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DO FRAGMENT MULTIPLICITY DISTRIBUTIONS OBEY ANY KIND OF SCALING?

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Existence of a new gas phase of nuclear matter and second-order phase transition that manifested themselves in high energy nuclear multifragmentation are considered in high energy physics starting from papers [1, 2] where it was found that mass spectra of light nuclear fragments obey a power law $A_f^{-\tau}$. This law follows from Fisher's theory [3] of a gas condensation at the critical point, and it seems reasonable to think that nuclear matter obeys such a law. To obtain the law in the nuclear theory some attempts took place [4-8]. The perspective one was made in the percolation model of the nuclear multifragmentation [9, 10] (see additional Refs. in [11, 12]). Now the criterions of the phase transition proposed in this approach [13] are used at experimental studies [14-17]. In this paper we show that there may be another simple criterion- the shape of the fragment multiplicity distribution as a function of mass of fragmenting system.

It is obvious that large system can fragment in larger number of fragments than the smaller one and the same is true for corresponding fluctuations. So for comparison of fragmentation of different systems the corresponding multiplicity distributions must be scaled. It was first proposed by Koba, Nielsen and Olesen in paper [18] at study of multiplicity distributions of produced particles in hadron-nucleon interactions (KNO-scaling). They supposed that scaled multiplicity distributions had not any dependence on energy of colliding hadrons. Now this scaling is well established by experimental data. An analogous scaling was observed for ^{4}He-fragment multiplicity distributions in the fragmentation of light nuclei [19-22]. We will try to understand the nature of this scaling in framework of percolation model.

In a simple approach in the percolation theory we have points connected with each other via bonds or links. Two neighboring points are connected if there is a bond or link between them. A subset of the points is called cluster if there is a way through connected points that linked two arbitrary points of the cluster. An isolated point is called cluster too. In an initial state it is assumed that all points belong to the one linked cluster. Very often the state is represented by network or lattice.

The bonds are then broken with probability p_b (bond percolation), or the points are ejected with probability 1- p_s (site percolation). This leads to a destruction of the lattice into many clusters. In infinite lattice (for infinite cluster) there is a critical probability p_c such that at p_s (p_b) greater than p_c the infinite cluster cannot exist. The change of the cluster structure of the lattice at $p \approx p_c$ has very sharp character like phase transition. So, we can say that there are two phases. One, when infinite cluster exists and (for example) electrical current in a metallic network can percolate from one end of the net to the other, and second, when there are many small clusters and the electrical current cannot percolate.

The number of points in a cluster is called cluster size - S. Above critical point of percolation parameter p the cluster size distribution obeys power law $S^{-\tau}$, where τ depends on dimension of the lattice space.

As one can see the percolation theory deals with set of objects very like on the set observed in nuclear fragmentation reactions, and it seems reasonable to use the percolation theory to fit the experimental data. It was first proposed in papers [9, 10].

Now we have three variants of the percolation model of nuclear multifragmentation. First - the bond percolation model [9, 12, 13, ,18] where it is assumed that in an initial state the nucleons occupied the site of a finite cubic lattice. Each nucleon has 6 neighbors and 6 bonds. Due to interaction the bonds are broken with probability p_b, which depends on impact parameter [23]. In this approach the nuclear fragmentation is simulated by thermal destruction of a solid state. The second one is the site percolation model (see Refs. in [11]). Here one assumes that in an initial state the nucleons occupied the site of the lattice. Due to interaction some nucleons are ejected and vacancies appear in the lattice and destruct it. The ratio of the ejected nucleon number to the lattice size gives the probability $1-p_s$, which must depend on impact parameter [11]. The third one is the aggregation model [10, 24-29]. Here one assumes that the nucleons after fast cascade stage of interaction have randomly space positions in any volume. If two nucleons are at the distance lower than any r_c they are considered as connected nucleons. It is natural that first stage is described in some dynamical approaches (in cascade evaporation model [26, 29] or in a time - dependent Thomas Fermi approach [27] and so on).

All percolation models reproduce the mass yield curve of nuclear fragment. Some of them [11, 12, 26, 29] can describe fragment momentum distribution. But a question about a scaling of the fragment multiplicity distribution was not considered until now. The main characteristics of the fragment multiplicity distributions below, near and above critical point were studied in percolation model in papers [23,24]. It was shown that at critical point the fluctuations of the multiplicity became large.

To study the fragment multiplicity distribution more carefully we choose bond / site percolation models for finite flat square lattice. We were varying p from 0.05 to 1 with step 0.05. The lattice was destructed with the help of Monte Carlo method. 10,000 samples were collected for each lattice size and each value of p.

Fragment multiplicity distributions were studied in many experimental papers (see for example [19 -22, 30]). Most of them were performed using nuclear photoemulsion. In photoemulsion one can determine the charge of spectator fragments of projectile nuclei measuring ionization of secondary tracks and find the number of charged fragments - N_f. The frequency distribution of events with given N_f is called multiplicity distribution - $P_f(N_f)$. $P_f(N_f)=N_{ev.,N_f}/N_{tot}$, here $N_{ev.,N_f}$ is the number of events with N_f fragments and N_{tot} is the total number of events.

The average number of fragments is determined as

$$\langle N_f\rangle = \sum_{n=0}^{\infty} n\, P_f(n).$$

The second and third normalized moments are

$$C_2 = \sum_{n=0}^{\infty} n^2\, P_f(n)/\langle N_f\rangle^2, \qquad C_3 = \sum_{n=0}^{\infty} n^3\, P_f(n)/\langle N_f\rangle^3.$$

KNO scaling means that C_2, C_3....etc. have not any dependence on energy and the same is true for corresponding scaled multiplicity distributions - $\psi(N_f/\langle N_f\rangle)=\langle N_f\rangle P_f(N_f/\langle N_f\rangle)$. But these scaled functions can/cannot depend on mass number of fragmenting nucleus. The last possibility seems to us more competitive. Independence of ψ from energy is quite natural at high energy - nucleon-nucleon cross sections are independent of energy (E > 4-5 GeV) and so-called limiting fragmentation of nuclei takes place. Below we try to answer the question - Do the fragment multiplicity distributions obey any kind of KNO scaling?

Before going to calculation results let us make simple estimation of the behavior of $<N_f>$ as function of percolation parameter -p. It is obvious that at large values of p for site percolation model we have mainly one big fragment, so

$$p_f(0)=0, \quad p_f(1)=1-p_f(2), \quad p_f(2), \quad p_f(3)=p_f(4)=....=0,$$

where $p_f(n)$ is the probability to have n-fragment and $p_f(2)\simeq(1-p)^2$ for two dimensional square lattice. In this case

$$<N_f>=1+p_f(2), \qquad C_2\simeq1+p_f(2), \qquad C_3\simeq1+4p_f(2).$$

In the other limiting case $p\rightarrow0$ we have

$$p_f(0)=1-p_f(1), \quad p_f(1)\simeq1-p, \quad p_f(2)=p_f(3)=....=0,$$

$$<N_f>=p_f(1)\simeq1-p, \quad C_2\simeq1/p_f(1)\simeq1/(1-p), \quad C_3\simeq1/p_f^2(1)\simeq1/(1-p)^2.$$

If p is determined as a ratio of the number of occupied sites to the total number of sites (we used this model), C_2, $C_3 \rightarrow 1$ at $p\rightarrow0$. It *follows* that $<N_f>$ *(C_2 and C_3) increases with increasing* p to reach maximum value and then falling down to zero (unity).

In bond percolation model analogous relations are

$$p\rightarrow 0, \ p_f(0)=0, \ p_f(1)=1-p_f(2), \ p_f(2)\simeq p^2, \ p_f(3)=p_f(4)=....=0,$$

$$<N_f>=1+p_f(2)\simeq1+p^2, \quad C_2\simeq1+p^2, \quad C_3\simeq1+4p^2 \ ,$$

$$p\rightarrow 1, \ p_f(0)=0, \ p_f(1)=0, \ p_f(4)=....=0, \ ..., \ p_f(N_s-1)\simeq1-p,$$

$$p_f(N_s)=1-p_f(N_s-1),$$

$$<N_f>=N_s-p_f(N_s-1)\simeq N_s-1+p, \ C_2\simeq1+p_f(N_s-1)/N_s^2, \ C_3\simeq1+3p_f(N_s-1)/N_s^2,$$

where N_s is the total number of sites.

In Fig. 1 the mean number of all fragments is represented as *function of percolation parameter p for different lattice sizes.* As one can see the behavior of the curves is the same as we were expected. It is observed that the number of fragments is very restricted in site percolation approach. However, in the bond percolation model the number of fragments can reach maximum value N_s.

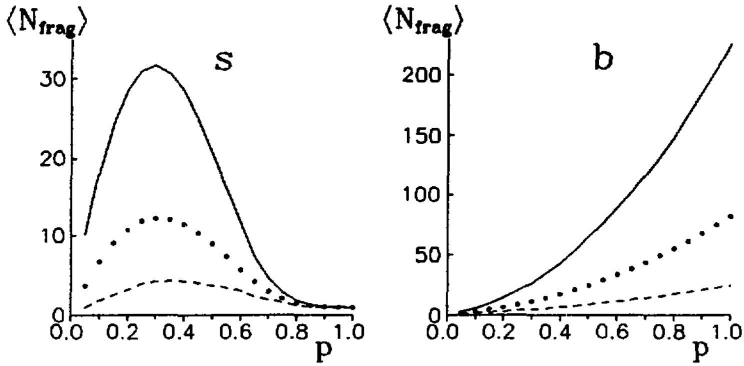

Fig. 1. Mean number of all fragments as function of percolation parameter p calculated for 5x5 (---), 9x9 (...) and 15x15 (——) lattice sizes using site (left) and bond (right) percolation models

In Fig. 2 the second and third moments of total fragment multiplicity distribution as function of percolation parameter in the site and bond percolation models are shown. As one can see in site percolation model C_2 and C_3 have different dependence on lattice size above and below critical value of p (p_c=0.593). Below critical value normalized moments decrease with increasing lattice size at given p, and inverse situation we have at $p > p_c$. At P=0.7 there can be KNO scaling because at this point C_2 and C_3 for different lattices are nearly the same.

In bond percolation model all moments must decrease with increasing lattice size, except in the region of p≃ 0.1. At p≃0.1 KNO scaling exists. Because mass yield curves were described at critical value of percolation parameter, we predict that moments will decrease with increasing mass number of fragmenting nuclei.

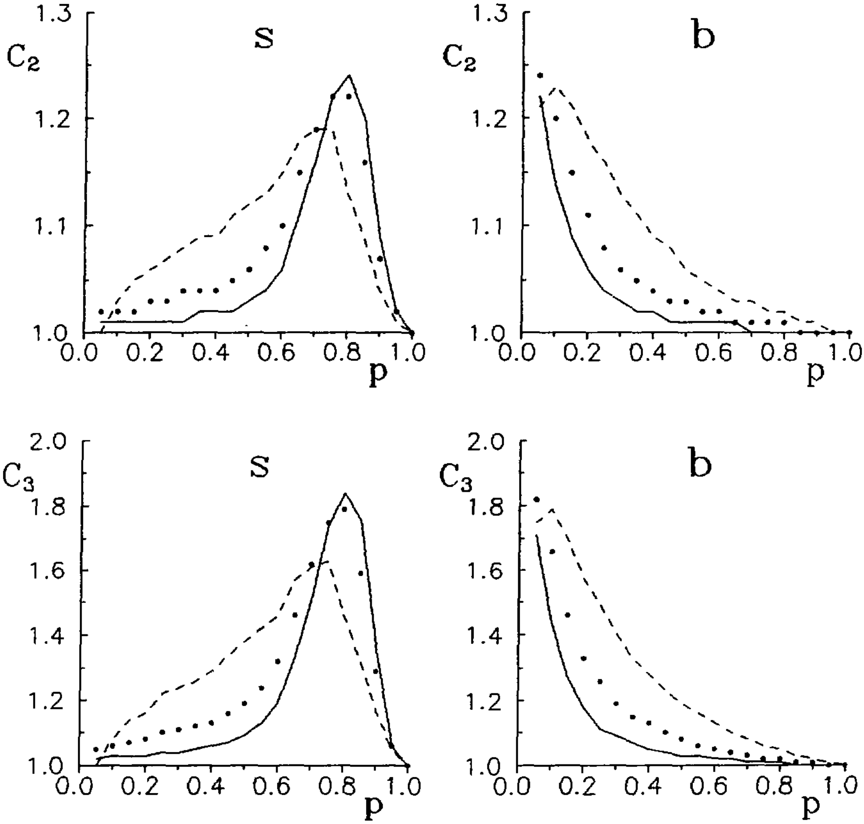

Fig. 2. Second and third moments as a function of p for different lattice sizes (notations are the same as on fig, 1) calculated using site and bond percolation models

Summing up, it is clear that a study of multiplicity distributions of all charged fragments can give information about a state of fragmenting system. We think that this can be tested using existing experimental data.

Let us go to the scaling of α-particle multiplicity distributions. The corresponding experimental data were obtained and reported in papers [19-22] (see Fig.3).

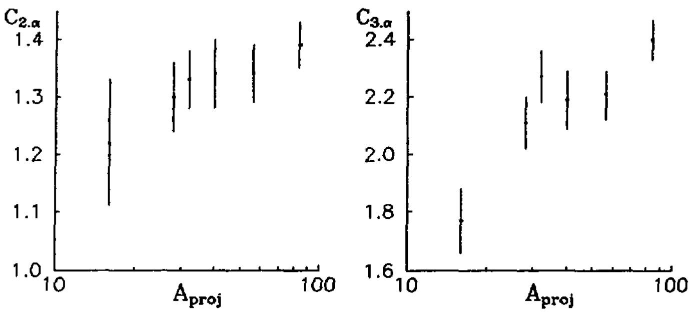

Fig. 3. Experimental second (left) and third (right) normalized moments of α-particles multiplicity distributions [21] as function of projectile mass number

The authors of these papers have used the following determination of average multiplicity and normalized moments

$$\langle N_\alpha\rangle = \sum_{n_\alpha=1}^{\infty} n_\alpha P_\alpha(n_\alpha) / \sum_{n_\alpha=1}^{\infty} P_\alpha(n_\alpha),$$

$$C_{2,\alpha} = \sum_{n_\alpha=1}^{\infty} n_\alpha^2 P_\alpha(n_\alpha) / \langle N_\alpha\rangle^2, \quad C_{3,\alpha} = \sum_{n_\alpha=1}^{\infty} n_\alpha^3 P_\alpha(n_\alpha) / \langle N_\alpha\rangle^3.$$

In order to compare our calculations with experimental data these determinations were used and fragments with length from 4 to 6 were considered as "α"-fragments. The reason for this is that we did not take into account the charge of nucleons and other nuclear binding effect.

In Fig. 4 the average multiplicity of "α"-fragments as a function of percolation parameter is given. According to experimental data of Jain et al. [21] for light nuclei fragmentation $\langle N_\alpha \rangle \simeq 1.7$. This value is reached in the site percolation approach only at p=0.55. But at the same time experimental value for Kr fragmentation into α-particles cannot be reached. So, the site percolation model cannot be applied. The same conclusion holds if one looks at normalized moments presented in Fig.5.

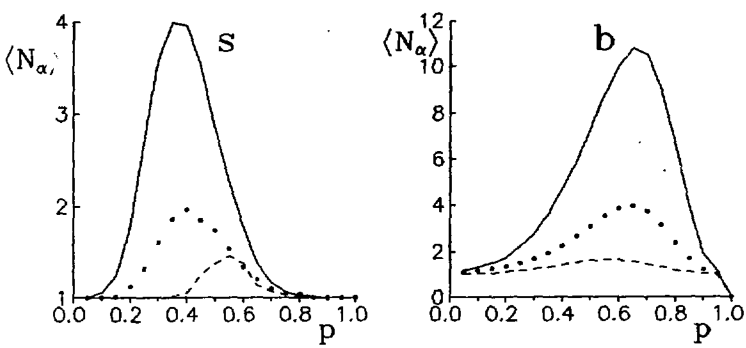

Fig. 4. Mean number of "α"-fragments as a function of p for site and bond percolation models

The experimental values for the average number of α-particles can be reached in the bond percolation model at p≃0.6 and 0.5 for light and Kr nuclei respectively (see Fig. 5). In the former case the corresponding value for $C_{2\alpha} \simeq 1.2$ which is just near experimental values for light nuclei [21]. However, for Kr $C_{2,\alpha}$ reaches only 1.25 which is lower than the experimental value [19]. In the case of third moments we have 1.75 and 1.82 for light and Kr nuclei,

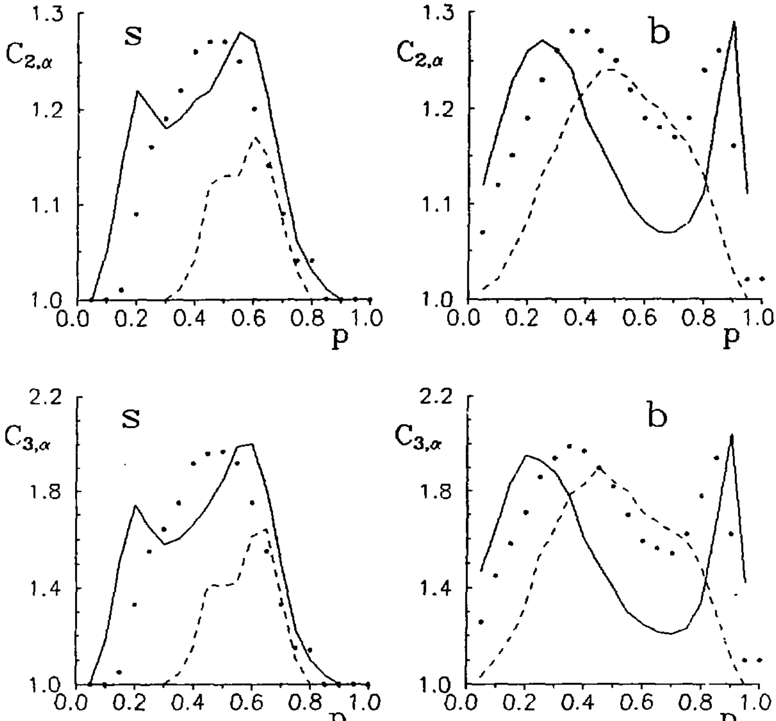

Fig. 5. Second and third moments of "α"-fragments multiplicity distributions as a function of p for site (s) and bond (b) percolation models

respectively. Although these values of second and third moments do not agree exactly with the experimental data, it has the tendency to increase slightly with increasing lattice size just as the experimental tendency of the data- cf. Figs. 3 and 5. Note that, moments become lower for larger lattices. It seems to us that the new experimental data about Au fragmentation will be very useful in this situation. According to our calculations for Au fragmentation, we predict that the values of $C_{2\alpha}$ and $C_{3\alpha}$ will be lower than those of Kr, as shown on fig 5, and $<N_{\alpha}>$ will be nearly 4-6 (see Fig. 4). These predictions can be tested in Au + Em interactions at Brookhaven energy. We think that the usage of more refined percolation models will not change our results drastically.

SUMMARY

The Scaling property of the fragment multiplicity distributions is studied in the framework of two-dimensional percolation models. It was shown that:

1- In site percolation model the multiplicity distributions of all fragments have something like KNO scaling at p=0.7. Below critical value normalized moments decrease with increasing lattice size at a given p and vice versa in the case p> p_C. In bond percolation model all moments must decrease with increasing lattice size.

2- $<N_{\alpha}>$ cannot be described in site percolation model.

3-In bond-percolation model, the second and third moments of α-particles produced from light and Kr nuclei have the tendency to increase slightly with increasing lattice size as the experimental data have. However, in the case of heavier nuclei (e. g. Au) we expect that the second and third moments will be lower than Kr nucleus.

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