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# RENORMCHARGES IN V-A THEORIES

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## I. Introduction

Renormgroups [1] serve as a powerful means for calculating the dependence of charges in a theory on momentum transfers. The charge, here, must be a scalar and must be invariant under transformations such as $P$, $C$, etc. This is indeed so, if one deals with a vector theory. But in the case, when the theory is not a vector theory, this invariance may be violated. One of such theories is the standard V-A theory of weak interactions. Such models, also, are the unification models [2], in which the weak interaction occurs as a constituent part.

We shall now proceed to examine the properties of the renormcharge in the theories indicated.

## II. The Renormcharges in Standard Vector Theories ($U(1)$ and $SU(N)$)

We shall first consider electrodynamics (the $U(1)$ theory). When the loop diagram is taken into account, we have [1,3].

$$e^2 \bar{u}(k')\gamma^\mu u(k)\left[g^\sigma_\mu - \frac{e^2 g^{\lambda\sigma}}{q^2}\times\right.$$

$$\left.\times\int \frac{d^4p'' \, [\bar{u}(p'')\gamma_\mu u(p''-q)]\,[\bar{u}(p''-q)\gamma_\lambda u(p'')]}{(2\pi)^4 (p''^2-M^2)\,[(p''-q)^2-M^2]}\right]\bar{u}(p')\gamma_\sigma u(p) \qquad (1)$$

($u(p)$, $\gamma^\mu$, $g_{\mu\lambda}$, $M$, $\alpha$ are standard notations), which can be rewritten, upon performing appropriate calculations, in the following form:

$$e^2\,[1 - \frac{\alpha}{3\pi}\ln(\frac{\Lambda^2}{-q^2})]\,(\bar{u}(k')\gamma^\mu u(k))\,(\bar{u}(p')\gamma_\mu u(p)). \qquad (2)$$

The term in square brackets can be reduced, applying the standard scheme [1], to the renormalized coupling constant (renormcharge) $\alpha(q^2)$:





$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 + \frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{\mu^2}{-q^2}\right)}, \qquad (3)$$

$\mu^2$ is the mass shell on which the coupling constant is determined. If the existence of leptons and quarks is taken into account, formula (2) acquires the form:

$$\alpha(q^2) = \frac{\alpha(\mu^2)}{1 + n\frac{\alpha(\mu^2)}{3\pi}\ln\left(\frac{\mu^2}{-q^2}\right)},$$

$$n = n_e + 3\left(\frac{4}{9}\right) n_q + 3\left(\frac{1}{9}\right) n_q. \qquad (4)$$

$n_e$ is the number of leptons, $n_q$ is the number of quarks of charge +2/3, $n_{q'}$ is the number of quarks with charge −1/3.

In the case of non-Abelean $SU(N)$ theories [4] the renormcharge will have this form, but with the substitution

$$n \to -\frac{11}{3}N + n; \quad \mu^2 \to \Lambda^2_{str}, \qquad (5)$$

where $N$ is the number of colours, and $n$ is the number of flavours.

Now, consider transformation of expression (1) with respect to parity $P$ making use of

$$U(P)\, u_L(k)\, U^{-1}(P) = \xi\gamma^0 u_R(-\mathbf{k},k_0),$$

$$U(P)\, A_\mu\, U^{-1}(P) = (A_0, -\mathbf{A}). \qquad (6)$$

The second part of expression (1) in square brackets is invariant under transformation (6). If one turns to expression (2), the above signifies that the charge part

$$e^2\left[1 - \frac{\alpha}{3\pi}\ln\left(\frac{\Lambda^2}{-q^2}\right)\right]$$

is invariant under transformation with respect to parity (the renormcharge in vector theories is invariant under transformation with respect to parity).

## III. Renormcharges in V-A Theories

Consider transformation of the spinors $u_L$ and $u_R$ with respect to parity:

$$U(P)\, u_L(k)\, U^{-1}(P) = \xi\, \gamma^0\, u_R\, (-\mathrm{k}, k_0)\,,$$

$$U(P)\, u_R(k)\, U^{-1}(P) = \xi\, \gamma^0\, u_L\, (-\mathrm{k}, k_0)\,. \qquad (7)$$

Under this transformation expression (1)

$$u_{L,R} \rightarrow u_{R,L}\,, \quad \left( u = u_L + u_R\,,\; u_L = \frac{1-\gamma_5}{2}\, u \right)$$

remains invariant.

If one considers the V-A $U(1)$ theory, the right components of the spinors, $u_R$, $\bar{u}_R$, are absent in this theory. Therefore, the term in square brackets under the integral in an expression such as (1) will acquire the form:

$$F_{L,\mu,\lambda}(\ldots) = [\bar{u}_L(p)\, \gamma_\mu\, u_L\, (p-q)]\, [\bar{u}_L\, (p-q)\, \gamma_\lambda\, u_L\, (p)]\,. \qquad (8)$$

This expression is not invariant under $P$ transformation and

$$U(P)\, F_{L,\mu,\lambda}(\ldots)\, U^{-1}(P) = 0\,.$$

A similar situation will occur, also, in the casee of the $SU(N)$ V-A theory. If one, now, makes use of an expression like (3), like it is usually done, as the expression for the renormcharge, one can see that this expression does not take into account the left-handed nature of the V-A theory. Evidently, the indicated fact should be taken into account in calculating the renormcharge in such theory.

Generally, the equation of renormgroups [1] is $P$ and $C$ invariant and it is formed for left-right symmetric theories, and its direct application for V-A theory is, probably, not quite correct. Most likely, one can try to derive the correct equation of renormgroups for V-A theory. But in doing so one must take into account that this interaction is left-handed, and so the renormcharge must be asymmetric with respect to the direction of propagation of the fermion.

In principle, we can formally take advantage of the standard expression for the equation for the renormcharge [1—4], as it is usually done. In this case the mass parameter $\mu^2$ (or $\Lambda^2_{str}$) must be used for choosing the normalization of the charge. But, if for the weak interaction we make use of the standard expression for the renormgroup and renormcharge, there arises a problem that has to be



solved. The part of the electroweak interaction that is responsible for the weak interaction is a non-Abelean $SU(2)$, so within this theory we must perform renormtransformations by analogy with QCD. There must, then, appear a dimensional constant that is determined from the weak interaction. However, unlike QCD, the theory of weak interaction is a theory with left interaction, and in this theory no dimensional parameter $\Lambda_{weak}$ can arise [5]. Thus, we must apparently draw the conclusion that within the framework of weak interactions no renormtransformation of charge should exist (i.e., the charge should remain constant). The dimensional parameters ( $M_W$, $M_Z$, ... ) that do occur in the theory of weak interaction arise outside the framework of this interaction (in the standard theory of electroweak interaction masses are due to the Higgs mechanism, while in extended theories they are due to the theory of technicolor [6]). Apparently, within the framework of the theories indicated, where dimensional parameters are present, there will already exist corresponding renormtransformations of charge. But, this is already outside the scope of the theory of weak interaction (the theory involving exchange of $W$, $Z$-bosons).

In connection with the above conclusion it seems appropriate to mention the analysis carried out in refs. [7], where no significant correction effects related to weak interaction were found.

## IV. Conclusion

If one formally considers the problem of charge and renormcharge in gauge theories, then, depending on the type of theory, there may exist three types of charge:

1. The charge $g_L$ in gauge V-A theories;
2. The charge $g_R$ in gauge V-A theories.

The charges $g_L$ and $g_R$ can exist in gauge V-A and V+A theories with separated left-handed and right-handed interactions.

3. The charge $g$ in gauge vector theories (i.e., theories with left-right symmetric interaction).

To be strict, the charges $g_L$, $g_R$, $g$ and are in no way related in gauge theories. In this sense, the problem exists in grand unification theories [2] of relating the charges $e$ and $g$ of the respective electromagnetic and strong interactions to the charge $g_L$ of weak interaction. Moreover, as indicated earlier, if no dimensional parameter $\Lambda_{weak}$ determining the scale of the renormtransformation exists in the theory of weak interaction, then unification of these theories encounters one more difficulty.

The main purpose of the present work is to draw attention to the following essential problems:

a) in the theory of weak interaction there may not exist any group of the renormtransformation of charge (i.e., the charge $g_L$ remains constant), or if such a group exists, it is then realized for the charge $g_L$, and it is not related to the charge $g$ in the vector theory;

b) in connection with the above there arises the problem of relating the charges $g_L$ and $g$ when constructing the unified theory of interaction in the standard way [2].

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