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EXACT SOLUTION OF 2D POINCARE GRAVITY COUPLED WITH FERMION MATTER

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1. The numerous recent attempts to formulate the theory of gravity in the framework of a consistent gauge approach resulted in constructing the gauge gravity models for the de Sitter and Poincaré groups (for a review see, e.g., [1]). The independent variables are now vielbeins $e^a = e^a_\mu dx^\mu$ and Lorentz connection one-form $\omega^a{}_b = \omega^a{}_{b,\mu} dx^\mu$. These methods being appplied in two dimensions, give us an dynamical description of 2D gravity. It was argued that investigation of simple two-dimensional model leads to a better understanding of four-dimensional gravity and its quantization [2]. It was shown in [2] that the Lagrangian $L = \gamma R^2 + \beta T^2 + \lambda$ is the most general one quadratic in curvature R and torsion T, and containing a cosmological constant λ. The classical equations of motion for this type of two-dimensional gravity were analyzed in conformal gauge [3] and in light cone gauge [4] and their exact integrability was demonstrated. The various aspects of quantization of the model were recently considered in [5]. In ref.[6] was shown that the formulation of the model on the language of differential forms is very useful. This allows to find exactly the solution of vacuum gravitational equations using an appropriate (and rather natural) coordinates on the 2D space-time. The resulting metric can be written in the Schwarzschild-like form and describes asymptotically de Sitter black hole configuration [6]. Using this method in [7] one proves the integrability of the general 2D Poincaré gauge gravity with Lgrangian being an arbitrary (not necessary quadratic) function of curvature and torsion and demonstrates that the field equations is again of the black hole type.

The coupling with matter in general case breaks this exact integrability. One exceptional case noted in [6,7] is the 2D Yang-Mills field. In this letter we consider the coupling the 2D Poincaré gauge gravity with 2D massless Dirac fermions and show that the resulting field equations are exactly integrated by means the method of ref.[6].

2. We begin with brief description the Poincaré gauge gravity and Dirac spinors in two dimensions [1]. In this letter we follow notations

[1]The exhausted introduction to 2D Dirac spinors one can find in [7]

of paper [6]. The 2D gauge gravity is described in terms of zweibeins $e^a = e^a_\mu dz^\mu, a = 0,1$ (the 2D metric on the surface M^2 has the form $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$, $\eta_{ab} = diag(+1,-1)$) and Lorentz connection one-form $\omega^a{}_b = \omega\varepsilon^a{}_b$, $\omega = \omega_\mu dz^\mu$ ($\varepsilon_{ab} = -\varepsilon_{ba}$, $\varepsilon_{01} = 1$). The curvature and torsion two-forms are:

$$R = d\omega, \quad T^a = de^a + \varepsilon^a{}_b \omega \wedge e^b \tag{1}$$

With respect to the Lorentz connection ω one can define the covariant derivative ∇ which acts on the Lorentz vector A^a as follows

$$\nabla A^a := dA^a + \varepsilon^a{}_b \omega \wedge A^b$$

The Dirac matrices γ^a, $a = 0,1$ in two dimensions satisfy the relations:

$$\gamma^a \gamma^b = \eta^{ab} - \varepsilon^{ab}\gamma_5 \tag{2}$$

where $\gamma_5 = \gamma^0\gamma^1$, $(\gamma_5)^2 = 1$. The following identities are also useful:

$$\gamma^a\gamma_5 + \gamma_5\gamma^a = 0 \tag{3}$$

and

$$\gamma^a\gamma_5 = \varepsilon^a{}_b \gamma^b \tag{4}$$

In further consideration we use explicit realization of γ-matrices:

$$\gamma^0 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \quad \gamma^1 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} \quad \gamma_5 = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix} \tag{5}$$

The Dirac spinors in two dimensions have two complex components:

$$\Psi = \begin{pmatrix} \psi_1 \\ \psi_2 \end{pmatrix} \tag{6}$$

and under local Lorentz rotation (on angle Ω) transform as follows

$$\Psi \to \Psi' = S\Psi, \quad \bar\Psi \to \bar\Psi' = \bar\Psi S^{-1} \tag{7}$$

where the Dirac conjugated spinor is defined as $\bar{\Psi} = \Psi^+\gamma^0$. Matrix S realizing the spinor representation of 2D Lorentz group is given by

$$S = \cosh[\frac{\Omega}{2}] - \gamma_5 \sinh[\frac{\Omega}{2}] \tag{8}$$

One can see that components ψ_1 and ψ_2 transform independently:

$$\psi_1' = e^{\frac{\Omega}{2}}\psi_1, \quad \psi_2' = e^{-\frac{\Omega}{2}}\psi_2 \tag{9}$$

This means that left(right)-chiral spinors defined as

$$\gamma_5\Psi = \mp\Psi \tag{10}$$

give us the irreducible representations of the Lorentz group.

It is useful to define the covariant spinor derivative ∇ as differential operator acting on the field Ψ considered as zero-form with values in two-dimensional complex spinor space:

$$\nabla\Psi := d\Psi + \frac{1}{2}\omega\gamma_5\Psi, \quad \nabla\bar{\Psi} := d\bar{\Psi} - \frac{1}{2}\omega\bar{\Psi}\gamma_5 \tag{11}$$

This definition means that operator ∇ acts on spinor biliniear combinations, such as $\bar{\Psi}\Psi$, $\bar{\Psi}\gamma^a\Psi$, $\bar{\Psi}\gamma^{[a}\gamma^{b]}\Psi$, as usual covariant derivative on Lorentz scalar, vector and bivector correspondingly. One can see from (11) that spinor covariant derivative ∇ acts on components of spinor field (6) as follows

$$\nabla\psi_1 = d\psi_1 - \frac{1}{2}\omega\psi_1 \; ; \; \nabla\psi_2 = d\psi_2 + \frac{1}{2}\omega\psi_2$$

3. The dynamics of 2D gravitational (e^a, ω) and fermion (Ψ) variables is determined by the action:

$$S = S_{gr} + S_{fer}, \tag{12}$$

where

$$S_{gr} = \int_{M^2} \frac{\alpha}{2} * T^a \wedge T^a + \frac{1}{2} * R \wedge R - \frac{\lambda}{4}\varepsilon_{ab}e^a \wedge e^b \tag{13}$$

is standard action of 2D Poincaré gauge gravity quadratic in curvature and torsion; $*$ is the Hodge dualization and α, λ are arbitrary constants.

The action for 2D Dirac fermions in terms of differential forms can be written as follows:

$$S_{fer} = \int \frac{i}{2}\varepsilon_{ab}e^a \wedge (\bar{\Psi}\gamma^b\nabla\Psi - \nabla\bar{\Psi}\gamma^b\Psi) \tag{14}$$

Notice that in this letter we consider only the massless fermions. One can see that due to identity (3) the Lorentz connection ω is dropped out from expression (14) and really one can use the usual external derivative d instead of ∇ in (14).

Instead the curvature R and torsion T^a two-forms let us consider the dual zero-forms $\rho = *R,\ q^a = *T^a$.

The variation of (12) with respect to the Lorentz connection ω and zweibeins e^a gives the following equations

$$d\rho = -\alpha q^a \varepsilon_{ab} e^b \tag{15}$$

$$\nabla q^a = -\frac{1}{2\alpha}\Phi(\rho, q^2)\varepsilon^a{}_b e^b + J^a. \tag{16}$$

where $q^2 = q^a q^b \eta_{ab}$. In (16) the following notation was introduced: $\Phi(q^2, \rho) = \rho^2 + \alpha q^2 - \lambda$. The matter one-form J^a takes the form:

$$J^a = -\frac{i}{2}\varepsilon^a{}_b(\bar{\Psi}\gamma^b\nabla\Psi - \nabla\bar{\Psi}\gamma^b\Psi) \tag{17}$$

It should be noted that $J^a = J^a_\mu dx^\mu$ is related with matter energy-momentum tensor: $T_{\mu\nu} = \frac{1}{2}(\varepsilon_\mu{}^\alpha J^a_\alpha e^a_\nu + \varepsilon_\nu{}^\alpha J^a_\alpha e^a_\mu)$.

Variation of action (12) with respect to fermion field Ψ gives equation:

$$(e^a\varepsilon_{ab}\gamma^b) \wedge (\nabla\Psi) = \frac{1}{2}T^a\varepsilon_{ab}\gamma^b\Psi \tag{18}$$

From (1) we obtain that

$$\omega = \tilde{\omega} - q_a e^a \tag{19}$$

where $\tilde{\omega}$ is torsionless part of the Lorentz connection:

$$de^a + \varepsilon^a{}_b \tilde{\omega} \wedge e^b = 0 \tag{20}$$

Using (19) and identity (4) the equation (18) can be rewritten as follows

$$(e^a \varepsilon_{ab} \gamma^b) \wedge (d\Psi + \frac{1}{2}\tilde{\omega}\gamma_5 \Psi) = 0 \tag{21}$$

i.e. the torsion is dropped in the Dirac equation. Taking the Hodge dualization of (21) one can transform (21) to more standard form of the Dirac equation:

$$\gamma^\mu(\partial_\mu + \frac{1}{2}\tilde{\omega}_\mu \gamma_5)\Psi = 0$$

where $\gamma^\mu = e^\mu_a \gamma^a$.

Using the Dirac equation (21) one can show that one-forms J^a satisfy following identities:

$$J_a \wedge e^a = 0, \quad \varepsilon_{ab} J^a \wedge e^b = 0 \tag{22}$$

Really (23) are consequences of invariance action (14) under local Lorentz and conformal transformations correspondingly [8].

The components of the spinor field (6) can be written as $\psi_i = e^{\chi_i}$, $i = 1, 2$, where $\chi_1 = \beta + iv$, $\chi_2 = \gamma + iu$ are complex fields. Then the one-forms J^a (17) take the form

$$J^0 = [e^{2\gamma} du - e^{2\beta} dv], \quad J^1 = [e^{2\gamma} du + e^{2\beta} dv], \tag{23}$$

while the Dirac equation (18) reads

$$\begin{aligned} (e^0 - e^1) \wedge (idu + d\gamma + \frac{1}{2}\omega)e^\gamma &= \frac{1}{2}(T^0 - T^1)e^\gamma \\ (e^0 + e^1) \wedge (idu + d\gamma - \frac{1}{2}\omega)e^\beta &= \frac{1}{2}(T^0 + T^1)e^\beta \end{aligned} \tag{24}$$

4. Assume that the orthonormal basis $\{e^a\}$ takes the conformal-Lorentz form:

$$e^a = e^\sigma(n^a d\tau - \varepsilon^a{}_b n^b dx) \tag{25}$$

where $n^a, a = 0,1$ is unite Lorentz vector, $n^2 = n^a n_a = \pm 1$. By means of diffeomorphism transformations in two dimensions arbitrary basis $\{e^a\}$ always can be transformed to the form (26).

The corresponding metric $ds^2 = \eta_{ab} e^a_\mu e^b_\nu dx^\mu dx^\nu$ takes the conformally flat form:

$$ds^2 = n^2 e^{2\sigma}(d\tau^2 - dx^2)$$

By means the identity

$$A \wedge e^a = \varepsilon^a{}_b e^b \wedge (*A), \tag{26}$$

where A is arbitrary one-form, we get for differential of (26):

$$de^a = \varepsilon^a{}_b(- * (d\sigma) + n^\alpha \varepsilon_{\alpha\beta} dn^\beta) \wedge e^b. \tag{27}$$

Inserting (29) into (20) we obtain for $\tilde{\omega}$:

$$\tilde{\omega} = *(d\sigma) - n^a \varepsilon_{ab} dn^b. \tag{28}$$

Assuming for definitness that $n^2 = 1$, components n^a can be written as $n^0 = \cosh\theta$, $n^1 = \sinh\theta$. So we have that $n^a \varepsilon_{ab} dn^b = d\theta$. Under local Lorentz rotation on angle Ω variable θ transforms as $\theta \to \theta - \Omega$. So the last term in (30) is pure gauge part of the Lorentz connection.

Substituting the expression (30) into the Dirac equation (21) and using identities (4) and (28) we get

$$(e^a \varepsilon_{ab} \gamma^b) \wedge (d + \frac{1}{2} d\sigma - \frac{1}{2} d\theta \gamma_5)\Psi \tag{29}$$

or in spinor components (6):

$$\begin{aligned} (e^0 + e^1) \wedge (d + \frac{1}{2} d\sigma \psi_1 + \frac{1}{2} d\theta)\psi_1 = 0 \\ (e^0 - e^1) \wedge (d + \frac{1}{2} d\sigma \psi_1 - \frac{1}{2} d\theta)\psi_2 = 0 \end{aligned} \tag{30}$$

For basis (26) we have

$$(e^0 \mp e^1) = e^{(\sigma \mp \theta)}(d\tau \pm dx)$$

Taking this into account, the equations (32) are easily solved and we obtain for the spinor field:

$$\Psi = e^{-\frac{\sigma}{2}} \begin{pmatrix} e^{-\frac{\theta}{2}} e^{iv(x^-)} p(x^-) \\ e^{\frac{\theta}{2}} e^{iu(x^+)} f(x^+) \end{pmatrix} \tag{31}$$

where v, p and u, f are arbitrary functions of the light-cone coordinates $x^- = \tau - x$ and $x^+ = \tau + x$ correspondingly.

Thus the Dirac equation (18),(21), taken separately, is exactly solved in the conformal-Lorentz gauge (26) and general solution takes the form (34). However, now one must put the (34) in the gravitational eqs.(15),(16) and find the joint solutions of the coupled gravitaty-Dirac system.

5. As in vacuum case [6], there are two types of solutions of eqs.(15)-(18). The first one is characterized by that the torsion squared is zero on two-dimensional space-time, $q^2 \equiv 0$. One can see from eqs.(15)-(18) that it is possible only in the case when torsion is identically zero: $q^a \equiv 0,\ a = 0, 1$, the space-time has constant curvature [2] : $\rho^2 = \lambda$, and the one-forms (17) vanishe: $J^a \equiv 0,\ a = 0, 1$.

If zweibeins are taken in the form (26) the constant curvature condition: $*(d\omega) = \rho = const$, gives us the equation for conformal factor σ: $*d*(d\sigma) = \rho = const$, which is equivalent to the Liouville equation:

$$2\partial_-\partial_+\sigma = \frac{\rho}{2} e^{2\sigma}, \tag{32}$$

where $\rho = \pm\sqrt{\lambda}$. The general solution of the Liouville equation is well-known. By means the coordinate changing it can be transformed to the form:

$$2\sigma = -\ln(1 - \frac{\rho}{4} x^+ x^-)^2.$$

Correspondingly, we have for metric

$$ds^2 = \frac{dx^+ dx^-}{(1 - \frac{\rho}{4} x^+ x^-)^2}$$

[2]Note that only if $\lambda \geq 0$ there exists the constant curvature solution.

and for Lorentz connection (30):

$$\omega = \frac{\frac{\rho}{4}}{1 - \frac{\rho}{4}x^+x^-}(x^+dx^- - x^-dx^+) - d\theta$$

The one possible solution for the Dirac field is trivial, $\Psi = 0$. ($e^\gamma = e^\beta = 0$). The non-trivial Ψ with vanishing forms J^a(24) is given by (34) where u and v are constant functions:

$$\Psi = (1 - \frac{\rho}{4}x^+x^-)^{\frac{1}{2}} \begin{pmatrix} e^{-\frac{\rho}{2}} e^{iv} p(x^-) \\ e^{\frac{\rho}{2}} e^{iu} f(x^+) \end{pmatrix} \tag{33}$$

6. Let us now assume that $q^2 \neq 0$ identically on 2D space-time. We begin the analysis with the case when $J^a = 0$, $a = 0, 1$. Then the gravitational field equations (15), (16) completely decouples from the Dirac equation (18). One sees from (24) that J^a vanish if e^γ, e^β are zero or/end the imaginary part of χ_i, u and v, are constant functions. The gravitational equations reduce to the vacuum case. The general vacuum solution was obtained in [6] (for more accurate definitions see [9]). It is essential that one uses the variable ρ as one of the space-time coordinates. Introducing ϕ as additional, orthogonal to ρ, coordinate, we can write the vacuum solution for the zweibeins:

$$e^a = q^a e^{-\frac{\rho}{\alpha}} d\phi - \frac{1}{\alpha q^2}\varepsilon^a{}_b q^b d\rho \tag{34}$$

and for the Lorentz connection:

$$\omega = -\frac{1}{q^2}q^a \varepsilon_{ab} dq^b - \frac{\alpha}{2}(q^2)'_\rho e^{-\frac{\rho}{\alpha}} d\phi, \tag{35}$$

where q^2 is known function of ρ:

$$q^2(\rho) = -\frac{1}{\alpha}(\rho + \alpha)^2 + \Lambda + \epsilon e^{\frac{\rho}{\alpha}}, \tag{36}$$

where $\Lambda = \lambda/\alpha - \alpha$, ϵ is integrating constant.

The corresponding metric

$$ds^2 = q^2 e^{\frac{-2\rho}{\alpha}} d\phi^2 - \frac{1}{\alpha^2 q^2} d\rho^2 \tag{37}$$

was shown to describe the asymptotically de Sitter black hole configuration with ADM mass proportional to ϵ. The zeros of q^2 are points of the horizons [6].

It is worth observing that (40) takes the form (26) if we identify: $n^a = \frac{q^a}{q}$, $e^\sigma = qe^{-\frac{\rho}{\alpha}}$, $\tau = \phi$, $x = \int^\rho \frac{e^{\frac{\rho'}{\alpha}}}{\alpha q^2(\rho')} d\rho'$. For definitness we assume that $q^2 > 0$, then $q \equiv \sqrt{q^2}$.

Indeed, in coordinates (ϕ, x) the metric (41) is conformally flat:

$$ds^2 = q^2(\rho)e^{-2\frac{\rho}{\alpha}}(d\phi^2 - dx^2) \tag{38}$$

where ρ can be, in principle, expressed as function of x. Note again that the first term in (41) is pure gauge: $\frac{1}{q^2}q^a\varepsilon_{ab}dq^b = d\theta$.

Since the solution of the Dirac equation (18), (21) for zweibeins taken in the form (26) is already known (34), we obtain the following expression for the fermion field:

$$\Psi = q^{-1/2}e^{\frac{\rho}{2\alpha}} \begin{pmatrix} e^{-\frac{\theta}{2}}e^{iv}p(x^-) \\ e^{\frac{\theta}{2}}e^{iu}f(x^+) \end{pmatrix} \tag{39}$$

where u and v are constants, and $x^\mp = \phi \mp x$. We see that Ψ (42) divergences at points where q^2 has zeros. Remember that these points are regular horizons of the vacuum metric (43). Neverless, nothing singular happens at these points since the energy-momentum tensor for the spinor configuration (45) is identically zero. The fermion field Ψ also divergences at the point $e^{-\frac{\rho}{\alpha}} = 0$, where the black hole singularity is located (see [6]), while it tends to zero, $\Psi \to 0$, if $e^{-\frac{\rho}{\alpha}} \to \infty$.

7. Let us assume that $q^2 \neq 0$ identically on 2D space-time. The fermion action (14) is invariant under (global) chiral (γ_5) transformations: $\Psi \to \Psi' = exp[\mu\gamma_5]\Psi$. Therefore for simplicity we may restrict ourselves by consideration only the fermions of fixed chirality:

$$\gamma_5\Psi = \Psi.$$

In this case the fermion field has only one non-zero component: $\psi_1 = 0$, $\psi_2 = e^\chi$, where $\chi = \gamma + iu$ is complex field.

Then only the first of equations (25) is non-trivial. It gives us, in particular, that $du \sim (e^0 - e^1)$. In Lorentz invariant form it can be written as follows:

$$q_a e^a - q^a \varepsilon_{ab} e^b = B du, \tag{40}$$

where B is still unknown scalar function. As it is seen from (9), only the real part of χ transforms under Lorentz group: $\gamma \to \gamma - \frac{\Omega}{2}$. So the imaginary part, u, is Lorentz invariant.

One can see from (15) and (46) that variables ρ and u can be naturally chosen as coordinates on 2D space-time. Then basis of one-forms e^a is expressed in terms of $(d\rho,\ du)$:

$$e^a = \frac{q^a}{q^2}\left(-\frac{d\rho}{\alpha} + B du\right) - \frac{1}{\alpha q^2}\varepsilon^a{}_b q^b d\rho \tag{41}$$

The metric $ds^2 = \eta_{ab} e^a_\mu e^b_\nu dx^\mu dx^\nu$ correspondingly takes the form:

$$ds^2 = \frac{1}{q^2}\left(B du - \frac{d\rho}{\alpha}\right)^2 - \frac{1}{\alpha^2 q^2} d\rho^2 \tag{42}$$

In terms of the field $\chi = \gamma + \imath u$ the one-form J^a (24) has the components:

$$J^0 = J^1 = e^{2\gamma} du$$

It is convenient to introduce the one-form

$$J = \frac{2}{q^0 + q^1} e^{2\gamma} du$$

Assuming for definiteness that $q^2 > 0$ let us introduce variable θ: $q^0 = q\cosh\theta,\ q^1 = q\sinh\theta,\ q \equiv \sqrt{q^2}$. Then we have for J:

$$J = \frac{2}{q} e^{2\gamma - \theta} du$$

Under local Lorentz rotation on angle Ω variable θ transforms as: $\theta \to \theta - \Omega$. So the combination $(2\gamma - \theta)$ is really Lorentz invariant.

Multiplying eq.(16) on q^a and $q^b\varepsilon_{ba}$ separately we obtain equations:

$$dq^2 = \frac{\Phi}{\alpha^2}d\rho + q^2 J \tag{43}$$

$$\omega + d\theta = -\frac{\Phi}{2\alpha q^2}q_a e^a + \frac{1}{2}J \tag{44}$$

where we used that $\frac{1}{q^2}q^a\varepsilon_{ab}dq^b = d\theta$. The Lorentz connection ω with respect to Lorentz rotations transforms as $\omega \to \omega + d\Omega$. So that $(\omega + d\theta)$ is again Lorentz invariant. The eq.(52) gives us q^2 as function of ρ and u, while (53) is equation on Lorentz connection ω. The (52) is equivalent to

$$\partial_\rho q^2 = \frac{\Phi}{\alpha^2}(\rho, q^2), \quad \partial_u q^2 = 2q e^{2\gamma-\theta} \tag{45}$$

It follows from the first eq. (54) that q^2 as function of ρ has the same form as in vacuum case [6] (see eq.(42)). However, the ϵ now is is function of u, $\epsilon = \epsilon(u)$, which is found from the second eq.(54). Taking into account that $\partial_u q^2 = \partial_u \epsilon e^{\rho/\alpha}$ we get

$$\partial_u \epsilon = 2q e^{-\rho/\alpha} e^{2\gamma-\theta} \tag{46}$$

Since the left hand side of eq.(56) is function of only variable u we obtain that $(2\gamma - \theta)$ must have the following form:

$$2\gamma - \theta = -\ln q + \frac{\rho}{\alpha} + 2\ln f(u). \tag{47}$$

where $f(u)$ is an function of variable u related with $\epsilon(u)$ by means of equation:

$$\partial_u \epsilon = 2f^2(u). \tag{48}$$

Acting now by external differential d on both sides of eq.(46) we obtain:

$$B'_\rho d\rho \wedge du = (\frac{\Phi}{\alpha} - q^2)V + J_a \wedge e^a - \varepsilon_{ab}J^a \wedge e^b \tag{49}$$

From (47) we have $d\rho \wedge du = \frac{\alpha}{B}q^2 V$. Then using (23) and (54) the eq.(59) gives us the equation on function B:

$$\frac{B'_\rho}{B} = \frac{1}{q^2}\partial_\rho q^2 - \frac{1}{\alpha} \tag{50}$$

From this we finally find:

$$B = B_0(u)q^2e^{-\frac{\rho}{\alpha}} \tag{51}$$

where B_0 is an arbitrary function of u. Now inserting (61) into eq.(53) we obtain the expression for Lorentz connection.

$$\omega + d\theta = -\frac{\alpha}{2}\partial_\rho q^2 e^{-\frac{\rho}{\alpha}} B_0(u)du + \frac{1}{2q^2}\partial_\rho q^2 d\rho + \frac{1}{q^2}e^{\frac{\rho}{\alpha}} f(u)du.$$

Taking into account eq.(58) we finally obtain

$$\omega + d\theta = -\frac{\alpha}{2}\partial_\rho q^2 e^{-\frac{\rho}{\alpha}} B_0(u)du + \frac{1}{2}d(\ln q^2) \tag{52}$$

It should be noted that modulo exact forms this expression for ω takes the same form as in vacuum case [6].

Now it is easy to check the self-consistency condition: $*(d\omega) = \rho$. Really this procedure is the same as in vacuum case.

Let us again consider the Dirac equation (25). It is easy to see from (47) that

$$e^0 - e^1 = B_0 q e^{-\frac{\rho}{\alpha}} e^{-\theta} du$$

Inserting this and eq.(63) into the first equation (25) we obtain

$$B_0 e^{-\frac{\rho}{\alpha}} du \wedge (d\gamma - \frac{1}{2}d\theta + \frac{1}{2q}\partial_\rho q d\rho) = -\frac{1}{2}V \tag{53}$$

Using the obtained expressions for γ (57) and eq.(58) we obtain that (64) holds identically.

This completes the proof of exact integrability of equations (15)-(18). The complete solution is given by expression

$$e^a = \frac{q^a}{q^2}(-\frac{d\rho}{\alpha} + Bdu) - \frac{1}{\alpha q^2}\varepsilon^a{}_b q^b d\rho$$

for zweibeins; expression (62) for the Lorentz connection ω and

$$\Psi = q^{-1/2} e^{\frac{\theta}{2}} e^{\frac{\rho}{2\alpha}} \begin{pmatrix} 0 \\ e^{iu} f(u) \end{pmatrix} \tag{54}$$

for the chiral fermion field. The q^2 in known function of ρ and u:

$$q^2(\rho) = -\frac{1}{\alpha}(\rho+\alpha)^2 + \Lambda + \epsilon(u)e^{\frac{\rho}{\alpha}},$$

where

$$\epsilon(u) = 2\int^u f^2(u')du'$$

Note that up to this moment everything was Lorentz invariant. As result, the general solution depends on arbitrary field θ that is reflection of underlying Lorentz symmetry. Now one can fix the gauge, say $\theta = 0$ (see [9]).

The solution also depends on arbitrary function $f(u)$ which is not determined from the field equations and is found from initial conditions for fermion field.

In the case when fermions of both chiralities present the eqs.(15)-(18) can be integrated in the same manner but solution takes more complicated form.

8. The sense of found solution becomes more transparent if we consider δ-like impulse of fermion matter:

$$f^2(u) = \frac{E}{2}\delta(u-u_0),\ E>0 \tag{55}$$

Then equation (53) is easily solved:

$$\epsilon(u) = \epsilon_0 + E\theta(u-u_0) \tag{56}$$

where $\theta(x)$ is step function. In regions $u < u_0$ and $u > u_0$ taken separately $\epsilon(u)$ is constant and one can consider here new variable v:

$$v = u - \int^\rho \frac{2e^{\frac{\rho'}{\alpha}}}{\alpha q^2(\rho')}d\rho' \tag{57}$$

Then in coordinates (u,v) the metric (26) takes the vacuum conformally flat form (44):

$$ds^2 = q^2 e^{\frac{-2\rho}{\alpha}} dudv \tag{58}$$

For $u < u_0$ we have vacuum black hole solution (40)-(44) with mass $\epsilon = \epsilon_0$. The fermion impulse with energy E falls into this space-time along the line $u = u_0$. In result, for $u > u_0$ we again obtain vacuum black hole solution but with mass $\epsilon = \epsilon_0 + E$.

It was shown in [6] that the sapce-time structure of the vacuum solution (40)-(44) critically depends on value of the constant ϵ. The falling fermion matter leads to re-construction of initial vacuum accordingly to new value of ϵ. It should be noted that in this aspect the found solution is similar to that of the 2D dilaton gravity coupled with scalar (conformal) matter [10]. However, there are some essential differences. The flat space-time is one of solutions in 2D dilaton gravity. The falling of the scalar matter into the flat space-time leads to formation of the black hole. In the case under consideration there is no such a solution describing the black hole formation from regular space-time (in our case it is the de Sitter one) due to fermion matter. The "bare" vacuum black hole configuration is necessary. The reason is that the vacuum constant curvature solution is not obtained from the black hole one (40)-(44) for an value of integrating constant ϵ, i.e. these solutions are not parametrically connected [3] . Instead, in 2D dilaton gravity [12] the flat space-time is obtained as zero mass black hole solution.

9. In conclusion, we studied the 2D Poincaré gauge gravity coupled with 2D massless Dirac fermions and showed that the classical equations are exactly integrated. As in vacuum case, there are two types of solutions. The solution of the first type is space-time of constant curvature ($\rho^2 = \lambda$) and zero torsion, $q^a = 0$, $a = 0, 1$. The corresponding fermion field can take trivial ($\Psi = 0$) and non-trivial configurations. The solution of second type is characterized by that torsion is not identically zero. The space-time is of the vacuum black hole type with mass dependent on in-coming fermion matter energy.

[3]Really this situation is typical for 2D gravity described by action polynomial in curvature [11].

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References

[1] A.A.Tseytlin, Phys.Rev. D26 (1982), 3327; V.N.Ponomarev, A.O.Barvinski and Yu.N.Obukhov, "Geometrodynamical Methods and Gauge Approach to Gravity Interactions", Energoatomizdat, Moscow, 1985.

[2] M.O.Katanaev, I.V.Volovich, Ann.Phys. **v.197** (1990), 1;

[3] M.O.Katanaev, J.Math.Phys. **v.31** (1991), 2483. M.O.Katanaev, J.Math.Phys. **v.34** (1993), 700.

[4] W.Kummer, D.J.Schwarz, Phys.Rev. **D45** (1992), 3628; H.Grosse, W.Kummer, P.Presnajder and D.J.Schwarz, J.Math.Phys. **v.33** (1992), 3892.

[5] W.Kummer, D.J.Schwarz, Nucl.Phys. **B382** (1992), 171. P.Schaller, T.Strobl, *Canonical quantization of non-Einsteinian gravity and the problem of time*, TUW-92-13. M.O.Katanaev,

[6] S.Solodukhin, JETP Lett., **57** (1993), 329; Phys.Lett. **B319** (1993), 87.

[7] E.W.Mielke,
F.Gronwald, Yu.N.Obukhov, R.Tresguerres and F.W.Hehl, Phys.Rev. **D 48** (1993), 3648.

[8] Yu.N.Obukhov, S.N.Solodukhin, Class.Quant.Grav. **7** (1990), 2045.

[9] S.Solodukhin, *On exact integrability of 2D Poincaré gravity*, Preprint JINR, March 1994.

[10] C.G.Callan, S.B.Giddings, J.A.Harvey and A.Strominger, Phys.Rev. **D45** (1992), R1005.

[11] S.Solodukhin, *On higher derivative gravity in two dimensions*, Preprint, JINR, March 1994.

[12] G.Mandal, A.Sengupta and S.Wadia, Mod.Phys.Lett. **A6** (1991), 1685; E.Witten, Phys.Rev. **D44** (1991), 314.

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