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# INCOMPATIBILITY OF THE GENERALLY ACCEPTED DEFINITION OF A MOVING SCALE LENGTH WITH INTERVAL LORENTZ INVARIANCE

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According to Minkowski's figure of speech, the relativity theory has established: "The three-dimensional geometry becomes a chapter of four-dimensional physics" /1/. The sense of this quotation is in fact that according to the relativity theory a time component is added to three space ones. In other words, the scale must be described by a four-fold quantity:

$$l^i = (c\Delta t, \Delta x, \Delta y, \Delta z) = (c\Delta t, \Delta x, 0, 0) \tag{1}$$

depending on the coordinate difference of its ends. Here we take into account that one space size is much greater than two others, and so they can be neglected. In addition to this, for simplicity the rod is oriented along the X-axis and the directions of the corresponding axes of different reference systems coincide.

The considerations that the longitudinal sizes of bodies change in motion appeared at least at the end of the last century. At the present time this effect is considered as a main result of relativity theory. This means that in the reference system, when the scale is moving, the length $l = \Delta x$ figuring in (1) must depend on its speed $v$. Therefore we rewrite (1) in the form

$$l^i = (c\Delta t, l^* f(\beta), 0, 0),$$

where $l^*$ is the rod length at rest, $\beta = v/c$.

If $l^i$ is a 4-vector, then, as known, its scalar square reads as

$$l^i l_i = -s^2 = c^2\Delta t^2 - l^{*2} f^2(\beta), \tag{2}$$

where $s$ is an interval. As we know, the interval is an invariant quantity, i.e., its value does not change, remains constant with changing the speed of motion of the reference system. In other words,

$$\Delta t = c^{-1}[s^2 + l^{*2} f^2(\beta)]^{1/2} = (l^*/c)[s_1^2 + f^2(\beta)]^{1/2} = f_1(\beta), \tag{3}$$

where $s_1 = s/l^*$, i.e., the time component $l^0$ must be with necessity the function of speed in order to ensure interval constancy.

1. From the standpoint of the above-said let us consider the generally accepted definition of a moving scale. Remind that according to this definition, the length $l_c$ of a moving scale is named the distance between simultaneous position of its ends. As known, its consequence is the contraction formula. The four-component quantity corresponding to this "time-space configuration" takes the form

$$l_c^n = (0, l_c, 0, 0) = [0, l^*(1-\beta^2)^{1/2}, 0, 0], \tag{4}$$

whence in accordance with (3), we have

$$l_c^0 = l^*[1 + s_1^2 - \beta^2]^{1/2} \tag{3'}$$

instead of $l_c^0 = 0$. Hence it appears that the traditional simultaneity condition of end position marks ($\Delta t = 0$) [1] of a moving scale, when its length is measured, contradicts eq. (3′).

Thus, one can say that the four-component quantity (4) is not a 4-vector /2/ [2], and thereby the generally accepted definition does not satisfy the Lorentz-covariance condition.

In the framework of the alternative definition (see, e.g.,/3/) based on the radar method of distance measurement we have

$$l_r^i = (\beta\gamma l^*, \gamma l^*, 0, 0), \tag{5}$$

where $\gamma = (1-\beta^2)^{-1/2}$. From here it follows with evidence that the demand of interval Lorentz invariance is undoubtedly fulfilled in this case. Or otherwise, the radar "time-space configuration" (4-length) is indeed a 4-vector.

2. Certainly, one could come much earlier to the conclusion of the non-covariance of contracted length leaning upon the long ago known fact of non-covariance expressions for energy ($G^0$) and momentum (G) of the electromagnetic field of a moving charge. Remind that for this we have an explicitly relativistic covariant equation:

$$G^i = \int T^{ik} dV_k . \tag{6}$$

Here $T^{ik}$ ik is the energy-momentum tensor of the electromagnetic field, $dV_k$ is a four-fold quantity representing the relativistic generalization of a space volume element (see, e.g., /4/). As the Lorentz covariance of $T^{ik}$ is beyond doubt, the non-covariance of $G^i$ in the left side of (6) must mean the non-covariance of $dV_k$ and, consequently, the non-covariance of longitudinal contracted size (length) forming its basis.

This difficulty (the "4/3 problem") is usually solved by introducing mechanical tensions. Otherwise, the non-covariant expressions:

$$G^0 = (1 + \frac{1}{3}\beta^2)\mathcal{E}^*\gamma, \quad \mathbf{G} = \frac{4}{3}\beta c\mathcal{E}^*\gamma \tag{7}$$

should be added by the corresponding expressions for mechanical quantities. They, evidently, should take the form

$$G_m^0 = -\frac{1}{3}\beta^2\mathcal{E}^*\gamma, \quad \mathbf{G}_m = -\frac{1}{3}\beta c\mathcal{E}^*\gamma. \tag{8}$$

Thus, we are obliged to introduce another "different" non-covariant quantiy $G_m^n$ that resembles a space-like 4-vector with a negative energetic component!

Note that similar difficulties do not arise at all in the framework of the concept of relativistic (radar) length (see, e.g.,/5/).

[1] Exactly, more general condition ($\Delta t = const$).

[2] Or otherwise, the contracted length $l_c$ is not a 4-vector component.



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