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D.I.Kazakov[1]

# ON A POSSIBLE EXPLANATION OF THE ORIGIN OF THE QUARK MASS SPECTRUM*

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[1]E-mail address: KazakovD@thsun1.jinr.dubna.su
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# 1 Introduction

The miracle of the quark and lepton mass spectrum has been a real challenge for theorists for many years. In the Standard Model the masses appear as a result of spontaneous symmetry breaking and have the form

$$m_i = y_i \cdot v \quad , \tag{1}$$

where $y_i$ are the corresponding Yukawa couplings to the Higgs field and $v$ is the Higgs field vacuum expectation value. Thus, the quark and lepton mass spectrum is actually that of the Yukawa couplings.

How to get this spectrum and what is the reason of its nondegeneracy? The answer to this question is still lacking. One should, however, have in mind that the masses of quarks and leptons in eq.(1) as well as all the parameters in the Standard Model are the running ones. Therefore, one usually talks about the value of the quark mass at the scale of the mass itself, i.e. $m_q^2 = \bar{m}_q^2(m_q^2)$. Thus we are interested in the spectrum of the Yukawa couplings at the scale of quark masses.

Following the hypothesis of Grand Unification that the symmetry increases at high energy it would be natural to assume that all Yukawa couplings are equal at the unification scale and then split when coming to lower energy scale thus defining the mass spectrum for quarks and leptons. Indeed, this phenomenon really takes place for the masses of the superpartners in the MSSM [1]. However, the renormalization group equations for the Yukawa couplings are different. Even if one assumes that the flavour symmetry is slightly broken at high energy it will restore at lower energy scale, i.e. the global flavour symmetry has the property opposite to the Grand Unification of the local symmetries [2]. Therefore, to get the nondegenerate spectrum of quark and lepton masses one has to input it at high energy.

One of the most interesting attempts of this kind is the one discussed in ref. [3], where the values of the Yukawa couplings as well as the Kobayashi-Maskawa mixing matrix at the unification scale are given in the form of the so-called *textures*, and then evolve to the observed values at low energies. The textures themselves are chosen according to maximal simplicity and symmetry while the needed parameters are fitted. Without denying this possibility to get the quark mass spectrum, in this paper we would like to suggest an alternative approach, which naturally arises in attempts to construct SUSY GUTs free from ultraviolet divergences [4], [5], [6].

## 2 SUSY GUT Scenario

While the Standard Model exploits the minimal version of the Higgs mechanism with only one Higgs doublet to provide masses to all quarks and leptons simultaneously, already in the minimal supersymmetric extension of the SM, the so-called MSSM, one needs at least two doublets. One doublet then provides masses to up quarks, while the other - to down quarks and leptons. Thus, we have two vacuum expectation values and their ratio $\tan\beta \equiv v_2/v_1$ is the free parameter of the model.

In the standard minimal SUSY GUT scenario [1] the theory possesses both the supersymmetry and the unified gauge symmetry at the unification scale with soft SUSY breaking terms arising from a supergravity. At this scale all quarks and leptons are massless and their superpartners all have the same mass. Going down to lower energies the superpartners masses run according to the RG equations, split due to different interactions and, thus, give us the mass spectrum at Tev scale. This is accompanied also by the radiative spontaneous symmetry breaking, which leads to the reconstruction of the vacuum state. The latter, according to the usual Higgs mechanism, provides us with the masses for quarks, leptons and $SU(2)$ gauge bosons and additional mass terms to their superpartners.

Quarks and leptons themselves are not involved in this process, since they are relatively light. Their mass spectrum remains completely arbitrary due to the arbitrariness of the corresponding Yukawa couplings. Having two Higgs vacuum expectation values with arbitrary $\tan\beta$ fitted by experiment does not change the situation. However, already here the value of $\tan\beta$ can be found from the minimization of the potential for neutral Higgses, if the parameters are known, and differs from unity [1]. Thus, we can get a hierarchy if the potential has various minima, though it is not essential when the Yukawa couplings remain arbitrary.

## 3 Finite SUSY Models

Whence we have already enlarged the number of Higgses, we can go further and consider some non-minimal model. At first sight this looks absolutely hopeless because of increasing number of arbitrary parameters. However, there is one exception. This is a SUSY GUT model which though non-minimal still remains almost as rigid as the minimal one. It is distinguished by its ultraviolet properties being absolutely UV finite to all orders of perturbation theory [4], [5]. Let us remind the main properties of a finite SUSY GUT:

- the number of generations is fixed by the requirement of finiteness,
- the representations and the number of the Higgs fields are fixed,
- all the Yukawa couplings are expressed in terms of the gauge one,
- the various realistic possibilities are given by $SU(5), SU(6), SO(10)$ and $E(6)$ gauge groups with few generations.

We consider below the simplest case of the gauge group $SU(5)$ [6]. Then only three generations are allowed and the Higgs sector contains one 24 representation, which breaks $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$, and four pairs of Higgses in 5 and $\bar{5}$ representations. Thus we have three extra pairs of Higgses compared to the MSSM.

After spontaneous breaking of $SU(5)$ one naturally achieves that one pair of Higgses obtains the mass of the order of $M_X$, while the other three split into doublets and triplets under $SU(2)$. Triplets become heavy while the doublets remain light due to the fine tuning. As a result below $M_X$ we get three pairs of light Higgs doublets: one for each generation.

Equation (1) for the quark masses is now modified. As we have already mentioned, all the Yukawa couplings are uniquely defined by the requirement of finiteness at the GUT scale. In the leading order of PT one has:

$$y_{D_i} = const \cdot g, \quad y_{U_i} = \sqrt{\frac{4}{3}}\, const \cdot g, \quad y_{L_i} = const \cdot g\ , \tag{2}$$

where $y_{D_i}, y_{U_i}$ and $y_{L_i}$ are the Yukawa couplings of down and up quarks and leptons, respectively, and g is the $SU(5)$ gauge coupling. In higher orders one has the calculable corrections to eq.(2) as a power series over $g$.

These values of the Yukawa couplings serve as the boundary conditions for the RG equations. Since the interactions are flavour symmetrical, the values of the Yukawa couplings at $M_Z$ are also flavour degenerate.

Then eq.(1) takes the form

$$m_{U_i} = y_U \cdot v_i, \quad m_{D_i} = y_D \cdot \bar{v}_i, \quad m_{L_i} = y_L \cdot \bar{v}_i\ , \tag{3}$$

where $v_i$ and $\bar{v}_i$ $(i = 1,2,3)$ are the v.e.v.s of the Higgs fields in 2 and $\bar{2}$ representations, respectively.

# 4 Quark Mass Spectrum

As one can see from eq.(3) the mass spectrum of quarks and leptons is now defined by the v.e.v.s rather than by the Yukawa couplings. In its turn the v.e.v.s themselves are the solutions of the minimization conditions for the Higgs potential.

At the GUT scale we start with the potential

$$V(H_i, \bar{H}_i) = V_{SUSY} \ + \ V_{Soft} \ , \tag{4}$$

which has a discrete symmetry of interchange $H \leftrightarrow \bar{H}$ and the generation symmetry. However, when running the parameters to the lower energies where spontaneous breaking of $SU(2)$ gauge invariance takes place, both these symmetries are destroyed. The reasons for this are two fold: different renormalization of $H$ and $\bar{H}$ fields and the Higgs mixing matrix $\mu_{ij}$, which is analogous to the Kobayashi-Maskawa mixing matrix of quarks but in the Higgs sector.

At the electroweek scale the potential for the neutral Higgs components takes the form:

$$\begin{aligned} V &= m_1^2|H_i|^2 + m_2^2|\bar{H}_i|^2 - 2\bar{H}_i\mu_{ij}H_j \\ &+ \frac{g^2+g'^2}{8}\left(|\bar{H}_i|^2 - |H_i|^2\right)^2, \qquad (i=1,2,3). \end{aligned} \tag{5}$$

Looking for the minima of the potential (5) one can find several solutions which are degenerate and spontaneously break the generation symmetry. Since this symmetry due to the presence of the mixing matrix $\mu_{ij}$ becomes discrete, no Goldstone bosons appear. Each solution creates the hierarchy of v.e.v.s and, hence, the hierarchy of masses.

This phenomenon happens even if $\mu_{ij}$ is diagonal, but asymmetry of $\mu_{ij}$ is needed to avoid the global $SO(3)$ generation invariance and appearance of Goldstone bosons.

To illustrate the idea we consider the simplified example with two Higgses, just like in the MSSM [1]. One has:

$$V = m_1^2H^2 + m_2^2\bar{H}^2 - 2\mu\bar{H}H + \frac{g^2}{8}(\bar{H}^2 - H^2)^2. \tag{6}$$

The minimization conditions are

$$\frac{\delta V}{\delta H} \ = \ 2m_1^2H - 2\mu\bar{H} - \frac{g^2}{2}(\bar{H}^2 - H^2)H = 0,$$

$$\frac{\delta V}{\delta \bar{H}} = 2m_2^2\bar{H} - 2\mu H + \frac{g^2}{2}(\bar{H}^2 - H^2)\bar{H} = 0 \tag{7}$$

Introducing the vacuum expectation values

$$\begin{aligned} < H > &= v_1 = v\cos\beta, \\ < \bar{H} > &= v_2 = v\sin\beta, \end{aligned}$$

the solution to eqs.(7) has the form

$$v^2 = \frac{4}{g^2}\frac{m_1^2 - m_2^2\tan^2\beta}{\tan^2\beta - 1}, \quad \sin 2\beta = \frac{2\mu}{m_1^2 + m_2^2}, \tag{8}$$

or

$$\tan\beta \equiv \frac{v_2}{v_1} = \frac{m_1^2 + m_2^2 \pm \sqrt{(m_1^2 + m_2^2)^2 - 4\mu^2}}{2\mu}. \tag{9}$$

The sign in eq.(9) depends on relative values of $m_1^2$ and $m_2^2$. One takes (+) if $m_1^2 > m_2^2$ and (−) in the opposite case. One of the solitions being the inverse of the other. When $m_1^2 = m_2^2$ only the trivial solution, $v_1 = v_2 = 0$, exists. Evolving the difference between $m_1^2$ and $m_2^2$ we spontaneously break the discrete flavour symmetry in the Higgs sector and create the hierarchy and, hence, the mass spectrum. Therefore, even in a symmetrical original potential at the GUT scale one can have asymmetric solutions.

The Higgs particles also obtain masses which are given by the diagonalization of the matrix of the second derivatives of the potential (6) and have the form:

$$\begin{aligned} m_{H_{1,2}}^2 &= \frac{1}{2}\Big[m_1^2 + m_2^2 + M_Z^2 \\ &\pm\sqrt{(m_1^2 + m_2^2 + M_Z^2)^2 - 4(m_1^2 + m_2^2)M_Z^2\cos^2 2\beta}\Big], \end{aligned} \tag{10}$$

where $M_Z^2 = g^2v^2/2$. One of the Higgses can be light while the other is heavy.

The same mechanism works in a realistic model. All the parameters defining the spectrum of masses, like $m_1^2, m_2^2, \mu_{ij}$, etc are then determined from the requirement of consistency of the model as in the MSSM [1]. The lightest Higgs particle plays the role of a single Higgs of the SM.

# 5 Conclusion

We have demonstrated that it is possibile to obtain a quark mass spectrum which arises as a result of spontaneous breaking of $SU(2)$ symmetry by different v.e.v.s of the Higgs fields. The generation symmetry is reduced to the discrete one by the mixing in the Higgs sector and then is spontaneously broken. One does not need to introduce the quark spectrum "by hand" either at low energy, or at the GUT scale. Since all the Yukawa couplings in the finite model are known at the unification scale and then run according to the known RG equations, the only free parameters coming from the Higgs potential, the mixing matrix $\mu_{ij}$ and the soft SUSY breaking terms. The number of arbitrary parameters does not exceed that of the MSSM and can be even less if the mixing in the Higgs sector is correlated (or identified) with that in the quark sector.

The detailed analysis within the SUSY GUT scenario is in progress and will be published elsewhere.

# References


[1] H.E.Haber and G.L.Kane, Phys.Rep. 117 (1985) 75;
R.Barbieri, Riv.Nuovo Cim. 11 (1988) 1;
L.E.Ibanez and G.G.Ross, Nucl.Phys. B368 (1992) 3;
G.G.Ross and R.G.Roberts, Nucl.Phys. B377 (1992) 571;
R.Arnowitt and P.Nath, Phys.Lett. 287B (1992)89, 289B (1992) 368;
J.Ellis, S.Kelly and D.V.Nanopoulos, Nucl.Phys. B373 (1992) 55.

[2] B.Pendleton and G.G.Ross, Phys.Lett. 98B (1981) 291;
D.I.Kazakov, JINR Preprint E2-82-880, Dubna 1982.

[3] P.Ramond, R.G.Roberts and G.G.Ross, Nucl.Phys. B406 (1993) 19;

[4] A.Parkes and P.West, Phys.Lett. 138B (1984) 99;
P.West, Phys.Lett. 137B (1984) 371;
D.R.T.Jones and L.Mezincesku, Phys.Lett. 138B (1984) 293, 136B (1984) 242;
S.Hamidi, J.Patera and J.H.Schwarz, Phys.Lett. 141B (1984) 349;

[5] A.V.Ermushev, D.I.Kazakov and O.V.Tarasov, Nucl.Phys. B281 (1987) 72;

D.I.Kazakov, Mod.Phys.Lett. A9 (1987) 663;
D.R.T.Jones, Nucl.Phys. B279 (1986) 153 .

[6] S.Hamidi and J.H.Schwarz, Phys.Lett. 147B (1984) 301;
J.E.Bjorkman, D.R.T Jones and S.Raby, Nucl.Phys. B259 (1985) 503;
J.Leon, J.Perez-Mercader, M.Quiros and J.Ramirez-Mittelbrunn, Phys.Lett. 156B (1985) 66

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