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SYMMETRY, WIGNER FUNCTIONS AND PARTICLE REACTIONS

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Симметрия, функции Вигнера и реакции частиц

В работе рассматривается важнейший принцип физики — симметрия — и некоторые идеи, связанные с ним, выдвинутые выдающимся физиком Юджином Вигнером. Мы обсуждаем понятия симметрии и спина, изучаем задачу разделения кинематики и динамики в бинарных процессах. Используя вигнеровские функции (отражающие свойства симметрии) при разложении спиральных амплитуд и кроссинг-симметрию между спиральными амплитудами (содержащую те же вигнеровские функции), получаем удобный общий формализм для описания реакций с участием частиц с произвольными массами и спинами. Рассматриваются также некоторые приложения предложенного формализма.

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Symmetry, Wigner Functions and Particle Reactions

In this paper we consider the great principle of physics — symmetry — and some ideas, connected with it, suggested by a great physicist Eugene Wigner. We will discuss the concept of symmetry and spin, study the problem of separation of kinematics and dynamics in particle reactions. Using Wigner rotation functions (reflecting symmetry properties) in helicity amplitude decomposition and crossing-symmetry between helicity amplitudes (which contains the same Wigner functions) we get convenient general formalism for description of reactions between particles with any masses and spins. We also consider some applications of the formalism.

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I. SYMMETRY

In this report we consider a great principle of physics and some ideas, connected with it, suggested by a great physicist. The great principle is symmetry. The great physicist is Eugene Wigner.

We will discuss the concept of symmetry and spin, study the problem of separation of kinematics and dynamics in particle reactions. Using Wigner rotation functions (reflecting symmetry properties) in helicity amplitude decomposition and crossing–symmetry between helicity amplitudes (which contains the same Wigner functions), we get convenient and general formalism for description of reactions between particles with any masses and spins. We also consider some applications of the formalism.

In [1] Wigner says that there are three levels of Knowledge:

1. Observation (Galilei);

2. Equation (Newton, Maxwell, Schroedinger, ...);

3. Symmetry (Einstein, Poincare, ...). The author of this paper would add: Wigner, ... all modern particle physicists).

There is some hierarchy of knowledge, and on the highest level of this hierarchy is symmetry.

Symmetry means harmony, beauty, order. Symmetry explicitly shows itself in architecture, for example, in Oxford, Salamanca, Goslar – the cities where some of the last conferences on symmetry were held.

In physics, symmetry has three levels:

1. Coordinate systems, frames (spherical system, inertial systems);

2. Variables. (For example, for binary processes we have two independent variables, energy and angle, or invariant variables – s and t);

3. Functions. If we consider reactions with particles with spin

$$s_1 + s_2 \rightarrow s_3 + s_4, \qquad (1)$$

we have $N = (2s_1+1)(2s_2+1)(2s_3+1)(2s_4+1)$ functions to describe the process, and we must choose the optimal set of these functions.

In particle physics E.Wigner considered three types of symmetry:

1. Space-time symmetries;

2. Intrinsic symmetries;

3. "Intermediate" symmetry: crossing.

The language of symmetry is mathematical theory of groups and their representations. We have rotation, Lorentz and Poincare invariance and the corresponding groups with their representations. Poincare invariance has two Kasimir operators, two invariants. These invariants are connected with two fundamental properties of elementary particles: mass and spin. Their existence is connected with symmetry. Mass is both a classical and a quantum quantity, whereas spin is a pure quantum object.

Symmetry is connected with fundamental conservation rules — conservation of energy, momentum and angular momentum (the latter is the sum of spin and orbital momentum).

So, in elementary particle physics we have the particle with mass m, spin s, energy E, momentum p. It is often convenient to consider also helicity, the projection of spin in the direction of motion.

We have two types of symmetry: global and local (gauge). If we suggest symmetry (Lorentz) and the spin of the particle, we can write a free particle Lagrangian – L_0. If we suggest the gauge symmetry for a free particle Lagrangian, we get with necessity the particle which takes interactions (photon, gauge W and Z bosons, gluon) and even the interaction Lagrangian – $L_{Int.}$.

Generalization of the spin in the "intrinsic" direction is isospin [2]. The isospin is connected with the group $SU(2)$. Wigner [3] suggests

first generalization of $SU(2)$, $SU(4)$. The revolution in physics was made by suggesting $SU(3)$ and quarks [4], color [5], unified theory of electroweak interactions and quantum chromodynamics.

Due to the symmetry in particle physics (quantum field theory), we have a Lagrangian of a definite form which depends on a small number of masses and interaction constants. This is in sharp contrast with quantum mechanics where interactions are considered as arbitrary functions(potentials) for every pair of particles. The symmetry does not admit arbitrary functions.

Today we have the following succession:

$$\text{Symmetry} \rightarrow \text{group} \rightarrow \text{particle interaction.}$$

So $SU_c(3)$ symmetry and corresponding group give us quantum chromodynamics; Symmetry and Group $U(1)XSU(2)$ — electroweak interactions; We have the standard 1X2X3 model and other unification schemes. These unifications are realized at very high energies, which were realized at the earliest stages of our Universe in Big Bang theory, so symmetry gives us the key to the Universe.

Symmetry between fermions and bosons creates supersymmetry, a theory which predicts new particles – supersymmetric partners of old ones. These particles are: gravitino (with spin 3/2), photino and so on.

So, symmetry gives us the characteristics of particles (mass, spin momentum and so on), the particles which carry interactions (gauge particles) and the interaction Lagrangian. This in principle must be the full theory.

But, in reality, there are some difficulties.

For example, QCD is a good theory, describes a lot of effects, but in QCD there exists a problem of confinement; QCD works on the quark level, with the subprocesses, not in a full region of variables; there is "spin crisis"; perturbative QCD has difficulties in explanation of polarization effects at high-energy large fixed angle proton-proton scattering.

Another excited theory, SUSY, has mathematical problems with dimensions, compactification and so on.

So today we have no full and final theory. Thus there exists a problem which has its own history: problem of direct investigation of processes with elementary particles, based on the general symmetry principles and independent of the explicit form of Lagrangian – spin kinematics (or amplitude kinematics). Another piece of theory that is also general and represents an alternative approach to particle theory is the S-matrix approach with its analytical properties, singularities, dispersion relations, sum rules and so on. We will consider these things, and the role of symmetry in particle physics, and in particular, the role of Wigner's d-functions in the description of spin-particle reactions.

II. SPIN, PARTICLE REACTIONS AND WIGNER FUNCTIONS

Most of the particles have a nonzero spin. We are going to consider binary reactions with particles of arbitrary spins. The spin-particle reactions are convenient to describe in the helicity amplitude formalism [6]. Helicity amplitudes $f_{\lambda_3,\lambda_4;\lambda_1,\lambda_2}(s,t)$ have clear physical meaning, observables are expressed by them in a simple way. Helicity amplitudes contain all the information about the considered process. But helicity amplitudes have kinematic singularities.

Scattering of spinless particles is described by one amplitude. Considering this amplitude as a function of invariant variables, we have the function $A(s,t)$. This amplitude has some singularities. They are called the dynamic singularities. The analytic properties of the amplitude are connected with causality and unitarity and this amplitude obeys dispersion relations.

For spin-particles, the process is described by several functions, several helicity amplitudes. And they have additional, so-called kinematic, singularities. So helicity amplitudes do not fulfill simple dispersion relations. It is necessary to find and separate kinematic singularities. So, helicity amplitudes are expressed via a set of other amplitudes without kinematic singularities. For a lowest spin it is convenient to introduce invariant amplitudes.

Let us consider the simplest non-trivial reaction: π-N scattering, elastic scattering of a spin-zero particle with the mass μ on the spin-1/2 particle of mass m. Using the Dirac equation one can find the following connection between the helicity and invariant amplitudes (in standard designation):

$$f^s_{0,\lambda_4;0,\lambda_2}(s,t) = \bar{u}^{\lambda 4}(p_4)\{A(s,t) + \hat{Q}B(s,t)\}u^{\lambda 2}(p_2). \qquad (2)$$

Here A(s,t) and B(s,t)are invariant amplitudes. Properly defined invariant amplitudes have no kinematic singularities.

For the general case of scattering of particles with spins s_i we have

$$f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \sum_{n=1}^{N} a^n_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)A_n(s,t). \qquad (3)$$

Kinematic singularities of $f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$ are contained in the coefficient functions $a^n(s,t)$.

This procedure is nice for low spins. It is difficult to construct such an expansion for high spins. For all $s_i = 3/2, N = 256$ and for $s_i = 11/2, N \simeq 20000$. Besides, the main difficulty is in finding such a decomposition in a way that coefficients of invariant amplitudes do not contain "secret singularities" rather than in dimensions. So, in describing the Compton effect for several years people used a decomposition suggested in [7], but then it appeared that those invariant amplitudes had additional singularities, and later a more complicated decomposition [8] was suggested.

Besides technical difficulties for spins more than 1, a nontrivial question of uniqueness of such decomposition arises and since for higher spins the invariant amplitude decomposition is not unique, the "secret" singularities, additional and noncontrollable kinematic constraints appear.

There exists another way which uses symmetry principles and is connected with the use of representations of a rotation group — Wigner's d-functions. If we use d-functions in the s-channel; then use d-functions in the t-channel and finally connect channels also by d-functions, we can get a result much more convenient than (3).

The helicity amplitudes in the center-of -mass system of s-channel obey the rotation symmetry (this symmetry is connected with the conservation of angular momentum). Because of this symmetry it is convenient to expand helicity amplitudes over the representation of a rotation group, over Wigner's functions:

$$f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \sum_J (2J+1) f^J_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s) d^J_{\lambda\mu}(\cos\theta). \quad (4)$$

Here, we have infinite summation. Wigner's functions have the form

$$d^J_{\lambda\mu}(\cos\theta) = g\left(\sin\frac{\theta}{2}\right)^{|\lambda-\mu|}\left(\cos\frac{\theta}{2}\right)^{|\lambda+\mu|} P^{|\lambda-\mu|,|\lambda+\mu|}_{J-M}(\cos\theta), \quad (5)$$

where $P^{mn}_k(\cos\theta)$ are Jacobi polynomials (see, for example, [9]). $M = max(|\lambda|, |\mu|)$, and $N = min(|\lambda|, |\mu|)$ and

$$g = \sqrt{\frac{(J+M)!(J-M)!}{(J+N)!(J-N)!}}.$$

The crossing relations between the s- and t-channel helicity amplitudes look as follows [10]:

$$f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \sum_{\mu_1\mu_2\mu_3\mu_4} \alpha d^{s_1}_{\lambda_1\mu_1}(\chi_1) d^{s_2}_{\lambda_2\mu_2}(\chi_2)$$
$$d^{s_3}_{\lambda_3\mu_3}(\chi_3) d^{s_4}_{\lambda_4\mu_4}(\chi_4) f^t_{\mu_3\mu_4,\mu_1\mu_2}(s,t). \quad (6)$$

The crossing relations also contain the Wigner functions. Here the summation is over helicity values and it is restricted.

III. KINEMATICS AND DYNAMICS. SUGGESTED FORMALISM

A lot of people worked in this direction by considering spin-kinematics and decomposition of helicity amplitudes in terms of other sets of amplitudes [11]. Combining some approaches and modifying others we suggest a new variant of formalism which has all advantages of different approaches, differs from all of them, is based on the symmetry and conservation laws, is general and simple.

Symmetry imposes restrictions on amplitudes. When one has additional symmetries in definite directions, the number of independent amplitudes in such "symmetrical directions" is reduced. Such situations occur for forward and backward scattering.

Consider the reaction in the s-channel described by the helicity amplitudes. Introduce the quantities $\lambda = \lambda_1 - \lambda_2$ and $\mu = \lambda_3 - \lambda_4$. Two particles in the center-of-mass system are moving in the opposite directions and thus λ and μ are projections of the total spin in the directions of motion prior to and after collision. Owing to the conservation of the projection of the total angular momentum, the amplitudes in the forward direction, $\theta_s \to 0$, should vanish in all cases except for $\lambda = \mu$. Analogously, for backward scattering, $\theta_s \to \pi$, the amplitudes should vanish for the same reasons in all cases except for $\lambda = -\mu$.

For forward scattering we have

$$f^{forward}_{\lambda_3\lambda_4,\lambda_1,\lambda_2} = \begin{cases} f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}, & when \quad \lambda = \mu, \\ 0, & when \quad \lambda \neq \mu, \end{cases} \quad (7)$$

whereas for backward scattering

$$f^{backward}_{\lambda_3\lambda_4,\lambda_1,\lambda_2} = \begin{cases} f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}, & when \quad \lambda = -\mu, \\ 0, & when \quad \lambda \neq -\mu. \end{cases} \quad (8)$$

Two questions arise:

Can the helicity amplitudes be parametrized so as to satisfy the conditions (7) and (8) automatically?

Can kinematic singularities of helicity amplitudes be found and separated in a simple way?

The answer is "Yes".

Using (4), for the spinless case we get the decomposition via the Legendre polynomials, depending on $\cos\theta$. By definition in the spinless case we have no kinematic singularities.

In the nonzero spin case, helicity amplitudes are splitted into two parts; one part is defined by the symmetry properties and enters into the functions $d^J_{\lambda\mu}(\cos\theta)$ that make the conservation laws of the angular momentum valid, and the other part has a dynamic nature and enters into the partial helicity amplitudes $f^J_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s)$.

In (4) all the t-dependence is contained in d-functions via $\cos\theta_s$. At the points $\cos\theta_s = \pm 1$ the d-function has kinematic singularities on the t-variable, which can be separated explicitly.

These singularities do not depend on J and we can separate the common singular factors. The rest sum will contain decomposition by polynomials on the t-variable. So we can define dispersion amplitudes for any binary processes:

$$f^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = A^{|\lambda-\mu|} B^{|\lambda+\mu|} \bar{f}^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t), \qquad (9)$$

here

$$A = \frac{\sqrt{L^2 - a^2}}{(m_1 + m_2)(m_3 + m_4)}, B = \frac{\sqrt{L^2 + a^2}}{(m_1 + m_2)(m_3 + m_4)},$$

$$L^2 = \{[s - (m_1 + m_2)][s - (m_1 + m_2)] [s - (m_3 + m_4)][s - (m_3 + m_4)]\}^{1/2},$$

$$a^2 = 2st + s^2 - s \sum m_k^2 + (m_1^2 - m_2^2)(m_3^2 - m_4^2).$$

The mass factors in the denominators make A and B dimensionless without introducing additional singularities in the variable s. Under this parametrization, the conditions (7) and (8) are fulfilled automatically. All kinematic singularities in variable t are separated explicitly and no false singularities in s are introduced. The amplitudes $\bar{f}^s_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t)$ suit well for studying the analytic properties of the amplitudes at fixed s because they obey dispersion relations. Therefore, we call them the dispersion amplitudes [12]. They still may have the kinematic singularities in the variable s.

Dispersion amplitudes remind reduced amplitudes [11], but they have no additional s-variable false singularities.

For t-channel processes the corresponding dispersion amplitudes are free from kinematic singularities in the variable s. Expressing the dispersion amplitudes of the s-channel in terms of the dispersion amplitudes on the annihilation channel, we obtain the connection between the amplitudes having kinematic singularities in s with the amplitudes which are free from them. So kinematic singularities of the s-channel helicity amplitudes are in crossing coefficients in crossing relations between s- and t-channel amplitudes. The number of coefficients is restricted and we do know the singularities of these coefficients; indeed these coefficients are Wigner's functions, and we do know their singularities!

So, using crossing symmetry we can find kinematic singularities of the s-channel dispersion amplitudes also in the variable s, separating these singularities we determine a new set of functions describing binary processes — dynamic amplitudes. Dynamic amplitudes for elastic processes have the following relations with the helicity amplitudes [13]:

$$f_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t) = \left(\frac{\sqrt{-t}}{m+\mu}\right)^{-|\lambda-\mu|} \left(\frac{\sqrt{L^2+st}}{(m+\mu)^2}\right)^{-|\lambda+\mu|}$$
$$\left(\frac{L}{(m+\mu)^2}\right)^{-2(s_1+s_2)} D_{\lambda_3\lambda_4,\lambda_1,\lambda_2}(s,t). \quad (10)$$

Dynamic amplitudes are in fact modified regularized helicity amplitudes, they differ from the reduced amplitudes by dimensions: all dynamic amplitudes have the same dimensions, whereas the dimensions of regularized amplitudes depend on spins and helicities.

IV. APPLICATIONS

The dynamic amplitude formalism is interesting for studying general characteristics of particle reaction theory and it also suits for exploring concrete processes. This approach provides an analysis where kinematics is fully taken into account and is clearly separated from dynamics. The observable quantities are simply expressed via the helicity amplitudes.

As we have already mentioned, the helicity amplitudes have a clear physical meaning, and physical observables (polarization cross sections, asymmetries, etc.) are simply expressed via them. As for elastic processes, the connection between the helicity and dynamic amplitudes is one-to-one, every helicity amplitude for elastic scattering is expressed in terms of one dynamic amplitude. Hence, it follows that all attractive features of the helicity amplitudes — a clear physical meaning, simple relations with observables, and equal dimensions — are also inherent in the dynamic amplitudes. The formalism of dynamic amplitudes is simple for low spins and remains such also for higher spins: the formalism is simple for any spins.

The differential cross section for elastic scattering, when one measures the helicity of each particle, is expressed via helicity, invariant and dynamic amplitudes in the following form:

$$\frac{d\sigma}{dt}(\lambda_3\lambda_4,\lambda_1\lambda_2) \sim | f_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)|^2=$$
$$=|\sum_1^N a^n_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)A_n(s,t)|^2=$$
$$=|\left(\frac{\sqrt{-t}}{m+\mu}\right)^{-|\lambda-\mu|} \left(\frac{\sqrt{L^2+st}}{(m+\mu)^2}\right)^{-|\lambda+\mu|}$$
$$\left(\frac{L}{(m+\mu)^2}\right)^{-2(s_1+s_2)} D_{\lambda_3\lambda_4,\lambda_1\lambda_2}(s,t)|^2. \quad (11)$$

The first relation in outward appearance is simplest, but helicity amplitudes contain kinematic singularities and the conservation laws are not fulfilled automatically, so kinematics and dynamics are not separated. Here we have one term. In the second equation there is a sum of all invariant amplitudes. Here we have N terms. For the spins equal to 3/2 there are 256, and for the spins equal to 11/2 more than 20 000 terms. In each term we have kinematic–dynamic separation, but there are so many such terms. In the parametrization via dynamic amplitudes we have no summation! The differential cross section is expressed only via one dynamic amplitude with the kinematic factors which contain all kinematic singularities. We have only one term.

Other quantities such as P, A_{nn}, A_{ll}, A_{ss} in terms of the helicity amplitudes have the form [14]

$$\sim \frac{\sum c_{mn} f_m f_n^*}{\sum | f_m |^2}.$$

Here m and n represent sets of helicity indices, $c_{mn} = \pm 1$. The sum is taken for all values of helicities. Obviously, the expressions will be most convenient in terms of dynamic amplitudes.

To compare the usefulness of different sets of amplitudes we suggest the following table:

Amplitudes	Helicity	Invariant	Dynamic
Observables	+	–	+
Phys. meaning	+	–	+
Same dimens.	+	–	+
Kinem. singul.	–	+	+
Conserv. laws	–	–	+

In the line "Observables" the sign "+" means simplicity, "–" denotes complexity. In the next line "+" stands for existence of clear physical meaning, whereas "–" means its absence. Amplitudes with the same dimensions are signed by "+". When we have no kinematic singularities the table shows "+", and that sign in the last line means automatic fulfillment of consequences from angular momentum conservation.

In the framework of the general spin formalism based on the symmetry properties ("dynamic amplitude" approach) obligatory kinematic factors arise in the expressions of observables. These spin structures for high energies give a small parameter that orders the contributions of helicity amplitudes to observables. Such a "kinematic hierarchy " predicts for *pp* elastic scattering at high energies and a large fixed angle (90^o) a simple connection between asymmetry parameters and even numerical values for them [15].

The spin kinematics allows one to obtain the low-energy theorems for photon-hadron processes [16] and gravitino scattering on spin-0

target. For the latter process at low energies the helicity amplitudes up to $0(E^3)$ are determined by their t-channel Born terms with the photon exchange [17].

The dynamic amplitudes, or more simply the t-channel dispersion amplitudes, can be used to prove model–independent dispersion inequalities for the Compton effect on the pion and nucleon target, including the case of the polarized photon scattering [18].

Here, we have mentioned other possible applications of dynamic amplitudes. These are the dispersion relations for individual helicity amplitudes for any elastic scattering and sum rules (especially dual sum rules) also for any elastic scattering.

REFERENCES

1. E.P. Wigner. *Symmetries and Reflections*, Indiana University Press, Bloomington–London, 1970.
2. W.Heisenberg, *Z. Physik.* 77 (1932) 1.
3. E.P.Wigner, *Phys. Rev.*, 51 (1964) 105.
4. M.Gell-Mann, *Phys. Rev.*, 125 (1962) 1067;
   Y.Neeman, *Nuclear Phys.*, 26 (1961) 222.
5. N.N.Bogoliubov, B.V.Struminsky, A.N.Tavkhelidze, *JINR Preprint D-1968*, Dubna, 1965;
   M.Y.Han, Y.Nambu, *Phys. Rev.* 139 (1965) 1006;
   H.Fritzsch, M.Gell-Mann, In: Proceedings of the XVI International Conference on High Energy Physics, vol.2, p.135. Chicago, 1972.
6. M.Jacob, G.C.Wick, *Ann. of Physics*, 7 (1959) 404.
7. A.C.Hern, E.Leader, *Phys. Rev.*, 126 (1962) 789.
8. W.A.Bardeen, Wu-Ki Tung, *Phys. Rev.*, 173 (1968) 1423.
9. D.A.Warshalovich et al., *Quantum Theory of Angular Momentum*, Nauka Publisher, Leningrad, 1975.
10. Ya.A.Smorodinsky, *JINR Preprint E-1227*, Dubna, 1963;
   T.L.Truman, G.C.Wick, *Ann. of Physics*, 26 (1964) 322;
   Y.Hara, *Phys. Rev.*, 136B (1964) 507.
11. H.Joos, *Fortsh. Physik*, 10 (1962) 65;

D.N.Williams, *Preprint UCRL-11113*, Berkeley, California, 1963; M.Gell-Mann et al. *Phys. Rev.*, 133B (1964) 145;

L.L.Wang, *Phys. Rev.*, 142 (1966) 1187;

G.Cohen-Tannoudji, A.Morel, H.Nvelet, *Ann. of Physics*, 46 (1968) 239;

J.P.Ader, M.Capdeville, H.Navelet, *Nuovo Cimento*, 56A (1968) 315;

T.L.Truman, *Phys. Rev.* 173 (1968) 1684 and so on.

12.M.P.Chavleishvili, *Polarization Dynamics in Nuclear and Particle Physics*, Proceedings of the International Symposium, Trieste, 1992;

M.P.Chavleishvili, *Ludwig-Maximilian University Preprint LMU-02-93*, München, 1993.

13.M.P.Chavleishvili, *Ludwig-Maximilian University Preprint LMU-03-93*, München, 1993;

M.P.Chavleishvili, *Soviet Journal of Nuclear Physics*, 40 (1984) 243;

M.P.Chavleishvili, *Soviet Journal of Nuclear Physics*, 41 (1985) 1055.

14.C.Bourrely, E.Leader, J.Soffer, *Phys. Reports*, 59 (1980) 96.

15.M.P.Chavleishvili, *High Energy Spin Physics* , Proceedings of the 8th International Symposium, Minneapolis, 1988. Ed.K.J.Heller. New York, 1989, vol 1, p.123;

M.P.Chavleishvili, *Ludwig-Maximilian University Preprint LMU-05-93*, München, 1993.

16.R.M.Muradyan, M.P.Chavleishvili, *Soviet Journal of Theor. and Math. Physics*, 8 (1971) 16;

M.P.Chavleishvili, *JINR Preprint P2-88-179*, Dubna, 1988.

17.M.P.Chavleishvili, *JINR Preprint E2-87-69*, Dubna, 1987;

M.P.Chavleishvili, *High Energy Spin Physics* , Proceedings of the 9th International Symposium, Bonn, 1990. Eds. K.-H.Althoff, W.Meyer. Springer-Verlag, Berlin, 1991, vol 1, p.489.

18.M.P.Chavleishvili, *Soviet Journal of Nuclear Physics*, 37 (1982) 680;

M.P.Chavleishvili, *Soviet Journal of Nuclear Physics*, 43 (1986) 385.

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