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GAUGELESS REDUCTION OF GRAVITY AND EVOLUTION OF UNIVERSE

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The problem of the quantization of gravity from the beginnings until last time meets with the difficulty of definition of physical degrees of freedom of gravitational field [1]- [10]. The procedure of identification of physical variables and their separation from nonphysical ones has been called the reduction procedure. There are two ways for the realization of the reduction in the classical and quantum theories: *the gaugeless and the gauge-fixing*. In the former, independent physical variables are constructed by the explicit resolution of constraints. To avoid the difficulties with the resolution of very complete constraints in gravity commonly used, the latter, gauge –fixing, method is based on the introduction into the theory of some new " gauge constraints" [1], [9]. However, such coordinate fixation due to the nonlinear character of gravitation meets with the problem of determination of the class of "admissible gauges", which allows us to obtain gauge independent results [11]. Recall that the gauge equivalence theorem has been proved only for the asymptotical flat space time [7]. It seems to us that the last problem of definition of admissible gauges is not easier than the problem of resolution of constraints.

In the present paper we try to follow the gaugeless reduction [12]- [17] of gravity, based on explicit resolving of the classical Hamiltonian constraints for nonphysical field momenta and the corresponding fields coordinates .

The application of the gaugeless approach to the relativistic particle model is quite simple. The resolving of the mass-shell constraints for relativistic particle

$$\mathcal{H} = \frac{1}{2}(-p_0^2 + p_i^2 + m^2) = 0 \,, \tag{1}$$

leads to the notion of particle energy

$$p_0 = \pm\omega \,; \quad \omega = \sqrt{p^2 + m^2} \,, \tag{2}$$

and resolution of motion equation of corresponding coordinate gives us the definition of observable time. It is very attractive to transfer these clear notions of energy and

observable time for a relativistic particle to the case of gravity. We shall deal with this analogy and show that the resolution of constraints and corresponding equations of general gravity leads to the new notion of "spectral energy" $\mathcal{E}_s$ of the type of (2) and spectral time T_s as a variation of the reduced action with respect to the spectral energy

$$T_s = \frac{\partial W_s}{\partial \mathcal{E}_s} \,. \tag{3}$$

The main aim of our paper is to clear up the physical meaning of spectral energy and spectral time in gravity .

The paper is organized as follows. In section 2 we consider the method of gaugeless reduction for the examples of QED and relativistic quantum mechanics. We show that after the reduction both the theories contain only the observable gauge invariant variables (two transverse photons and the "time–reparametrization" invariant physical coordinates and spectral time, correspondingly). Section 3 is devoted to the calculation of the reduced action and spectral Hamiltonian for the system of gravitation and electromagnetic fields. The latter is used as the test of correct reduction. We investigate here the flat space–time limit of the reduced action. Section 4 is devoted to the spectral history of the quantum Universe.

1 Gaugeless reduction of Abelian gauge theory

Before considering a rather complicated case of gravity it is worth to illustrate the method of "gaugeless reduction " [12]- [17], by the simplest examples of free Maxwell field and relativistic mechanics.

Let us consider, first, the action for the Abelian gauge field

$$W[A] \;=\; -\int d^4x \frac{1}{4} F_{\mu\nu}F^{\mu\nu} \;=\; \frac{1}{2}\int d^4x [(\partial_0 A^i - \partial^i A_0)^2 - B_i^2] \,, \tag{4}$$

where

$$B_i = \epsilon_{ijk}\partial^j A^k \,.$$

There is Lagrangian constraint in the theory (Gauss law)

$$\Delta A_0 = \partial^i \partial_0 A_i. \tag{5}$$

It is easy to check that the electric tension on the constraint–shell (5) becomes functional from the gauge field

$$E_i[A] = \partial_0 A_i - \partial_i A_0[A] = \partial_0 A_i^T[A], \tag{6}$$

where

$$A_i^T[A] = \left(\delta_{ij} - \partial_i \frac{1}{\partial^2} \partial_j\right) A^j, \qquad \partial_i A_i^T[A] = 0. \tag{7}$$

The action (4) on the constraint-shell has the following form

$$W^{red}[A^T] = \frac{1}{2} \int d^4x [(\partial_0 A_a^T)^2 - (B_a(A^T))^2]. \tag{8}$$

As a result we obtain action in terms of the gauge invariant functionals $A^T[A + \partial\lambda] = A^T[A]$ and it contains only two observable transverse fields. Then we can pass to the Hamiltonian form for our theory by using the conventional Legandre transformation for physical coordinates $A_i^T[A]$. Thus we get the reduced phase space without exploiting any gauge, using the explicit resolving of constraint (5)

$$A_0[A_i] = \frac{1}{\Delta}(\partial^i A_i) \equiv -\frac{1}{4\pi} \int d^3y \frac{\partial^i \partial_0 A_i(y)}{|x-y|}. \tag{9}$$

The transversality constraint here arises as a result of reducing but not as an additional gauge requirement. As regards relativistic covariance, it is not manifest. It is easy to check that the Lorentz transformation of physical variables [15] has the form

$$A^T[A + \delta_L A] - A^T[\delta_L A] = \delta_L A^T + \partial_i \Lambda[A^T] \,,$$

where δ_L is the conventional Lorentz transformation with the parameter ε^k, which is supplemented by the gauge transformation:

$$\Lambda[A^T] = \varepsilon^k \partial_k \frac{1}{\Delta}(\partial_0 A^T) \,.$$

This form has been interpreted by Heisenberg and Pauli [18] (with reference to the unpublished note by von Neumann) as the transition from the Coulomb gauge with respect

to the time axis in the rest frame $\eta_\mu^0 = (1,0,0,0)$ to the Coulomb gauge with respect to the time axis in the moving frame

$$\eta_\mu = \eta_\mu^0 + \delta_L \eta_\mu^0 \,.$$

This fact reflects the Lorentz covariance of the reduced theory, and has been proved in quantum theory by B.Zumino [19].

For gravitational field there are more complicated Lagrangian constraints than (5). From this point of view it is useful to rewrite the initial action (4) in the first order formalism

$$W_I[A,E] = \int d^4x \left(E_i(\partial_0 A_i) + A_0 \partial_i E_i) - \frac{1}{2}((E_i)^2 + (B_i)^2) - \partial_i(A_0 E_i)\right). \tag{10}$$

In this form the action (10) describes the generalized Hamiltonian system, where E_i, A_i are the canonical conjugate variables, and A_0 is Lagrange factor. Conventionally, to fix this factor one uses a gauge constraint $F(A) = 0$. Instead of this we can repeat the described above procedure of the gaugeless reduction. It is clear that the Lagrangian constraint (5) can be got from the equation of motion for A_0

$$\frac{\delta W}{\delta A_0} = 0 \quad \Longrightarrow \quad \partial_i E_i = 0 \tag{11}$$

and the longitudinal part of the equation of motion for E_i

$$\frac{\delta W}{\delta E^i} = 0 \quad \Longrightarrow \quad E_i = (\partial_0 A_i - \partial_i A_0). \tag{12}$$

For the theory with vanishing the surface term $\partial_i(A_0 E_i)$, to remove all nonphysical components A_0, $\partial_i A_i$, $\partial_i E_i$ without gauge fixation it is enough to resolve explicitly only secondary constraint (11).

For two-dimensional Abelian theory it is known how to deal with the nonvanishing surface terms. In this case the explicit solutions of the constraint contain zero–modes as solutions of the corresponding homogeneous equations [13], [20]

$$E(x,t) = E_0(t) + E^T(x,t)\,, \quad \partial E^T = 0\,. \tag{13}$$

The existence of these zero–modes is connected with the nontrivial topological invariant of the Chern–Simons type. In quantum theory they correspond to the plane wave in the functional space of the type of the Coleman electric field [13] unlike the oscillator excitations of the transverse photons in QED.

2 Relativistic mechanics without gauge fixing

We have demonstrated the procedure of the gaugeless reduction for the singular theory with the Lagrangian constraint by example of free electromagnetic field. Now let us study this method for the singular theory without the Lagrangian constraint. The well-known example of such a theory is relativistic particle with the action

$$W[x] = -m \int_0^T d\tau \sqrt{\dot{x}_\mu^2}. \tag{14}$$

This action is invariant under reparametrization of time

$$\begin{aligned} \tau \to \tau' &= s(\tau), \\ x(\tau) \to x(\tau')' &= x(\tau) \end{aligned} \tag{15}$$

with $ds/d\tau > 0$. Therefore, there is an arbitrary function in the solution of equation of motion. So, beside fixing initial conditions, it is necessary to eliminate this function from the solution. The usual manner is gauge choosing. For example, the proper time fixing $x_0(\tau) = \tau$ leads to the instant form of dynamics for relativistic particle [4] . However, let us act in the spirit of the previous section and try to solve the problem without gauge fixing.

For our final aim – gaugeless reduction of the Einstein gravity – it is more transparent to rewrite action (14) in the following form

$$W[x,e] = 1/2 \int_0^T d\tau \left(\frac{\dot{x}_\mu^2}{e} + em^2 \right). \tag{16}$$

The phase space corresponding to the system (16) contains five variables (e, x_0, x_i) and their five canonical momenta (p_e, p_0, p_i). From (16) we obtain the primary constraint

$$p_e = 0 \, , \tag{17}$$

and the canonical Hamiltonian $H = e\mathcal{H}$, where $\mathcal{H}$ is defined from eq. (1). The Poisson bracket of this constraint and the Hamiltonian gives the secondary constraint

$$\{p_e, H\} = \mathcal{H} = 0. \tag{18}$$

Thus the initial action (16) is rewritten in the Hamiltonian form

$$W[x,e,p] = \int_0^T d\tau (p_0 \dot{x}_0 - p_i \dot{x}_i - e\mathcal{H}). \tag{19}$$

In accordance with the case of electromagnetic field let us express one of the momenta (p_0, p_i) in terms of the others

$$p_0 = \mp\omega(p); \qquad \omega(p) = \sqrt{p_i^2 + m^2}; \tag{20}$$

we shall call this quantity the "spectral energy", to distinguish it from (18) (in the Einstein theory of gravity (18) corresponds to the "energy density"). Note that the resolution of constraint with respect to p_0 corresponds to choice of the instant form of dynamics. In the present paper, we shall restrict ourselves only to this form of dynamics. As a result we get the following reduced action

$$W^{Red}[x_i, p_i] = \int_{x_0(0)}^{x_0(T)} dx_0 \left(\mp\omega(p) \; - \; p_i \frac{dx_i(x_0)}{dx_0} \right). \tag{21}$$

The reduced phase space contains only x_i, p_i as dynamical functions of x_0. The initial action (19), which is the functional from $x_0(\tau)$, transformed into the action (21) as the function from the boundary values $x_0(0), x_0(T)$. To elucidate this dependence, we can exploit the following equations of motion

$$\frac{\delta W^{Red}}{\delta x_i} = 0 \quad \Longrightarrow \quad \frac{d}{dx_0} p_i = 0. \tag{22}$$

The reduced action (21) on classical equations (22) is the function

$$W^C_\pm(X,p) = \mp\omega(p) X_0 + p_i X_i \tag{23}$$

of the global reparametrization invariants

$$X_\mu = x_\mu(T) - x_\mu(0). \tag{24}$$

From the point of view of the Hamilton – Jacobi theory just these invariants represent the observable time and coordinates. In the following we shall call them the spectral time and coordinates.

Now we can immediately write down the spectral representation for the wave function for our reduced system

$$\psi(X_i, X_0) = \int \frac{d^3\vec{p}}{(2\pi)^{3/2}\sqrt{2\omega}} \left\{ a^{(+)}(p) e^{iW^C_{(+)}} + a^{(-)}(p) e^{iW^C_{(-)}} \right\}. \tag{25}$$

Note that in the initial theory (14) there is a geometrical invariant of the proper time

$$dT_P = med\tau\ ; \quad (T_F(\tau') = T_F(\tau)), \tag{26}$$

which coincides with the spectral one for the lowest values of the spectral energy $\omega = m$. We would like to emphasize that the spectral time (unlike the mathematical one) has the absolute origin.

3 Gaugeless reduction of Einstein gravity

3.1 The Hamiltonian form

The previous examples give hope to fulfil the gaugeless reduction of gravitation theory without gauge fixation.

We start with the conventional scalar curvature action including the electromagnetic field to control the reduction procedure

$$W[\, g,\ A\,] = -\int d^4X\sqrt{-g}\left(\frac{1}{2\kappa^2}{}^{(4)}R(g) + \frac{1}{4}F_{\mu\nu}F^{\mu\nu}\right). \tag{27}$$

It is well known that the Einstein equations

$$\frac{\delta W}{\delta g_{0\mu}} = 0$$

are the Lagrange constraints. In the Hamiltonian approach they correspond to the secondary constraints, and the reduction consists in their explicit resolving with respect to the definite momentum and coordinate.

The Hamiltonian approach with the instant form of dynamics enforces us to assume that the space time manifold $\mathcal{M}$ can be represented as $\mathcal{M} = \mathcal{R}\times\Sigma$, where Σ is three-dimensional surface. The space time foliation is realized by introducing the so-called embedding variables $X(\mathbf{x}, t)$ [21] which are maps from a point $\mathbf{x}$ of the surface Σ to a space time point X of the manifold $\mathcal{M}$, and t labels the leaves of the foliation. This foliation leads to the well-known Dirac–Arnowitt–Deser–Misner (Dirac–ADM) metric [1], [4]

$$ds^2 = N^2(dt)^2 - a^2 h_{ik}(dx^i + N^i dx^0)(dx^k + N^k dx^0), \tag{28}$$

where N is the lapse function, N^i is the shift vector , a the "scale–space" component of metric, h_{ik} is the "graviton component" with determinant equal to unity:

$$\sqrt{-g} = Na^3\ , \quad \det(h_{ik}) = 1\ , \quad a \doteq \exp\mu\ . \tag{29}$$

The Einstein –Hilbert action (27) in terms of this metric possesses the manifest symmetry under the following group of transformations [22]:

$$\begin{aligned} t &\rightarrow t' = t'(t), \\ x^i &\rightarrow x^{i'} = x^{i'}(t, x^1, x^2, x^3)\ . \end{aligned} \tag{30}$$

Let us rewrite the action (27) in terms of the embeddings. The scalar curvature can be decomposed in three terms: a "kinetic" $\mathcal{K}$, the three-dimensional curvature ${}^{(3)}R$, and "surface" Σ:

$${}^{(4)}R = -\mathcal{K} + {}^{(3)}R + 2\Sigma, \tag{31}$$

$$\mathcal{K} = -6\frac{\overset{\odot}{\mu}{}^2}{N^2} + \frac{\overset{\odot}{h}{}^2}{4N^2}, \tag{32}$$

$${}^{(3)}R = \frac{4}{a^2}\left[h^{kl}\nabla_l\partial_k\mu + \frac{1}{2}\partial_k\mu\partial^k\mu\right] + \frac{1}{a^2}R(h), \tag{33}$$

$$\Sigma = \frac{1}{Na^3}\left[\partial_k\left[a\partial^k N + \frac{3a^3}{N}N^k\overset{\odot}{\mu}\right] - 3\partial_0\left(\frac{a^3\overset{\odot}{\mu}}{N^2}\right)\right], \tag{34}$$

where

$$\overset{\odot}{\mu} = \dot\mu - \frac{1}{3a^3}\partial_k(a^3N^k), \tag{35}$$

$$\overset{\odot}{h}{}^i_k = h^{il}\left(\dot h_{kl} - \nabla_l N_k - \nabla_k N_l + \frac{2}{3}h_{kl}\partial_i N^i\right), \tag{36}$$

$$R(h) = \frac{1}{4}\partial_i h^k{}_l(\partial^i h^l{}_k - 2\partial^l h^i{}_k) + \partial_k\partial_l h^{kl}. \tag{37}$$

The canonical momenta conjugated to μ, h, and A are the following

$$P_{(\mu)} = \frac{\partial\mathcal{L}}{\partial\dot\mu} = -\frac{6a^3\kappa^2}{N}\overset{\odot}{\mu}, \tag{38}$$

$$P^k_{(h)l} = \frac{\partial\mathcal{L}}{\partial\dot h^l_k} = \frac{a^3\kappa^2}{4N}\overset{\odot}{h}{}^k_l, \tag{39}$$

$$E_k = \frac{\partial\mathcal{L}}{\partial\dot A^k} = \frac{a}{N}\left(\dot A_k - \partial_k A_0 - N^l F_{lk}\right) = \frac{a}{N}\overset{\odot}{A}_k. \tag{40}$$

Here $\partial_j h^k{}_l = h^{ki}\partial_j h_{il}$, $N_l = h_{li}N^i$ and ∇_l is a covariant derivative in metric h_{ik}.

In terms of these variables the action (27) has the following form

$$W = \int d^3x dt\left[P_{(\mu)}\dot\mu + P^k_{(h)l}\dot h^l_k + E^k\dot A_k + A_0\partial_k E^k - N\mathcal{H} + N^k\mathcal{P}_k - \frac{\dot P_{(\mu)}}{2} - \partial_k S^k\right] \tag{41}$$

with the surface action term

$$S^k = +\frac{a}{\kappa^2}\partial^k N + 2P^k_{(h)l}N^l - \frac{1}{6}N^k P_{(\mu)} + A_0E^k. \tag{42}$$

In (41) $\mathcal{H}$ is the Einstein energy density:

$$\mathcal{H} = a^3\left[-\frac{\kappa^2}{2\cdot 6}\frac{P^2_{(\mu)}}{a^6} + \frac{4\kappa^2 P^2_{(h)}}{2a^6} + \frac{^{(3)}R}{2\kappa^2} + T^0{}_0(E)\right]; \tag{43}$$

and $\mathcal{P}$ is the momentum density:

$$\mathcal{P}_k = \frac{a^3}{3}\partial_k\left(\frac{P_{(\mu)}}{a^3}\right) + 2\nabla_l P^l_{(h)k} - E^l F_{lk}; \tag{44}$$

$T^0{}_0(E)$ is the electromagnetic energy density

$$T^0{}_0(E) = \frac{1}{2}\left(\frac{E_iE^i}{a^4} + \frac{F_{ij}F^{ij}}{2a^4}\right). \tag{45}$$

Now it is clear that the action (41) describes the generalized Hamiltonian dynamics for (μ, h_{kl}, A_k) and $P_{(\mu)}, P_{(h)kl}, E_k$ with Lagrange factors A_0, N_k, N, and the constraints

$$\mathcal{H} = 0, \quad \mathcal{P}_k = 0, \quad \partial_k E^k = 0. \tag{46}$$

Note that the action (41) differs from the ADM action by the surface terms, as will be seen later, they will be important for the definition of the spectral energy.

3.2 Reduction of phase space

We shall act in the direct analogy with the relativistic particle case and QED. As we verified, the resolution of constraints leads to the construction of gauge invariant variables (QED) and to the observable time as the global invariant of the reparametrization group (relativistic particle). This program for gravity has been realized in the framework of the cosmological perturbation theory on the level of the classical equations, with the choice of the conformal time [23]. Here we discuss the dynamical aspect of this program connected with the construction of " spectral Hamiltonian " and "spectral time". The main point is the resolution of the "energy" constraint $\mathcal{H} = 0$ against the space scale momentum

$$P_{(\mu)\pm} = \mp\mathbf{w}; \quad \mathbf{w} = \frac{a^3\sqrt{6}}{\kappa}\left[\frac{4\kappa^2 P^2_{(h)}}{a^6} + \frac{^{(3)}R}{\kappa^2} + 2T^0{}_0\right]^{1/2}. \tag{47}$$

The explicit resolution of constraints (46) generally allows one to express $P_{(\mu)}$ and μ as functionals from the physical variables $\Phi = (A, h), P_{(\Phi)} = (E, P_h)$ within the zero mode sector (compare with eq. (13))

$$\mu = \mu_0(t) + \mu_L[\Phi, P_{(\Phi)}]. \tag{48}$$

The explicit time dependence of μ is not defined in the same way as for the particle case, where the x_0 dependence remains unknown. Recall that the notion of observable time appears only after the resolving Hamiltonian constraints and motion equations.

On the constraints (46) the initial action (40) has the following reduced form

$$W^{Red}_\pm = \int d^3x\int_0^T dt\,[P_{(\Phi)}\dot\Phi \mp (\dot\mu\mathbf{w} - \frac{\dot{\mathbf{w}}}{2}) - \partial_k S^k]. \tag{49}$$

This expression is the basis for construction of the Hamiltonian scheme in terms of gauge invariant variables $P^I_{(\Phi)}, \Phi^I$

$$W^{Red}_{\pm} = \int d^3x \int_{T_s(0)}^{T_s(T)} dT_s \left[P^I_{(\Phi)} \frac{\partial \Phi^I}{\partial T_s} \mp \mathcal{H}_s[P^I{}_{(\Phi)}, \Phi^I] \right]. \qquad (50)$$

For the zero mode sector (in the homogeneous approximation) the global observable time can be introduced from the following condition

$$\frac{\partial \mathcal{H}_s}{\partial T_s} = 0 . \qquad (51)$$

The representation for the wave function of the reduced homogeneous system in terms of eigenfunctions of the spectral Hamiltonian

$$\mathcal{H}_s \Psi_s = \mathcal{E}_s \Psi_s$$

is

$$\Psi(\Phi^I) = \sum_s \left(A_s^{(+)} \exp\left(-i\mathcal{E}_s T_s(\mu_0)\right) \Psi_s(\Phi^I) + A_s^{(-)} \exp\left(i\mathcal{E}_s T_s(\mu_0)\right) \Psi^*{}_s(\Phi^I) \right). \qquad (52)$$

Here $T_s(\mu_0)$ describes the evolution of the quantum Universe with the absolute beginning of time $T_s(\mu_0)$. Below we consider the simplest examples of this evolution.

Before we would like to note that the surface term (34) (time derivative) and $\overset{\odot}{\mu} P_{(\mu)}$ on the constraints give us the following part of the spectral Hamiltonian

$$\overset{\odot}{\mu} P_{(\mu)} - \frac{1}{\kappa^2}\Sigma = \frac{6a^6 \overset{\odot}{\mu}}{\kappa^2 \mathbf{w}} \left[\frac{4\kappa^2 P_h^2}{\kappa^2 a^6} + T_0^0 \right]. \qquad (53)$$

In the flat space time limit from (53) we get the conventional action for electromagnetic field (8) in contrast with the ADM approach, where the full time derivative is neglected.

4 Spectral energy density and time

4.1 The Misner anisotropic Universe

In the limit of the small space scale factor a , (47) transforms to

$$\mathbf{w}(P_{(h)}) = 2\sqrt{6 P_{(h)}{}^2}. \qquad (54)$$

From (50) we conclude that

$$\mathcal{E}_s = \mathbf{w}(P_{(h)}) \quad \text{and} \quad T_s = \mu \qquad (55)$$

and the small earlier Universe is described by the action

$$W^{Red}_{(\pm)} = \int dx^3 \left[P_{(h)}{}^k_l \left(h^l{}_k(T,x) - h^l_k(0,x) \right) \mp \mathbf{w}(P_{(h)}) \left(\mu(T,x) - \mu(0,x) \right) \right] . \qquad (56)$$

The constraint $\mathcal{P}_k$ in this limit reduces to the condition of homogeneity

$$\nabla_l P^l_{(h)k} = 0 .$$

So, we get the Misner anisotropic model [24] –[25] with the following spectral representation for the wave function

$$\psi(\mu, h^l{}_k) = \int d^5(P^k_{(h)l}) \left[A^{(+)}_{(p)} e^{iW^{Red}_{(+)}} + A^{(-)}_{(p)} e^{iW^{Red}_{(-)}} \right] , \qquad (57)$$

The spectral time coincides with the logarithm of space scale

$$T_s = \log\left(\frac{a(T)}{a(0)} \right) ,$$

and has absolute beginning. The positive sign of time corresponds to the expansion of the Universe, and negative sign to the contraction of (anti) Universe. The "Observer" is seeing that the small Universe is created with finite volume and density and undergoes the inflation with respect to the "spectral time".

The inflation lasts till the size becomes so large that the next radiation term dominates.

4.2 Radiation dominance

At the radiation dominance stage

$$P_{(h)}{}^k{}_l = 0$$

the Universe is described by the following reduced action

$$W^{Red}_{(\pm)} = \int d^3x \int_0^T dt \left[E^i \dot{A}_i \mp \frac{E^2 + B^2}{2} \dot{T}_s \right], \qquad (58)$$

where the spectral energy density coincides with the conventional Hamiltonian electromagnetic density, and spectral time T_s in the homogeneous limit

$$^{(3)}R = -\frac{k}{a^2 r_0^2} \quad (k = \pm 1, 0) \ ; \partial_k a = 0,$$

is the conformal one η

$$\dot{T}_s = \frac{6a\dot{a}}{\kappa^2 \mathrm{w}} = r_0 \dot{\eta} \tag{59}$$

within the factor of the size of the Universe [16], [17].

In the flat space limit $\dot{T}_s = 1$ and the spectral Hamiltonian

$$H_s = \int d^3x \frac{E^2 + B^2}{2} \tag{60}$$

represents the generator of evolution . Recall that this Hamiltonian is obtained from $\mathrm{w}\dot{\mu} - \frac{\dot{\mathrm{w}}}{2}$ in the reduced action (49). The latter term is omitted in the ADM scheme.This is the reason why the flat limit for radiation cannot be reproduced in ADM approach.

4.3 The dust dominance

Finally, at the classical dust dominance stage

$$T_{(d)}{}_0^0 = \frac{M_{(d)}}{V_3(r_0)a^3}$$

it is easy to see [16],[17] that the reduced action (49) has the following form

$$W_{\pm}{}^{Red} = \mp \frac{M_{(d)}}{2} T_F(a). \tag{61}$$

Here the spectral time

$$T_F(a) = \int_0^T dt \frac{6a^2\dot{a}}{\kappa^2 \mathrm{w}} \tag{62}$$

coincides with the Friedmann (proper) time for any type of the Universe $k = 0, \pm 1$, as in the case of relativistic particle. Due to considering only the localizable part of the spectral energy in the representation (58) we got only one half the mass of the Universe in accordance with Tolman's result of 1930 [26]. In the quantum theory (52) the spectral energy $\mathcal{E}_s = 1/2M_{(d)}$ is a conserved quantity and represents the relict of the age of creation

of the Universe. It is naturally to suppose that at this moment the Universe had the size of its Compton length $M_{(d)}{}^{-1}$, which defined the minimal scale $a(0) = (\kappa M_{(d)})^{-1} \approx 10^{-60}$ in eq.(58).

5 Discussion

In the present paper, we have tried to fulfil the gaugeless reduction of the phase space of gravity. The main peculiarity of this reduction is the appearance of the concepts of the spectral energy and spectral time. These quantities have been obtained from all the surface terms of the initial Einstein action (including the total time derivative). In the quantum theory they correspond to the nonzero phases of the wave function. Just from these phases an " Observer" forms the spectral energy and spectral time.

We calculate these quantities for the set of simplest examples.

- In the limit of a small space scale component a of the metric, the "Observer" is observing that the creating Universe expands from the finite volume and density and it is filled with only the Misner anisotropic gravitons [24]. This Universe undergoes the inflation under the spectral time. The inflation corresponds to the energy density $1/a^6$, but not the de Sitter one.
- In the radiation stage, the reduced action has two correct limits. At the large cosmological scale limit the spectral time coincides with the conformal one. For the small scale (in the flat space time limit) the spectral energy is nothing else as the energy of transverse photons, in contrast with ADM scheme.
- In the stage of dust filled Universe, the "Observer" discovers that his spectral time, as the phase of the wave function, transforms to the classical Friedmann time, as the invariant interval (like the observable time for relativistic particle at rest transforms to the proper one).

The "Observer" sees the changes of the character of spectral time T_s in the process of the evolution of the Universe. Who is the "Observer ", whose conclusions strongly differ from the conclusions of a modern scientist [27]?

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