text{meson}} = \frac{e^2}{3\sqrt2}\frac{2f_\rho^V m_\rho}{m_\rho^2 - q_1^2}\int_0^1 dx\frac{1}{\bar p^4}\left[-a_{V1}\, f_\rho^V m_\rho\varphi_{\rho\perp}^{V1}(x) - f_\rho^A\,\varphi_{\rho\perp}^A(x)(1+2\bar x)\right], \tag{2.16}$$

where

$$\bar p^2 \equiv (q_2 + \bar x q_1)^2 = -q_1^2\, x\bar x + q_2^2\, x + p^2\,\bar x$$

is the virtuality of the hard quark written in the "parton" form (see (I)). The factor 2 in front of the formula emerges after one adds the contribution of the correlators L_n, L_n^5 and uses the symmetry properties of the wave functions (2.12). It should be noted that the wave functions $\varphi_{\rho\perp}^V$, $\varphi_{\rho\perp}^{V5}$ and $\varphi_{\rho\perp}^{V25}$ do not contribute to the form factor we are considering.

Deriving, in a similar way, the twist-5 contribution and applying the Borel transformation to the resulting amplitude, we obtain:

$$\Phi^{\text{bilocal},CF(I)}_{1,\rho-\text{meson}} = \frac{\sqrt2\alpha_{e.m.}4\pi}{3}\frac{f_\rho^V m_\rho}{m_\rho^2 + q^2}\int_0^1 dx\frac{1}{\bar x^2 M^4}\, e^{-Q^2x/M^2\bar x}\, e^{q^2x/M^2} \tag{2.17}$$
$$\times\left[-a_{V1}\, f_\rho^V m_\rho\left(\varphi_{\rho\perp}^{V1}(x) - \frac{4C_{V51}}{\bar x M^2}\varphi_{\rho\perp}^{V51}(x)\right) - f_\rho^A(1+2\bar x)\left(\varphi_{\rho\perp}^A(x) - \frac{4C_{A5}}{\bar x M^2}\varphi_{\rho\perp}^{A5}(x)\right)\right];$$

where $q^2 \equiv -q_1^2$ and $Q^2 \equiv -q_2^2$. As expected, the twist-5 contribution is suppressed by one power of $1/M^2$.

2.2 Continuum contribution

To begin with, we would like to remind that, in our basic OPE for the small-q_1^2 kinematics (see Fig.1 and Sec. 2 of (I)), one always deals with the difference between an "exact" bilocal correlator R (Fig.1d) and its perturbative analog R^{pert} (Fig.1i). An important observation is that the subtraction terms in the dispersion relation for R^{pert} coincide with those in the dispersion relation for R, because the ultraviolet behaviour of these two correlators is the same. Hence, there is no need to specify an explicit subtraction prescription for the correlators.

Now, incorporating our model for the bilocal correlators, in which the contribution due to higher excited states is approximated by the perturbative spectral density (see (2.4)), *i.e.*, by continuum starting at s_o, we can easily write down an expression for the difference between the continuum contribution to R and the perturbative bilocal R^{pert}. Then, substituting the result into the original expansion (2.2) and performing some straightforward calculations, we obtain:

$$\Phi^{\text{bilocal},CF(I)}_{1,\text{continuum}} - \Phi^{SD(II)}_{1,pt} = \frac{2\sqrt2\,\alpha_{e.m.}}{\pi}\int_0^1 dx\frac{1}{2M^2}\, e^{-Q^2x/M^2\bar x}\, e^{q^2x/M^2}$$
$$\times\left[\frac{2x}{M^2}\left(q^2\ln\frac{s_o+q^2}{q^2} - s_o\right) + \frac{x^2}{M^4}\left(q^4\ln\frac{s_o+q^2}{q^2} - q^2 s_o + \frac{s_o^2}{2}\right)\right] \tag{2.18}$$

The terms collected in the (...) brackets correspond to contributions due to operators with a definite twist, twist-3 and twist-5 in our case. Note, that these terms exactly cancel the logarithmic non-analyticities $q^2 \ln q^2, q^4 \ln q^2$ present in the term shown in Fig.1b, *i.e.* in the coefficient function of the unit operator for the usual OPE valid in symmetric kinematics (see (I.4.1)). As a result, the non-analytic terms are replaced by the combinations $q^2 \ln (s_o + q^2)$ and $q^4 \ln (s_o + q^2)$, which are "safe" in the $q^2 \to 0$ limit. In the opposite limit of large q^2, each (...) term vanishes like $1/q^2$. This observation should be confronted with the requirement that for large q^2 the usual OPE without additional terms must work, *i.e.*, the difference between "exact" correlator and its perturbative version must vanish faster than any power of $1/q^2$. To get the additional terms within our model for the spectral density, one should add the ρ-contribution to the difference displayed in eq. (2.18). The ρ-term also vanishes like $1/q^2$ for large q^2, so if the ρ-contribution is made perfectly dual to the perturbative spectral density, then the additional terms in this model would vanish like $1/q^4$. However, if such a duality is only approximate, there will remain a small $1/q^2$ term. In any case, using a rough model for the correlator, one should just be happy that the additional terms decrease as q^2 increases without relying too heavily on their extrapolation beyond the region $q^2 \leq m_\rho^2$.

2.3 Twist-3 bilocals for two-propagator coefficient functions

Next in complexity is the contribution related to the coefficient function formed by a product of two propagators (see Fig.1e) and (I.4.26)):

$$\mathcal{F}^{SD(II)}_{\alpha\mu\nu} = \frac{e^2}{3\sqrt{2}} \int d^4y\, e^{-iq_2 y}\, d^4z\, \frac{(y-z)^\delta}{2\pi^2 (y-z)^4} \frac{z^\xi}{2\pi^2 z^4} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} y^{\mu_1} \ldots y^{\mu_n} z^{\nu_1} \ldots z^{\nu_m} \tag{2.19}$$
$$\times \int d^4x\, e^{-iq_1 x} \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overleftarrow{\partial}_{\mu_1} \cdots \overleftarrow{\partial}_{\mu_n})\gamma_\nu\gamma_\delta\gamma_\gamma g_s(\overrightarrow{\partial}_{\nu_1} \cdots \overrightarrow{\partial}_{\nu_n} A^b_\gamma(0)) t^b \gamma_\xi\gamma_5\gamma_\alpha u(0)\}|0\rangle .$$

Here we explicitly extracted the bilocal correlator containing a composite operator composed of two quark and one gluonic field. Note, that the gluonic field $A^b_\gamma(z)$ here may be treated as taken in the Fock-Schwinger gauge, *i.e.*, it can be substituted by

$$A^b_\gamma(z) = z_\varphi \int_0^1 \alpha\, G^b_{\varphi\gamma}(\alpha z) d\alpha . \tag{2.20}$$

As a result, the ρ^o-meson contribution is determined by the following matrix elements:

$$\langle 0|\bar{u}(z_1)\gamma_\beta\gamma_5 g_s G^b_{\varphi\gamma}(z_3) t^b u(z_2)|\rho^o_\lambda; \vec{p}\rangle = p_\beta\, \epsilon_{\varphi\gamma\theta\kappa}\, p_\theta\, \varepsilon^{(\lambda)}_\kappa\, f^A_{3\rho}\, \phi^A_{3\rho}(z_i p, \mu^2) + \text{higher twist contributions} \tag{2.21}$$

$$\langle 0|\bar{u}(z_1)\gamma_\beta\, i g_s G^b_{\varphi\gamma}(z_3) t^b u(z_2)|\rho^o_\lambda; \vec{p}\rangle = p_\beta \left(p_\varphi \varepsilon^{(\lambda)}_\gamma - p_\gamma \varepsilon^{(\lambda)}_\varphi\right) f^V_{3\rho}\, \phi^V_{3\rho}(z_i p, \mu^2) + \text{higher twist contributions} . \tag{2.22}$$

In a standard way, we can introduce the momentum distribution amplitudes[1] $\varphi^{V,A}_{3\rho}(x_i)$:

$$\phi^{V,A}_{3\rho}(z_i p, \mu^2) = \int_0^1 [dx]_3\, \varphi^{V,A}_{3\rho}(x_i)\, e^{-i\sum x_i (z_i p)} . \tag{2.23}$$

[1] $[dx]_3 \equiv dx_1 dx_2 dx_3\, \delta(1 - \sum_i x_i)$

They have the following symmetry properties:

$$\varphi^A_{3\rho}(x_1, x_2; x_3) = \varphi^A_{3\rho}(x_2, x_1; x_3)\ ,\ \varphi^V_{3\rho}(x_1, x_2; x_3) = -\varphi^V_{3\rho}(x_2, x_1; x_3). \tag{2.24}$$

In our definition, the normalization constants $f^A_{3\rho} = 0.6 \cdot 10^{-2}\,\text{GeV}^2, f^V_{3\rho} = 0.25 \cdot 10^{-2}\,\text{GeV}^2$ [15] are factored out, so that the distribution amplitudes are normalized to unity:

$$\int_0^1 [dx]_3\, \varphi^A_{3\rho}(x_i) = 1\ ,\ \int_0^1 [dx]_3\, (x_1 - x_2)\, \varphi^V_{3\rho}(x_i) = 1\ . \tag{2.25}$$

Following the procedure described in Sec.2.1, we find the ρ^o-meson contribution:

$$\Phi^{\text{bilocal},CF(II)}_{1,\rho\text{-meson}} = \frac{\sqrt{2}\alpha_{e.m.} 8\pi}{3} \frac{f^V_\rho m_\rho}{m^2_\rho + q^2} \int_0^1 d\alpha\, \alpha \int_0^1 d\beta \int_0^1 [dx]_3\, e^{\,b/aM^2} \tag{2.26}$$
$$\times \left\{ f^A_{3\rho}\, \varphi^A_{3\rho}(x_1, x_2; x_3) \left[\frac{c_1}{a^2 M^4} - \frac{d_1}{2a^3 M^6}\right] - f^V_{3\rho}\, \varphi^V_{3\rho}(x_1, x_2; x_3) \left[\frac{c_2}{a^2 M^4} - \frac{d_2}{2a^3 M^6}\right]\right\},$$

where

$$\begin{aligned} a &= \alpha\beta x_3 + x_2, \\ b &= -q^2 \left(\alpha^2\beta x_3^2 + 2\alpha\beta x_2 x_3 - \alpha\beta x_3 + x_2^2 - x_2\right) + Q^2 \left(\alpha\beta x_3 + x_2 - 1\right) \end{aligned} \tag{2.27}$$

and

$$\begin{aligned} c_1 &= (\alpha\beta x_3)/(\alpha\beta x_3 + x_2), \\ d_1 &= c_1 \left(-q^2 \left(\alpha^2\beta x_3^2 + 2\alpha x_2 x_3 + \alpha\beta x_3 + x_2^2 + x_2\right) + Q^2\right), \\ c_2 &= (\alpha\beta x_3 + 2\beta x_2)/(\alpha\beta x_3 + x_2), \\ d_2 &= c_1 \left(q^2 \left(\alpha^2\beta x_3^2 + 2\alpha\beta x_2 x_3 + \alpha\beta x_3 + 2\beta x_2^2 - x_2^2 + x_2\right) + Q^2 (1 - 2\beta)\right). \end{aligned} \tag{2.28}$$

The perturbative spectral density for all of these correlators is suppressed by $O(\alpha_s/\pi)$-factor, and for this reason we neglect here the contribution due to higher states.

2.4 Twist-2 bilocals for three-propagator coefficient functions

The bilocals associated with the coefficient functions given by a product of three propagators can appear in the $\langle\bar{\psi}\psi\rangle^2$ quark condensate diagrams of the unmodified OPE (Figs.7a-r of (I)). Furthermore, it was pointed out there that at large and moderate q_1^2 only some of them contribute to the invariant amplitude F_1 we are interested in. The relevant diagrams are shown in Fig.2a-d. In fact, among these diagrams, only 2b and 2c produce bilocals with the three-propagator coefficient function (see also Fig.1f). After some algebra, we obtain

$$\mathcal{F}^{\text{bilocal},CF(III)}_{\alpha\mu\nu} = \frac{e^2}{3\sqrt{2}} \frac{16\pi\alpha_s\langle\bar{u}u\rangle}{9} \int dy e^{-iq_2 y} \left(\frac{p_\alpha}{p^2}\right) \frac{y_\beta}{8\pi^2 y^2} \sum_{n=0}^{\infty} \frac{1}{n!} y^{\mu_1} \ldots y^{\mu_n}$$
$$\times \int dx e^{-iq_1 x} \langle 0|T\{J_\mu(x)\, \bar{u}(0)(\overleftarrow{\partial}_{\mu_1} \cdots \overleftarrow{\partial}_{\mu_n})\gamma_\nu\gamma_\beta\gamma_5 u(0)\}|0\rangle. \tag{2.29}$$

Adding the charge conjugate contribution produces an extra factor of 2. The ρ-contribution into this correlator is determined by the matrix element

$$\langle 0|\bar{\psi}(0)\sigma_{\nu\beta}\psi(y)|\rho^o_\lambda;\vec{P}\rangle = i\,(\varepsilon^{(\lambda)}_\nu p_\beta - \varepsilon^{(\lambda)}_\beta p_\nu)\, f^T_\rho\, \phi^T_\rho(yp,\mu^2) + \text{higher twists} \quad (2.30)$$

where $\sigma_{\nu\beta} = \frac{i}{2}[\gamma_\nu, \gamma_\beta]$, $p = q_1 + q_2$.

The perturbative spectral density for this correlator is zero, so the simplest approximation is to model the correlator by the ρ^o-meson contribution only. For a more accurate estimate, one should use the two-resonance representation (see [5, 9]). This will not be attempted here.

Proceeding as described above, we get for the borelized structure function Φ_1:

$$\Phi^{\text{bilocal},CF(III)}_{1,\rho\text{-meson}} = -\frac{\sqrt{2}\alpha_{e.m.}64\pi^2}{27}\frac{\alpha_s\langle\bar{u}u\rangle}{M^6}\frac{m_\rho f^V_\rho f^T_\rho}{m^2_\rho + q^2}\int\!\!\int_0^1 dx\,d\beta\, e^{\beta(q^2x\bar{x}-Q^2x)/((1-x\beta)M^2)} \times \frac{\beta\,\varphi^T_\rho(x)}{(1-x\beta)^3} \quad (2.31)$$

3 Bilocals and contact terms

A special care must be taken about the correlators containing the Dirac operator $\gamma_\mu D^\mu$ acting on the quark field ψ. Since the correlator is a T-product of the electromagnetic current and a composite operator, applying the equation of motion one gets the $\delta^{(4)}(x)$-function, *i.e.*, the external vertices of the bilocal are contracted into a single point and it reduces to a q^2-independent constant.

Let us sketch a simple derivation for such terms (see, *e.g.*, [11]). Using the functional representation for the correlator

$$\langle 0|T\{\ldots\bar{\psi}(x)\hat{\nabla}\psi(0)\}|0\rangle = \int \mathcal{D}[\bar{\psi}]\,\mathcal{D}[\psi]\,\mathcal{D}[A]\,\{\ldots\bar{\psi}(x)\hat{\nabla}\psi(0)\}\,\exp\left(i\int\mathcal{L}(z)\,d^4z\right), \quad (3.1)$$

where $\mathcal{L}(z) = \bar{\psi}(z)\,i\hat{\nabla}\psi(z) + \cdots$, we can write

$$\hat{\nabla}\psi(0)\,\exp\left(i\int\mathcal{L}(z)\,d^4z\right) = -\frac{\delta}{\delta\bar{\psi}(0)}\exp\left(i\int\mathcal{L}(z)\,d^4z\right). \quad (3.2)$$

Integrating by parts in (3.1) results in

$$\int \mathcal{D}[\bar{\psi}]\,\mathcal{D}[\psi]\,\mathcal{D}[A]\left\{\cdots\frac{\delta\bar{\psi}(x)}{\delta\bar{\psi}(0)}\right\}\exp\left(i\int\mathcal{L}(z)\,d^4z\right). \quad (3.3)$$

It is the derivative $\delta\bar{\psi}(x)/\delta\bar{\psi}(0)$ that produces the $\delta^{(4)}(x)$ term mentioned above.

The contact terms play an important role in all applications of the QCD sum rules to low-momentum behaviour of hadronic form factors. In particular, without them, it is impossible to satisfy the Ward identities fixing the pion form factor normalization at zero momentum transfer [5, 10].

3.1 Separating short- and long-distance contributions

In our case, a nontrivial contact-term contribution can be obtained for the hard gluon exchange diagrams shown in Fig.2e and Fig.2f. In the SD(II)-regime, they produce bilocals associated with the three-propagator coefficient function. Their contributions are

$$2e \sim \frac{32i}{p^2q_1^2q_2^4}\,q_{1\mu}\,\epsilon_{\alpha\nu q_1q_2}, \qquad 2f \sim \frac{32i}{p^4q_1^2q_2^2}\,q_{1\mu}\,\epsilon_{\alpha\nu q_1q_2}, \quad (3.4)$$

i.e., they do not contribute to the invariant form factor F_1. However, as we will see below, in the SD(II)-regime (see Fig.1g,h), equations of motion "extract" the appropriate tensor structure, *i.e.*, these diagrams must be taken into account.

The relevant term from the three-point correlation function can be written as[2]

$$\mathcal{F}^{SD(II),2ef}_{\alpha\mu\nu}(q_1,q_2) = -\frac{16\pi e^2}{9\sqrt{2}}\frac{\alpha_s\langle\bar{u}u\rangle}{p^2q_2^2}\,\epsilon_{\alpha\nu q_1q_2}\int dz\,e^{ipz}\,\frac{1}{4\pi^2 z^2}\sum_{n=0}^{\infty}\frac{1}{n!}\,z^{\mu_1}\ldots z^{\mu_n} \times \int dx e^{-iq_1x}\,\langle 0|T\{J_\mu(x)\,\bar{u}(0)(\overleftarrow{\partial}_{\mu_1}\ldots\overleftarrow{\partial}_{\mu_n}).u(0)\}|0\rangle. \quad (3.5)$$

Extracting the bilocal term from (3.5), one should pick out the traceless combination of indices $\mu_1, \ldots \mu_n$, *i.e.*, the lowest twist term which gives the leading power contribution with respect to $1/p^2, 1/q_2^2$. Introducing the notation

$$\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1) = \int dx e^{-iq_1x}\,\langle 0|T\{J_\mu(x)\,\bar{u}(0)\{\overleftarrow{\partial}_{\mu_1}\ldots\overleftarrow{\partial}_{\mu_n}\}u(0)\}|0\rangle, \quad (3.6)$$

we can represent the correlator (3.6) in the following form:

$$\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1) = A^{(n)}(q_1^2)\,q_{1\mu}\,\{q_{1\mu_1}\cdots q_{1\mu_n}\} + B^{(n)}(q_1^2)\,\{q_{1\mu}, q_{1\mu_1}\cdots q_{1\mu_n}\} + {} + C^{(n)}(q_1^2)\,g_\mu\{_{\mu_1}q_{1\mu_2}\cdots q_{1\mu_n}\}, \quad (3.7)$$

where $\{\ldots\}$ denotes the traceless-symmetric part of a tensor. Because of the electromagnetic current conservation, we have the constraint $q_{1\mu}\,\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1) = 0$ which produces a relation between the invariant functions $A^{(n)}, B^{(n)}, C^{(n)}$. Using the formula from [19]

$$q_1^\alpha\,\{q_{1\mu_1}\cdots q_{1\mu_{n-1}}, q_\alpha\} = q_1^2\,\frac{n+1}{2n}\,\{q_{1\mu_1}\cdots q_{1\mu_{n-1}}\} \quad (3.8)$$

we obtain

$$\left(A^{(n)} + \frac{(n+2)}{2(n+1)}B^{(n)}\right)q_1^2 + C^{(n)} = 0. \quad (3.9)$$

Contracting (3.6) with $g_{\mu\mu_1}$ gives

$$\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1)\,g_{\mu\mu_1} = \left(A^{(n)}\,q_1^2\,\frac{(n+1)}{2n} + C^{(n)}\left(\frac{n+1}{n}\right)^2\right)\{q_{1\mu_2}\cdots q_{1\mu_n}\}. \quad (3.10)$$

[2]The charge conjugate contribution can be trivially added.

Furthermore, applying the technique symbolized by eqs. (3.1) - (3.3) (see Appendix C) we obtain, in the leading-twist approximation:

$$\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1)\, g_{\mu\mu_1} \simeq (-i)^{n-1}\,\{q_{1\mu_2}\ldots q_{1\mu_n}\}\times$$
$$\times\left[-2\langle\bar{u}u\rangle - \frac{1}{2}\, q_{1\epsilon}\int dx e^{-iq_1x}\,\langle 0|T\{J_\mu(x)\,\bar{u}(0)\{\overleftarrow{\partial}_{\mu_1}\ldots\overleftarrow{\partial}_{\mu_n}\}\sigma_{\mu\epsilon}u(0)\}|0\rangle\right], \quad (3.11)$$

where the first contribution inside the brackets is just the contact term (Fig.1,h), while the second one cannot be reduced to any other contact terms (Fig.1,g). Since the relevant perturbative spectral density vanishes, the bilocal can be approximated by the lowest resonance, *i.e.*, by the ρ-meson[3] (see Sec. 2.4 and discussion below). As a result, we get

$$\Pi_{\mu\{\mu_1\ldots\mu_n\}}(q_1)\, g_{\mu\mu_1} = (-i)^{n-1}\,\{q_{1\mu_2}\ldots q_{1\mu_n}\}\times$$
$$\times\left[-2\langle\bar{u}u\rangle + \frac{(f_\rho^V m_\rho)f_\rho^T}{m_\rho^2 - q_1^2}\,\frac{3}{2}\, q_1^2\int dx\, x^{n-1}\,\varphi_\rho^T(x)\right], \quad (3.12)$$

where $\varphi_\rho^T(x)$ is the twist-2 tensor wave function of the ρ-meson.

Using the formulas from Appendix B, one can easily perform the necessary contractions:

$$z_{\mu_1}\ldots z_{\mu_n}\,\Pi_{\mu\{\mu_1\ldots\mu_n\}} = A^{(n)}\, q_{1\mu}\,\tau^n\, C_n^{(1)}(\eta) +$$
$$+\frac{B^{(n)}}{n+1}\left[-z_\mu\frac{q_1^2}{2}\,\tau^{n-1}\, C_{n-1}^{(2)}(\eta) + q_{1\mu}\,\tau^n\, C_n^{(2)}(\eta)\right]$$
$$+\frac{C^{(n)}}{n}\left[z_\mu\,\tau^{n-1}\, C_{n-1}^{(2)}(\eta) - q_{1\mu}\,\frac{z^2}{2}\,\tau^{n-2}\, C_{n-2}^{(2)}(\eta)\right], \quad (3.13)$$

where $C_n^{(\lambda)}(\eta)$ are the Gegenbauer polynomials and the notation $\eta = i\,(q_1 z)/\sqrt{-z^2q_1^2}$, $\tau = -i\sqrt{-z^2q_1^2}/2$ is introduced.

The tensor structure ($\sim p_\alpha\,\epsilon_{\mu\nu q_1 q_2}$) we are interested in, can be produced in (3.13) only by the terms $\sim z_\mu$, hence, other terms can be ignored. Combining now eqs. (3.9) – (3.12), we get, modulo the next-twist contributions:

$$z_\mu\,\tau^{n-1}\, C_{n-1}^{(2)}(\eta)\left[-\frac{q_1^2\, B^{(n)}}{2(n+1)} + \frac{C^{(n)}}{n}\right] \simeq \quad (3.14)$$
$$\simeq\; z_\mu(-i\, q_1 z)^{n-1}\,\frac{2n^2}{(n+1)(n+2)}\left[-2\langle\bar{u}u\rangle + \frac{(f_\rho^V m_\rho)f_\rho^T}{m_\rho^2 - q_1^2}\,\frac{3}{2}\, q_1^2\int dx\, x^{n-1}\,\varphi_\rho^T(x)\right].$$

Note, that in the standard expansion, taken at the same accuracy,

$$\tau^{n-1}\, C_{n-1}^{(2)}(\eta) = (2\eta\tau)^{n-1}\, n - \tau^2\,(2\eta\tau)^{n-3}\,(n-1)(n-2) + \cdots \quad (3.15)$$

we are left with the first term only.

Substituting (3.14) into (3.5), integrating over dz and summing over n by using the generating function technique we get for the borelized contribution to Φ_1 :

$$\Phi_{1,\text{contact type}}^{\text{bilocal},2ef} = -\frac{\sqrt{2}\alpha_{e.m.}256\pi^2}{27}\,\frac{\alpha_s\langle\bar{u}u\rangle^2}{Q^2M^6}\int\!\!\int_0^1 d\beta dy\,\frac{\beta(\bar{\beta}-\beta)y}{(1-y\beta)^3}\, e^{y\beta(q^2\bar{\beta}-Q^2)/((1-y\beta)M^2)} \quad (3.16)$$

[3]Note, that in the chiral limit $m_u = m_d = 0$, the ρ- and ω-contributions cannot be distinguished and are equal to each other (cf. [1]).

$$\Phi_{1,\rho\text{-meson}}^{\text{bilocal},2ef} = -\frac{\sqrt{2}\alpha_{e.m.}256\pi^2}{27}\,\frac{\alpha_s\langle\bar{u}u\rangle}{Q^2M^6}\,\frac{(f_\rho^V m_\rho)f_\rho^T}{m_\rho^2+q^2}\,\frac{3}{4}\, q^2\times \quad (3.17)$$
$$\times\int\!\!\int\!\!\int_0^1 dxd\beta dy\,\frac{x\beta(\overline{x\beta}-x\beta)y}{(1-yx\beta)^3}\,\varphi_\rho^T(x)\; e^{yx\beta(q^2\overline{x\beta}-Q^2)/(1-yx\beta)M^2}$$

where $\overline{x\beta}\equiv 1-x\beta$, $\bar{\beta}\equiv 1-\beta$.

Analyzing the rest of the bilocal contributions capable of producing a coefficient function of the three-propagator type (see diagrams of Fig.7b,c,i,l,o,p of (I)), we found that the contact type terms are either absent or do not contribute to the desired tensor structure $\sim p_\alpha\,\epsilon_{\mu\nu q_1 q_2}\, F_1$. Contact terms are also absent for the bilocals corresponding to the coefficient functions of one- and two-propagator type.

4 QCD sum rule in the $|q_1^2| \ll |q_2^2| \geq 1$ GeV^2 kinematics

Collecting now all the contributions, we obtain the theoretical part (the modified OPE) of the QCD sum rule for the form factor $F_{\gamma^*\gamma^*\to\pi^\circ}(q_1^2, q_2^2)$ (see Fig.1):

$$\Phi_1(q^2,Q^2,M^2) = \Phi_{1,pt}(q^2,Q^2,M^2) + \Phi_{1,5b}^{\langle GG\rangle} + \Phi_{1,5c}^{\langle GG\rangle} + \Phi_{1,5f,g}^{\langle GG\rangle} +$$
$$+\Phi_{1,6g,h,i}^{\langle\bar{q}q\rangle} + \Phi_{1,6d,e,f}^{\langle\bar{q}q\rangle} + \Phi_{1,7a,b,c}^{\langle\bar{q}q\rangle} + \Phi_{1,7d.}^{\langle\bar{q}q\rangle} +$$
$$+\Phi_{1,\rho\text{-meson}}^{\text{bilocal},CF(I)} + \left(\Phi_{1,continuum}^{\text{bilocal},CF(I)} - \Phi_{1,pt}^{\text{bilocal},SD(II)}\right) - \Phi_{1,5b}^{SD(II)} - \Phi_{1,5c}^{SD(II)} - \Phi_{1,6d,f}^{SD(II)} +$$
$$+\Phi_{1,\rho\text{-meson}}^{\text{bilocal},CF(II)} - \Phi_{1,5f,g}^{SD(II)} - \Phi_{1,6e}^{SD(II)} - \Phi_{1,7a}^{SD(II)} +$$
$$+\Phi_{1,\rho\text{-meson}}^{\text{bilocal},CF(III)} - \Phi_{1,7b,c}^{SD(II)} + \left[\Phi_{1,\text{contact type}}^{\text{bilocal},2ef} + \Phi_{1,\rho\text{-meson}}^{\text{bilocal},2ef}\right], \quad (4.1)$$

where the first two rows correspond to the original OPE valid for symmetric kinematics (see the notations in (I) and eqs. (I.3.8), (I.3.11), (I.3.12)). Each of the next rows represents the additional terms corresponding to different types of the coefficient functions. As explained in (I), all the terms of the standard OPE, which are non-analytic in the $q^2 \to 0$ limit, are cancelled by the corresponding SD(II)-contributions. As a result, the coefficient functions of the SD(I)-regime are analytic functions of q^2 (compare with [6]). Substituting explicit expressions for all the terms which appear in eq.(4.1) gives

$$\Phi_1(q^2,Q^2,M^2) = \frac{\sqrt{2}\,\alpha_{e.m.}}{\pi}\,\frac{1}{M^2}\left\{\int_0^1 dx\; e^{-Q^2x/M^2\bar{x}}\left\{(1+\frac{q^2x}{M^2}\, e^{q^2x/M^2}) +\right.\right.$$
$$+e^{q^2x/M^2}\left[\frac{2x}{M^2}\left(q^2\ln\frac{(s_o+q^2)x}{M^2} - s_o\right) + \frac{x^2}{M^4}\left(q^4\ln\frac{(s_o+q^2)x}{M^2} - q^2s_o + \frac{s_o^2}{2}\right)\right] -$$
$$\left.-\sum_{n=1}^{\infty}\left(\frac{q^2x}{M^2}\right)^n\frac{\psi(n)(n+1)}{(n-1)!}\right\} +$$

$$+\frac{\pi^2}{9}\langle\frac{\alpha_s}{\pi}GG\rangle\left[\frac{1}{2M^2Q^2}+\frac{1}{M^4}\int_0^1 dx\,\frac{x}{\bar{x}^2}\,e^{-Q^2x/M^2\bar{x}}\sum_{n=1}^{\infty}\frac{1}{n!}\left(\frac{q^2x}{M^2}\right)^{n-1}\right]+$$

$$+\frac{64\pi^3}{243}\alpha_s\langle\bar{q}q\rangle^2\frac{q^2}{Q^4M^2}+\frac{64\pi^3}{27}\alpha_s\langle\bar{q}q\rangle^2\frac{1}{2Q^2M^4}+$$

$$+\frac{4\pi^2}{3}\frac{f_\rho^V m_\rho}{m_\rho^2+q^2}\int_0^1 dx\frac{1}{\bar{x}^2M^2}e^{-Q^2x/M^2\bar{x}}e^{q^2x/M^2}$$

$$\times\left[-a_{V1}f_\rho^V m_\rho\left(\varphi_{\rho\perp}^{V1}(x)-\frac{4C_{V51}}{\bar{x}M^2}\varphi_{\rho\perp}^{V51}(x)\right)-f_\rho^A(1+2\bar{x})\left(\varphi_{\rho\perp}^{A}(x)-\frac{4C_{A5}}{\bar{x}M^2}\varphi_{\rho\perp}^{A5}(x)\right)\right]$$

$$+\frac{8\pi^2}{3}\frac{f_\rho^V m_\rho}{m_\rho^2+q^2}\int_0^1 d\alpha\,\alpha\int_0^1 d\beta\int_0^1[dx]_3\,e^{\,b/aM^2}$$

$$\times\left\{f_{3\rho}^A\varphi_{3\rho}^A(x_1,x_2;x_3)\left[\frac{c_1}{a^2M^2}-\frac{d_1}{2a^3M^4}\right]-f_{3\rho}^V\varphi_{3\rho}^V(x_1,x_2;x_3)\left[\frac{c_2}{a^2M^2}-\frac{d_2}{2a^3M^4}\right]\right\}$$

$$-\frac{64\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle}{M^4}\frac{m_\rho f_\rho^V f_\rho^T}{m_\rho^2+q^2}\iint_0^1 dxd\beta\,\frac{\beta\varphi_\rho^T(x)}{(1-x\beta)^3}\,e^{\beta(q^2x\bar{x}-Q^2x)/((1-x\beta)M^2)}$$

$$-\frac{256\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle^2}{Q^2M^4}\iint_0^1 d\beta dy\,\frac{\beta(\bar{\beta}-\beta)y}{(1-y\beta)^3}\,e^{y\beta(q^2\bar{\beta}-Q^2)/((1-y\beta)M^2)}$$

$$-\frac{256\pi^3}{27}\frac{\alpha_s\langle\bar{u}u\rangle}{Q^2M^4}\frac{m_\rho f_\rho^V f_\rho^T}{m_\rho^2+q^2}\frac{3}{4}q^2\times$$

$$\times\iiint_0^1 dxd\beta dy\,\frac{x\beta(\overline{x\beta}-x\beta)y}{(1-xy\beta)^3}\varphi_\rho^T(x)\,e^{yx\beta(q^2\overline{x\beta}-Q^2)/((1-yx\beta)M^2)}\Bigg\}. \tag{4.2}$$

We use the following numerical values for the constants present in eq.((4.2)): $f_\rho^V = 0.2$ GeV, $m_\rho = 0.77$ GeV ; the constants $f_\rho^A = -f_\rho^V m_\rho/4$, $a_{V1} = 1/40$ are obtained from the equations of motion (see Appendix A), the values $f_{3\rho}^A = 0.6 \cdot 10^{-2}\,\mathrm{GeV}^2, f_{3\rho}^V = 0.25 \cdot 10^{-2}\,\mathrm{GeV}^2$ are taken from the QCD sum rule estimates given in ref. [15]. The quark and gluon condensate values are standard.

Finally, we write down the sum rule in the non-symmetric kinematics $|q_1^2| \ll |q_2^2| \geq 1\,\mathrm{GeV}^2$:

$$F_{\gamma^*\gamma^*\to\pi^0}(q^2,Q^2) = \frac{\sqrt{2}\alpha_{e.m.}}{\pi f_\pi}\left\{-2\int_{\sigma_o}^{\infty}d\sigma e^{-\sigma/M^2}\int_0^1 dx\frac{x\bar{x}(q^2x+Q^2\bar{x})^2}{[\sigma x\bar{x}+(q^2x+Q^2\bar{x})]^3}\right.$$

$$\left.+\ \Phi_1(q^2,Q^2,M^2)\frac{M^2\pi}{\sqrt{2}\,\alpha_{e.m.}}\right\}. \tag{4.3}$$

We recall that the value of $F^{C.A.}_{\gamma^*\gamma^*\to\pi^0}(0,0)$ is fixed by current algebra and axial anomaly: $F^{C.A.}_{\gamma^*\gamma^*\to\pi^0}(0,0) = \sqrt{2}\alpha_{e.m.}/\pi f_\pi$ [20].

For the continuum threshold in the ρ-channel we take the standard value $s_o \simeq 1.5\,\mathrm{GeV}^2$ obtained from the QCD sum rule for the ρ-decay constant f_ρ^V [1]. This value was also extracted from the QCD sum rule analysis of the first few moments of the ρ-meson wave functions [15]. However, when the moment number increases, the size of power corrections relative to the perturbative one also increases, and the thresholds obtained from a formal fitting of the relevant sum rules raise like $s_o^{(n)} \sim n$. As argued in [16, 17], an n-independent threshold is more natural from the physical point of view. One should realize that, for higher moments, the power series in $1/M^2$, in fact, "explodes" and one should sum up it in some way, *e.g.*, by using nonlocal vacuum averages ("nonlocal condensates") like $\langle\bar{q}(z)q(0)\rangle$, etc. [16]. The sum rule, derived within this approach for the pion wave function, works for rather high moments, and the corresponding thresholds $s_o^{(n)}$ are essentially independent of the moment's number. Moreover, the model wave function derived in this way is rather close to the asymptotic form.

Inspired by these results, we will take asymptotic forms for the following ρ-meson wave functions (see Appendix A):

$$\begin{aligned}
\varphi_{V1} &= \varphi_{V1}^{as} = 60x\bar{x}\,(2x-1),\\
\varphi_A &= \varphi_A^{as} = 6x\bar{x},\\
\varphi_{3A} &= \varphi_{3A}^{as} = 360x_1x_2x_3^2,\\
\varphi_{3V} &= \varphi_{3V}^{as} = 7!(x_1-x_2)x_1x_2x_3^2.
\end{aligned} \tag{4.4}$$

Numerically most important contributions in the theoretical part of the sum rule (4.2) come from: *a*) SD(I)-regime (first five rows of (4.2)) and *b*) ρ^o-meson contribution with leading twist wave functions (SD(II)-regime) in diagonal and nondiagonal correlators.

The tensor wave function, however, appears in a non-diagonal correlator. For this reason, instead of $\varphi_T^{as} = 6x\bar{x}$, we use

$$\varphi_T^{nondiag.\ corr.\ local\ contrib.} = \frac{1}{2}(\delta(x)+\delta(\bar{x}))$$

and take $f_\rho^T f_\rho^V m_\rho = -2\langle\bar{u}u\rangle$, which corresponds to the lowest-dimensional contribution to the nondiagonal correlator in (2.29). The one-resonance model with the ρ-term residue equal to $-2\langle\bar{u}u\rangle$ guarantees that at $q^2 \gg m_\rho^2$ the corresponding contribution in eq.(4.2) has the correct asymptotic behaviour, *i.e.*, it goes smoothly into that of diagrams Fig.2b,c. The accuracy of this estimate can be improved by *a*) using a more adequate model for the spectral density with two or more resonances [5, 9] and *b*) by incorporating the nonlocal condensates, *e.g.*, in a way outlined in ref. [18].

The terms associated with three-particle twist-3 wave functions are small: their contribution into the sum rule is of the order of a few percent. We expect that terms corresponding to the next-to-leading two-particle wave functions (twist-5) are suppressed as well. Contact-type power corrections are also small.

In Fig.3 we plot the $F_{\gamma^*\gamma^*\to\pi^0}(0,Q^2)$ form factor normalized by the value $F_{\gamma^*\gamma^*\to\pi^0}(0,0)$. We calculate it in the region $Q^2 \geq 1\mathrm{GeV}^2$ and compare our results with experimental data reported by CELLO collaboration [12]. The scale σ_o, the continuum threshold in the pion channel was obtained by an explicit fitting procedure. The resulting values lie in the interval $0.6 \le \sigma_o \le 0.85\,\mathrm{GeV}^2$, *i.e.*, they agree with existing estimates for the pion duality interval. The sum rule predictions are rather stable in the M^2-region $0.6\,\mathrm{GeV}^2 \le M^2 \le 1.3\,\mathrm{GeV}^2$ for different Q^2. Our results agree with experimental data within an accuracy of 15% – 20%, usual for the QCD sum rules.

Our sum rule (4.3) can be also used to calculate the form factor $F_{\gamma^*\gamma^*\to\pi^0}(q^2, Q^2)$ at small (but nonzero) momentum transfer $q^2 \le m_\rho^2$ and fixed $Q^2 \ge 1\mathrm{GeV}^2$. However, there are no experimental data for this region. A detailed analysis of the sum rule including a detailed study of the sensitivity to various choices of the ρ-meson wave functions will be given elsewhere.

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Appendix

A Equations of motion and ρ-meson wave functions

Here we demonstrate how one can use equations of motion to obtain relations between the moments of the ρ-meson wave functions of different twist. A similar analysis was done in refs. [13] and [14].

Consider the identity:

$$\langle 0|\bar{\psi}_\alpha(0)\,(i\hat{\nabla} - m)_{\beta\rho}\,\psi_\rho(z)|\rho\rangle = 0, \tag{A.1}$$

where $i\hat{\nabla}_{\beta\alpha} = (i\hat{\partial}_z + g\hat{A}(z))_{\beta\alpha}$. Applying the Fiertz transformation we rewrite (A.1) as

$$\begin{aligned}(i\hat{\nabla} - m)_{\beta\alpha} S(z) &+ \left[(i\hat{\nabla} - m)\,\gamma_5\right]_{\beta\alpha} P(z) + \left[(i\hat{\nabla} - m)\,\gamma_\mu\right]_{\beta\alpha} V_\mu(z) - \\ &- \left[(i\hat{\nabla} - m)\,\gamma_\mu\gamma_5\right]_{\beta\alpha} A_\mu(z) + \frac{1}{2}\left[(i\hat{\nabla} - m)\,\sigma_{\mu\delta}\right]_{\beta\alpha} T_{\mu\delta}(z) = 0.\end{aligned} \tag{A.2}$$

To obtain a relation between wave functions, one should substitute in (A.2) the expressions for the bilocal matrix elements like (2.10),(2.11),(2.21)(2.22),(2.30), differentiate with respect to z and put $z^2 = 0$. By contraction with $[\sigma_{\nu\rho}]_{\alpha\beta}$, we extract a combination of the V-, A- and T-projections. There are three independent tensor structures

$$z_\nu\varepsilon_\rho - z_\rho\varepsilon_\nu,\ (\varepsilon z)\,(z_\nu p_\rho - z_\rho p_\nu),\ p_\nu\varepsilon_\rho - p_\rho\varepsilon_\nu$$

and, as a result, we get three systems of equations:

$$\begin{aligned}f_\rho^A m_\rho^2 \langle x^{n+1}\rangle_A &= f_{3\rho}^A m_\rho^2 \int_0^1 d\beta\,\beta\, n\,\langle\, [x_3\beta + x_2]^{n-1}\rangle_{3A} + \\ &+ 2 f_\rho^A C_{A5}\, n\,\langle x^{n-1}\rangle_{A5} - 2C_{V5}\, f_\rho^V m_\rho\,\langle x^n\rangle_{V5} + \frac{3}{2} f_\rho^{V25}\,\langle x^n\rangle_{V25},\end{aligned} \tag{A.3}$$

$$f_\rho^A C_{A5}\,\langle x^n\rangle_{A5} = a_{V1}\, f_\rho^V m_\rho C_{V15}\,\langle x^n\rangle_{V15} + \frac{1}{2} f_\rho^{V25}\,\langle x^{n+1}\rangle_{V25}, \tag{A.4}$$

$$\begin{aligned}(n+2)\, f_\rho^A\,\langle x^n\rangle_A &= -f_\rho^V m_\rho\,\langle x^{n+1}\rangle_V - a_{V1}\, f_\rho^V m_\rho\,\langle x^n\rangle_{V1} - \\ &- f_{3\rho}^A \int_0^1 d\beta\,\beta\, n\,\langle\, [x_3\beta + x_2]^{n-1}\rangle_{3A} \\ &+ f_{3\rho}^V \int_0^1 d\beta\,\beta\, n\,\langle\, [x_3\beta + x_2]^{n-1}\rangle_{3V} + 4m_q\, f_\rho^T\,\langle x^n\rangle_T.\end{aligned} \tag{A.5}$$

Eq.(A.5) was derived also in [13], but the constant a_{V1} was missed there. In the chiral limit, we can neglect the last term in (A.5). Taking the infinite limit for the renormalization parameter, $\mu^2 \to \infty$, we obtain the equation relating the moments of the asymptotic twist-3 wave functions:

$$(n+2)\, f_\rho^A \langle x^n \rangle_A = -f_\rho^V m_\rho \langle x^{n+1} \rangle_V - a_{V1}\, f_\rho^V m_\rho \langle x^n \rangle_{V1}. \qquad \text{(A.6)}$$

Taking into account the normalization conditions (2.12), we conclude that there exists the only solution:

$$\varphi_V^{as} = \frac{3}{2}(1 - 2x\bar{x}),\ \varphi_{V1}^{as} = 60x\bar{x}\,(2x-1),\ \varphi_A^{as} = 6x\bar{x} \qquad \text{(A.7)}$$

with

$$f_\rho^A = -\frac{f_\rho^V m_\rho}{4}, \quad a_{V1} = \frac{1}{40}. \qquad \text{(A.8)}$$

Note, that $\varphi_V^{as}, \varphi_{V1}^{as}$, given by eqs. (A.7) and (A.8), obey the condition that for a longitudinally polarized ρ^o-meson (*i.e.*, when $\varepsilon_\sigma^{\lambda=0} \simeq i\, p_\sigma/m_\rho + \mathcal{O}(m_\rho/p_z)$, as $p_z \to \infty$), the leading-twist part in eq.(2.10) provides the well known asymptotic twist-2 vector wave function (cf. eq.(2.6)). The value of f_ρ^A was also calculated in the SR method [15], and the result is in a good agreement with that dictated by equations of motion.

Substituting eq.(A.8) into eq.(A.6), we get

$$\frac{(n+2)}{4} \langle x^n \rangle_A = \langle x^{n+1} \rangle_V + \frac{1}{40} \langle x^n \rangle_{V1}. \qquad \text{(A.9)}$$

B Some properties of traceless combinations

To construct the orthogonal projection operators $P_{(n)}$ onto the subspace of traceless symmetric Lorentz tensors of rank n, we use the techniques similar to those in [19]. Here we list some useful formulas concerning these projectors as well as some contractions that appear in the paper.

By definition, for an arbitrary Lorentz tensor T we have [19]:

$$\left[P_{(n)} T\right]^{\mu_1 \dots \mu_n} = P_{(n)}{}^{\mu_1 \dots \mu_n}_{\nu_1 \dots \nu_n} T^{\nu_1 \dots \nu_n}. \qquad \text{(B.1)}$$

It is straightforward to derive the formula

$$\left(P_{(n)} T\right)^{\mu_1 \dots \mu_n} = \frac{1}{n} \sum_{i=1}^{n} T^{\mu_1 \dots [\mu_i] \dots \mu_n \mu_i} - \frac{1}{n^2} \sum_{i<j}^{n} g^{\mu_i \mu_j}\, T^{\mu_1 \dots [\mu_i] \dots [\mu_j] \dots \mu_n \alpha \alpha}, \qquad \text{(B.2)}$$

where $T^{\nu_1 \dots \nu_{n-1} \alpha}$ is now traceless and symmetric in its first $n-1$ indices and $[\mu_i]$ means that the corresponding index is absent. Choosing $T^{\{\nu_1 \dots \nu_{n-1}\}\alpha} \equiv s^\alpha \{q_1^{\nu_1} \dots q_1^{\nu_{n-1}}\}$ we have:

$$\begin{aligned} \{s^{\mu_1} q_1^{\mu_2} \dots q_1^{\mu_n}\} &= \frac{1}{n} \sum_{i=1}^{n} s^{\mu_i} \{q_1^{\mu_1} \dots [q_1^{\mu_i}] \dots q_1^{\mu_n}\} - \\ &- \frac{1}{n^2} \sum_{i<j}^{n} g^{\mu_i \mu_j} s^\alpha \{q_1^\alpha q_1^{\mu_1} \dots [q_1^{\mu_i}] \dots [q_1^{\mu_j}] \dots q_1^{\mu_n}\}. \end{aligned} \qquad \text{(B.3)}$$

Making use of the Nachtmann's [21] contraction

$$z^{\mu_1} \dots z^{\mu_n} \{q_1^{\mu_1} \dots q_1^{\mu_n}\} = \left(\frac{q_1^2 z^2}{4}\right)^{n/2} C_n^{(1)}(\eta) \qquad \text{(B.4)}$$

and some recursion relations for the Gegenbauer polynomials $C_n^{(\lambda)}(\eta)$ [22], one can derive the formula:

$$\begin{aligned} z^{\mu_1} \dots z^{\mu_{n-1}} \{q_1^\alpha q_1^{\mu_1} \dots q_1^{\mu_{n-1}}\} &= \frac{1}{n} \frac{\partial}{\partial z^\alpha} z^{\mu_1} \dots z^{\mu_n} \{q_1^{\mu_1} \dots q_1^{\mu_n}\} = \\ &= \frac{1}{n} \left[\frac{z^\alpha}{z^2} \tau^n \left(-2\, C_{n-2}^{(2)}(\eta)\right) + q_1^\alpha \tau^{n-1} C_{n-1}^{(2)}(\eta) \right], \end{aligned} \qquad \text{(B.5)}$$

where $\eta = i\,(q_1 z)/\sqrt{-z^2 q_1^2}$, $\tau = -i\sqrt{-z^2 q_1^2}/2$.

Using (B.3) – (B.5), one gets for an arbitrary 4-vector s:

$$z^{\mu_1} \dots z^{\mu_n} \{s^{\mu_1} q_1^{\mu_2} \dots q_1^{\mu_n}\} = \frac{(zs)}{n} \tau^{n-1} C_{n-1}^{(2)}(\eta) - \frac{(q_1 s)}{n} \frac{z^2}{2} \tau^{n-2} C_{n-2}^{(2)}(\eta). \qquad \text{(B.6)}$$

C Contact terms

Here we derive eq.(3.11). Before considering the relevant contraction, let us note that, incorporating the relation

$$\{z\partial\}^n = (z\partial)\{z\partial\}^{n-1} - \frac{\partial^2 z^2}{4} \frac{(n-2)}{n} \{z\partial\}^{n-2}, \qquad \text{(C.1)}$$

and neglecting higher twist contributions, one can substitute the original correlator (3.6) by

$$\Pi_{\mu\{\mu_1 \dots \mu_n\}}(q_1) = \Pi_{\mu\mu_1\{\mu_2 \dots \mu_n\}}(q_1) + \cdots. \qquad \text{(C.2)}$$

As a result,

$$\begin{aligned} \Pi_{\mu\{\mu_1 \dots \mu_n\}}(q_1)\, g_{\mu\mu_1} &\simeq \Pi_{\mu\mu\{\mu_2 \dots \mu_n\}}(q_1) \\ &= \int dx e^{-iq_1 x} \langle 0| T\{J_\mu(x)\, \bar{u}(0) \overleftarrow{\partial}_\mu \{\overleftarrow{\partial}_{\mu_2} \dots \overleftarrow{\partial}_{\mu_n}\} u(0)\}|0\rangle \\ &= \frac{1}{2} \int dx e^{-iq_1 x} \langle 0| T\{J_\mu(x)\, \bar{u}(0) \overleftarrow{\hat{\partial}} \gamma_\mu \{\overleftarrow{\partial}_{\mu_2} \dots \overleftarrow{\partial}_{\mu_n}\} u(0)\}|0\rangle && \text{(C.3)} \\ &- \frac{1}{2} \int dx e^{-iq_1 x} \langle 0| T\{J_\mu(x)\, \bar{u}(0) \gamma_\mu \{\overleftarrow{\partial}_{\mu_2} \dots \overleftarrow{\partial}_{\mu_n}\} \overrightarrow{\hat{\partial}} u(0)\}|0\rangle && \text{(C.4)} \\ &- \frac{i}{2} q_{1\varepsilon} \int dx e^{-iq_1 x} \langle 0| T\{J_\mu(x)\, \bar{u}(0) \gamma_\mu \gamma_\varepsilon \{\overleftarrow{\partial}_{\mu_2} \dots \overleftarrow{\partial}_{\mu_n}\} u(0)\}|0\rangle, && \text{(C.5)} \end{aligned}$$

where we have made use of the identity:

$$\begin{aligned} \int dx e^{-iq_1 x} \left[\langle 0| T\{J_\mu(x)\, \bar{u}(0) \overleftarrow{\partial}_\varepsilon \hat{\Gamma} u(0)\}|0\rangle + \langle 0| T\{J_\mu(x)\, \bar{u}(0) \hat{\Gamma} \overrightarrow{\partial}_\varepsilon u(0)\}|0\rangle \right] &= \\ = -i\, q_{1\varepsilon} \int dx e^{-iq_1 x} \langle 0| T\{J_\mu(x)\, \bar{u}(0) \hat{\Gamma} u(0)\}|0\rangle. \end{aligned} \qquad \text{(C.6)}$$

Applying (3.3) to (C.3) and (C.4), and integrating by parts in (C.3) we get

$$(C.3) = -2(-i)^{n-1}\{q_{1\mu_2}\dots q_{1\mu_n}\}\langle\bar{u}u\rangle. \qquad (C.7)$$

Now, taking into account that $\langle 0|\bar{u}(0)\{\overleftarrow{\partial}_{\mu_2}\dots\overleftarrow{\partial}_{\mu_n}\}u(0)\}|0\rangle = 0$ for all n, we obtain:

$$(C.4) = 0. \qquad (C.8)$$

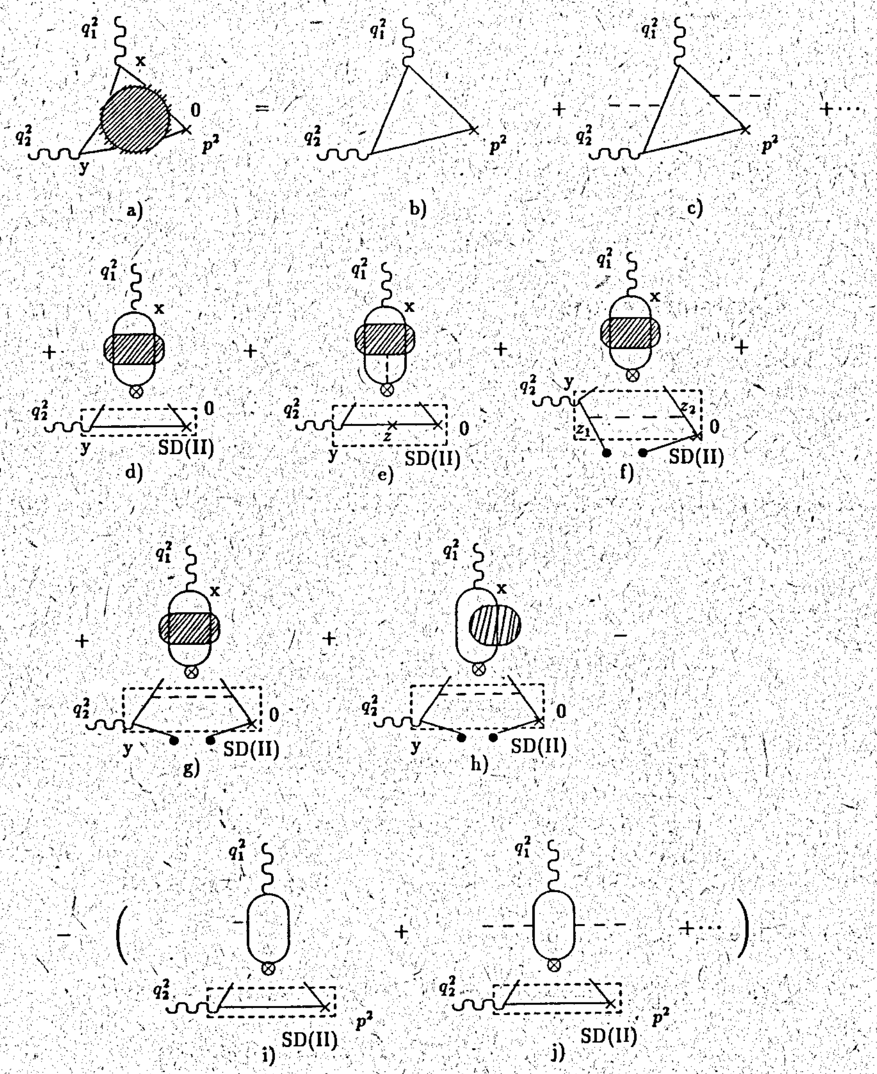

Fig. 1.

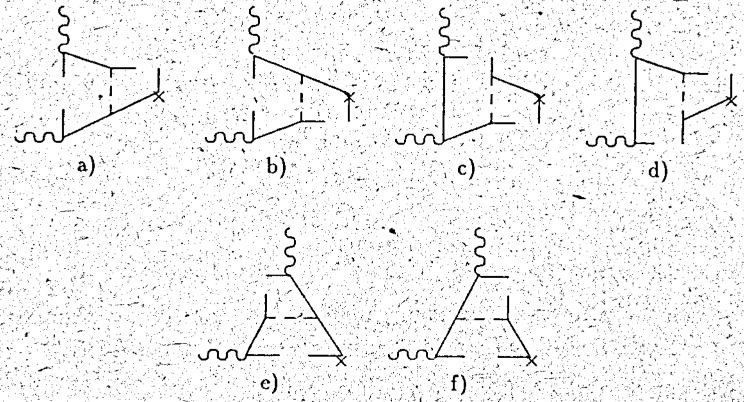

Fig. 2.

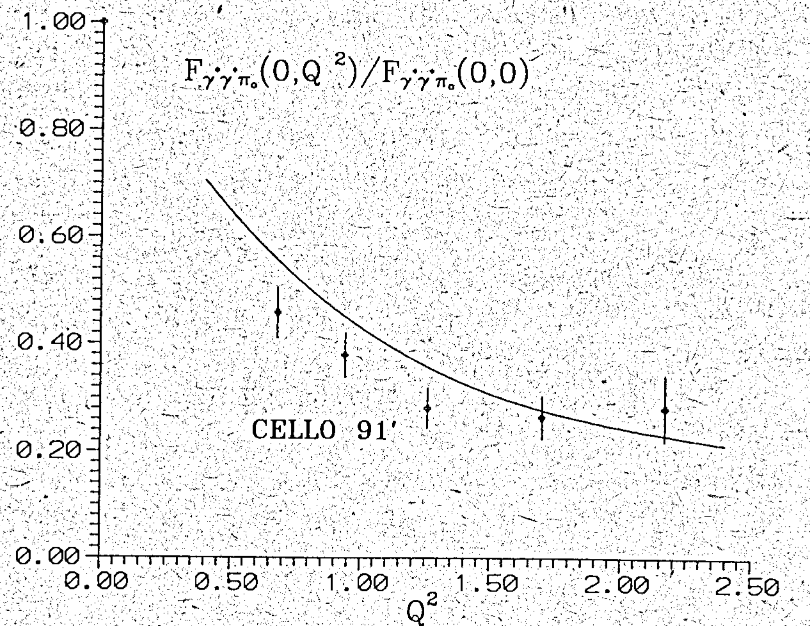

Fig. 3. The normalized form factor

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Формфактор процесса $\gamma^*\gamma^* \to \pi^0$ при малой виртуальности одного из фотонов и правила сумм КХД (II): правило сумм

Правила сумм КХД для формфактора $F_{\gamma^*\gamma^* \to \pi^0}(q_1^2,\ q_2^2)$ обобщаются в случае, когда одна из виртуальностей фотонов мала: $|q_1^2| << |q_2^2| \geq 1$ ГэВ^2. В этой области передач необходимо проделать дополнительное OPE для факторизации больших и малых расстояний. Вклад больших расстояний в трехточечную амплитуду факторизуется в двухточечные корреляторы (билокалы) электромагнитного тока и составных операторов двух нижайших твистов. Низкоэнергетическая зависимость корреляторов определяется из вспомогательных правил сумм КХД. Полученное правило сумм для формфактора регулярно в пределе $q_1^2 \to 0$. Наши оценки для $F_{\gamma^*\gamma^* \to \pi^0}(q_1^2 = 0,\ q_2^2)$ находятся в хорошем согласии с существующими экспериментальными данными.

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Radyushkin A.V., Ruskov R. E2-94-248

Form Factor of the Process $\gamma^*\gamma^* \to \pi^0$ for Small Virtuality of One of the Photons and QCD Sum Rules (II): Sum Rule

We extend the QCD sum rule analysis of the $F_{\gamma^*\gamma^* \to \pi^0}(q_1^2,\ q_2^2)$ form factor into the region where one of the photons has small virtuality: $|q_1^2| << |q_2^2| \geq 1$ GeV^2. In this kinematics, one should perform an additional factorization of short- and long-distance contributions. The extra long-distance sensitivity of the three-point amplitude is described by two-point correlators (bilocals), and the low-momentum dependence of the correlators involving composite operators of two lowest twists is extracted from auxiliary QCD sum rules. The sum rule obtained is regular in the limit $q_1^2 \to 0$. Our estimates for $F_{\gamma^*\gamma^* \to \pi^0}(q_1^2 = 0,\ q_2^2)$ are in good agreement with existing experimental data.

The investigation has been performed at the Bogoliubov Laboratory of Theoretical Physics, JINR.

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