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THE PHOTON STRUCTURE FUNCTION  $F_2$   
IN QCD WITH NONLOCAL VACUUM QUARK  
CONDENSATES

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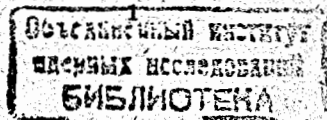
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1. Here we present the calculation of the hadronic part of the photon structure function (PhSF) by taking into account the nonlocal quark condensate and following the method suggested by Gorsky et al. [1]. We also compare our results with experiments and provide a brief discussion.

The approach to calculate the hadronic part of the PhSF  $F_2(x, P^2, Q^2)$  in photon-photon DIS ( $Q^2 \gg P^2$ ) in the region of moderate values of  $x$  and  $Q^2$  has been proposed in [1]. This approach is based on the operator product expansion (OPE) technique for a 4-current correlator with account of the non-zero vacuum condensate (VC) of gluon fields ( $\langle GG \rangle$ ). The authors of this paper have managed to reproduce rather well the existing experimental data in the region  $0.3 < x < 0.7$  for different values of  $Q^2$  ( $Q^2 = 4.3, 5.3, 9.2, 23.0 \text{ GeV}^2$ ) without introducing new phenomenological parameters. As to VC of quark fields, it was asserted that their contribution (through the operators of the lowest dimension, i.e.  $\langle \bar{q}q \rangle$ ) is proportional to the  $\delta(1-x)$  and thus, could be dropped out of the consideration of the region  $0.3 < x < 0.7$ . (The contributions of quark VCs to PhSF in this region are possible also from diagrams with radiative corrections to the usual box diagrams, which are of order  $\alpha_S$  and for this reason aren't considered.)

However, the use of the nonlocal quark condensate [2], which is equivalent to the summation of an infinite set of condensates of higher dimensions, leads to the result with new properties. We show that nonlocal quark VCs give rise to a smooth (over  $x$ ) contribution to PhSF. It is interesting to note, that essential diagrams are new. The magnitude of the corresponding contribution to PhSF ( $\Delta^{(\bar{q}q)} F_2$ ) is of the same order as the one from gluon VC ( $\Delta^{(GG)} F_2$ ) and is determined by the characteristic length of the quark vacuum correlations  $1/\lambda_q$ :

$$\Delta^{(GG)} F_2(x) \sim \frac{1}{m_p^4} \left\langle \frac{\alpha_S}{\pi} GG \right\rangle, \quad \Delta^{(\bar{q}q)} F_2(x) \sim \frac{\pi^4 \langle \bar{q}q \rangle^2}{\lambda_q^6}. \quad (1)$$



The latter estimate in (1) doesn't depend on the concrete form of a condensate distribution function [2] and reflects the essentially nonperturbative character of this contribution. Numerical values are:  $\Delta^{(\bar{q}q)}F_2 \approx 0.35$ , whereas  $\Delta^{(GG)}F_2 \approx -0.15$  in the central region of  $x$  for the standard values  $\lambda_q^2 \approx 0.4 \text{ GeV}^2$  ( $\lambda_q^2 \equiv \langle \bar{q}D^2q \rangle / \langle \bar{q}q \rangle \approx 0.4 - 0.6 \text{ GeV}^2$ ) [3]. Careful analysis shows the substantial improvement of agreement with experiment in all the region  $0.2 < x < 0.8$  just for these standard values of correlation length  $1/\lambda_q$ . For larger values,  $\lambda_q^2 \approx 1.2 \text{ GeV}^2$ , which are typical of instanton liquid models [4], the quark VC's contribution is of no importance as compared with the gluon one. It's clear however, that the value of  $\lambda_q$  couldn't be much less than the standard value because of the "explosion" of  $\Delta^{(\bar{q}q)}F_2$  for  $\lambda_q \rightarrow 0$ . One can conclude that the PhSF is rather sensitive to the parameters of condensate structure, therefore the extraction of  $F_2$  with high precision from experiments on photon-photon interactions provides the possibility of independent evaluation of the length of correlations in the nonperturbative QCD vacuum.

We also establish the breakdown of factorization theorem [5] for the discontinuities (i.e. imaginary parts, *Disc*) of diagrams with nonlocal VC on the cut line. This effect depends on the decay rate of vacuum correlations at large distances: e.g., the Gauss decay of quark VC ( $\langle \bar{q}(z)q(0) \rangle \sim \exp(-\gamma z^2)$  for  $|z| \rightarrow \infty$ ) does violate the factorization, but the exponential one ( $\sim \exp(-\gamma|z|)$ ) doesn't. At the same time, our conclusion about the necessity to take into account  $\delta$ -function distributions isn't related with the nonlocality of VCs: the distribution concentrated near the border of some area couldn't be used properly for treating local problems in the center of the area. In this case the appearance of a border-concentrated distribution like the  $\delta$ -function from the OPE signals about the deficiency of that expansion. Our method, that uses another type of distributions, avoids this problem.

2. The nonlocal VC seems to be introduced for the first time in [6], and the exponential decay in coordinate representation was obtained in lattice calculations [7]. The nonlocal VC was successfully employed to explain different dynamical hadron properties in exclusive processes [8]. We use a  $\delta$ -shaped Ansatz for the distribution functions  $f(\alpha)$  of nonlocal scalar ( $M(z)$ ) and vector ( $M_\mu(z)$ ) quark VCs [9]:

$$M(z) \equiv \langle \bar{q}(0)\hat{E}(0,z)q(z) \rangle = \langle \bar{q}(0)q(0) \rangle \int_0^\infty e^{\alpha z^2/4} f_S(\alpha) d\alpha; \quad (2)$$

$$M_\mu(z) \equiv \langle \bar{q}(0)\gamma_\mu\hat{E}(0,z)q(z) \rangle = -iz_\mu A \int_0^\infty e^{\alpha z^2/4} f_V(\alpha) d\alpha; \quad (3)$$

$$f_S^{mod}(\alpha) = \delta(\alpha - \Delta^2); \quad f_V^{mod}(\alpha) = \delta'_\alpha(\alpha - \Delta_V^2), \quad (4)$$

where  $A = 2/81\pi\alpha_s\langle\bar{q}q\rangle^2$ , and scales  $\Delta^2 \equiv \lambda_q^2/2$  and  $\Delta_V^2 \equiv a_V\lambda_q^2/2$  aren't always equal,  $\langle\sqrt{\alpha_S\bar{q}q}\rangle^{1/3} \approx 0.23 \text{ GeV}$ . We select the value  $a_V = 0.7$ , that is consistent with the Taylor expansion of the VC in the lowest orders [2].

Let us consider diagrams of fig.1 (crossed graphs are also understood; the contribution of diagram 1A is marked by index  $S$ , and that of 1B - by  $V$ ) and take into account

the contributions of the lowest twist, i.e. neglect terms like  $P^2/Q^2$ ,  $\exp(-Q^2/(\Delta^2 x))$ . Then, denoting

$$C_{norm} \equiv \frac{3\alpha_{em} \sum_q e_q^4}{\pi}; \quad \gamma_S \equiv \frac{2xP^2}{\Delta^2}; \quad \gamma_V \equiv \frac{\bar{x}P^2}{\Delta_V^2},$$

we obtain

$$\frac{1}{C_{norm}} \Delta_S^{(\bar{q}q)} F_2^T(x; P^2) = \frac{8\pi^4}{9\Delta^6} (\bar{q}q)^2 (x\bar{x}) \exp(-\gamma_S); \quad (5)$$

$$\frac{1}{C_{norm}} \Delta_S^{(\bar{q}q)} F_2^L(x; P^2) = \frac{8\pi^4}{9\Delta^6} (\bar{q}q)^2 \left(\frac{x^2}{\gamma_S}\right) \exp(-\gamma_S); \quad (6)$$

$$\begin{aligned} \frac{1}{C_{norm}} \Delta_V^{(\bar{q}q)} F_2^T(x; P^2) &= -\frac{32\pi^3}{81\Delta_V^6} \alpha_S (\bar{q}q)^2 x \exp(-\gamma_V) \times \\ &\times \left( x^2 + \bar{x}^2 - \gamma_V \left( x^2 + \frac{x+1}{2} \right) + \gamma_V^2 \frac{x}{2} \right); \quad (7) \end{aligned}$$

$$\begin{aligned} \frac{1}{C_{norm}} \Delta_V^{(\bar{q}q)} F_2^L(x; P^2) &= -\frac{32\pi^3}{81\Delta_V^6} \alpha_S (\bar{q}q)^2 x\bar{x} \exp(-\gamma_V) \times \\ &\times \left( \frac{2x-1}{2\gamma_V} - 3x - \frac{1}{2} + \gamma_V \frac{5x+1}{2} - \gamma_V^2 \frac{x}{2} \right). \quad (8) \end{aligned}$$

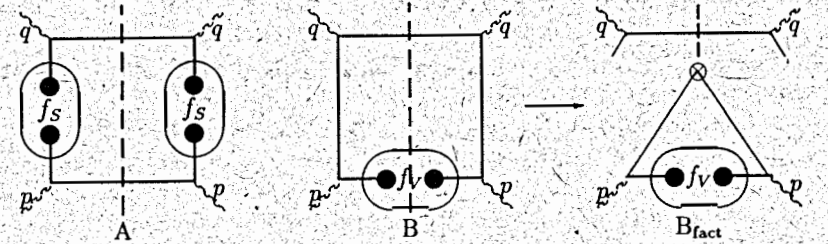


Figure 1: Diagrams with nonlocal VCs, essential for PhSF  $F_2(x)$ . To the left there is the diagram with scalar VCs; in the middle, the initial diagram with vector VC, to the right, its factorized form.

It should be noted that the methods used for calculating these contributions are different: for the scalar VCs the Cutkosky's approach was applied (it is equivalent to the double Borel transform method, for details see [10]), and for the vector VC we used the factorization of large and small distance contributions directly - the *Disc* of diagram is determined the by coefficient function of the process (i.e., at short distances). This difference is explained by the following. The exact calculation of the imaginary part of diagram 1B with the nonlocal vector VC with the  $\delta$ - Ansatz form (3) gives the exact zero. This is a simple consequence of the more general

**Proposition:**

If the weight  $f(\alpha)$  of some line of a diagram in  $\alpha$ -space is concentrated on the bounded<sup>3</sup> support, then the contribution to Disc of this diagram from the cut through this line is zero.

This can be proved by direct calculations in the case of "box" diagrams. The distribution  $f_V^{mod}(\alpha)$  (4) we use satisfies conditions of this proposition. Meanwhile there is a whole class of Ansatzes  $h_n(\alpha)$ , for which the nonlocal VCs:

- decay in the large  $|z|$  limit exponentially ( $\sim \exp(-\gamma|z|)$ );
- have the unbounded support in  $\alpha$ -space, e.g. :

$$h_n(\alpha) = \exp\left(\frac{-m_n^2}{\alpha}\right) \cdot \frac{\alpha^{-n}(m_n^2)^{n-1}}{\Gamma(n-1)};$$

- imitate the  $\delta$ -shaped Ansatz for large values of the parameter  $n$ ;

The imaginary part of diagram 1B for these Ansatzes isn't zero. But this quantity is essentially Ansatz-dependent. For this reason we use the factorization method, which is weak sensitive to the concrete choice of Ansatz (we could use the Ansatz  $h_n(\alpha)$  in all the calculations, but it would produce substantially complicated expressions and numerically not rather different results).

We see, that the nonlocal quark VCs gives smooth (over  $x$ ) contributions in all the region  $0 < x < 1$  and the parameter of nonlocality is located in the denominators of a common factors, which is a signal of nonperturbativeness of these corrections. We also want to emphasize that in the limit  $P^2 \rightarrow 0$  only contributions to  $F_2^L$  are singular whereas those related to  $F_2^T$  are regular.

3. For treating SF of a real photon it's necessary to realize in some way an extrapolation of the result obtained for  $P^2 \gg \Lambda_{QCD}^2$  to the region  $P^2 \rightarrow 0$ . First of all, we should remember that in this limit physical SF  $F_2^T$  and  $F_2$  coincide, and physical  $F_2^L \rightarrow 0$  (for details see [1]). So, we'll consider further only the transverse part of PhSF, i.e.  $F_2^T$ . Then, we can extrapolate to  $P^2 = 0$  our results for nonlocal quark VC contributions without any problem. Note that we don't pretend to describe the region of large  $P^2 \gg \lambda_p^2$  in (5)-(8), because this asymptotic regime is determined by the unknown details of distribution function  $f(\alpha)$  in large  $\alpha$  region. For treating singularities like  $1/P^4$  (from the gluon VC) and  $\log(Q^2/P^2)$  (from the perturbative part) which appear in OPE calculations in QCD, we used the method and model of [1]. By this method, the PhSF is represented via dispersion relations in  $p^2$  in terms of the contributions of physical states (vector meson ( $\rho$ ) + continuum) and the parameters of the model are chosen so that they correctly reproduce all the terms of OPE calculations.

<sup>3</sup>We are grateful to O. Teryaev for stimulating discussion on this point.

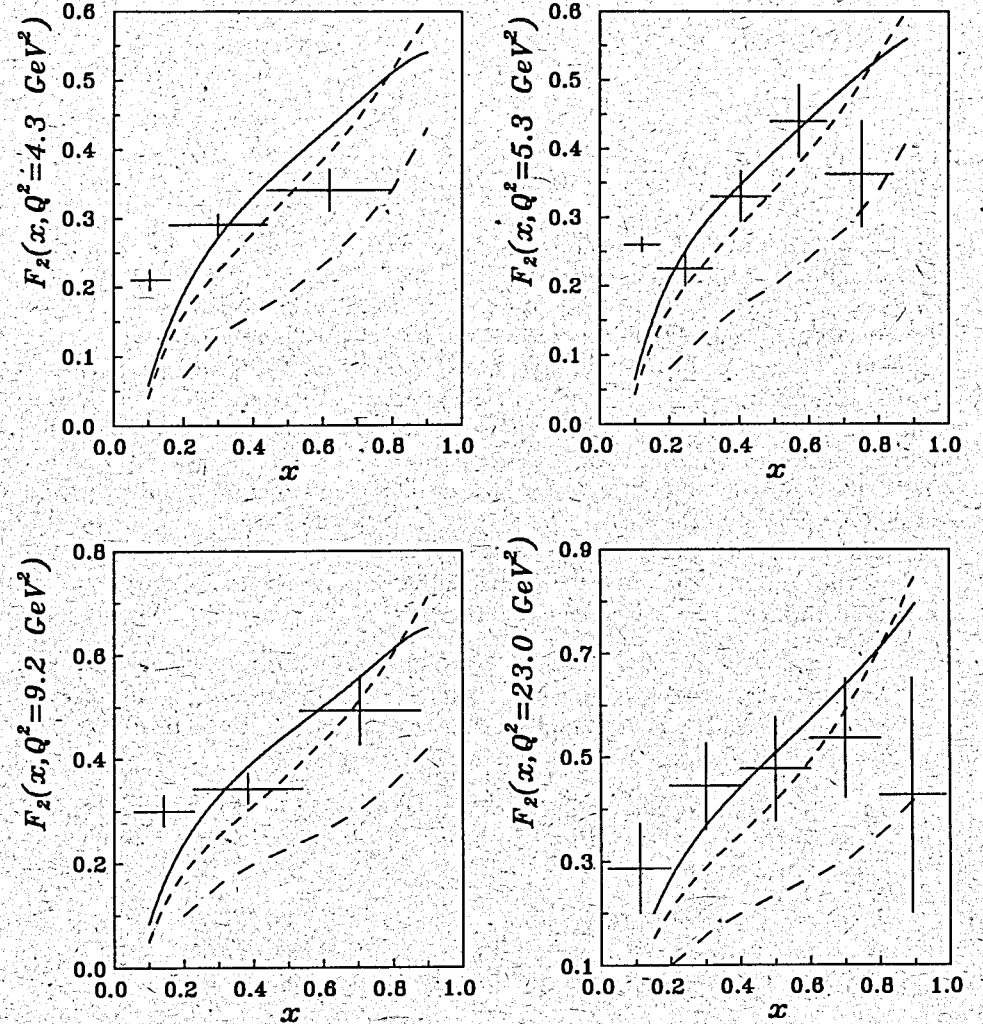


Figure 2: Comparison of the theoretical model predictions with experimental data for  $Q^2 = 4.3 \text{ GeV}^2$ ,  $Q^2 = 5.3 \text{ GeV}^2$  and  $Q^2 = 9.2 \text{ GeV}^2$  from [11] and for  $Q^2 = 23.0 \text{ GeV}^2$  from [12]. Solid line - our results; dashed - results of [1] and the lowest dashed (long dashes) - hadronic part contribution.

Then for the real PhSF we obtain:

$$\frac{1}{C_{norm}} F_2(x) = x \left\{ -1 + 6x\bar{x} + [x^2 + \bar{x}^2] \log \left( \frac{Q^2}{x^2 p_0^2} \right) + \left[ \frac{8\pi^4}{9\Delta^6} \langle \bar{q}q \rangle^2 \bar{x} - [x^2 + \bar{x}^2] \frac{32\pi^3}{81\Delta^6} \alpha_S \langle \bar{q}q \rangle^2 \right] - \frac{p_0^4}{2m_\rho^4} \left[ x^2 + \bar{x}^2 + \frac{8\pi^2}{27p_0^4 x^2} \left( \frac{\alpha_S}{\pi} GG \right) \right] \right\} \quad (9)$$

where  $p_0^2 \approx 1.5 \text{ GeV}^2$  is the standard value of continuum threshold in the QCD sum rule calculations of the  $\rho$ -meson properties and  $m_\rho$  is the  $\rho$ -meson mass. The quark contribution to hadronic part is shown in the first term of the second line in (9). To compare the new result (9) with experimental data [11], [12], we should include the evolution of the quark VC with  $Q^2$ :  $\langle \bar{q}q \rangle^2(\mu^2 \sim \text{GeV}^2) \rightarrow \langle \bar{q}q \rangle^2(Q^2)$ ,  $\langle \bar{q}g(\sigma G)q \rangle(\mu^2) \rightarrow \langle \bar{q}g(\sigma G)q \rangle(Q^2)$  following usual one loop evolution formulae (e.g. [13]). The quark terms lead to the growth of the hadronic part in the central region of  $x$  and happily works to the better agreement with the experiment, as represented in Fig. 2. Moreover, in the region of  $x \geq 0.8$  there is a weak tendency, due to quark VC corrections, to the lower growth of the curve. The best agreement with data is achieved for the value of  $\lambda_q^2 = 0.5 - 0.6 \text{ GeV}^2$ .

Nevertheless, the experimental errors should be reduced few times for comparing experiment with theory curve carefully. Further theoretical progress may be reached in the three ways:

1. To improve the hadronic model of PhSF (see [1]) by introducing new vector resonances. This step can improve the behavior of the theoretical curve at "large"  $x \geq 0.7$ .
2. To revise the expression for gluon corrections by taking into account the correlation length of gluon condensate [14], [15]. It will correct the theoretical curve in the region  $x \sim 0.2$ .
3. At  $x \sim 0$ , i.e. near the singularity in  $t$ -channel, the OPE series diverges [1]. But it may be corrected by reformation<sup>4</sup> of OPE in the way of cancellation of the "long distance" contribution following the approach in [16]

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<sup>4</sup>Authors are grateful to R.Ruskov for stressing this point.

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