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1. INTRODUCTION

Generally, there are two families of forward dispersion relations of the elastic scattering of two strongly interacting particles. They can be classified according to the relative position of an elastic threshold ω_{el} to the first inelastic or eventually anomalous threshold ω_0 on the physical sheet of Riemann surface.

The first family includes the forward dispersion relations without the unphysical cut, i.e., the following inequality $\omega_0 > \omega_{el}$ between the corresponding thresholds is fulfilled. In this case the forward dispersion relations can be in principle used to relate the real part of forward scattering amplitude with experimental data on total cross section without any admixture of a model dependence at all energies. In case that an eventual appearance of pole terms in unphysical region were misleading (provided that one does not want to utilize the method of determination of corresponding residues based on the use of analyticity in $\cos\theta$ plane [1]) we stress that by using symmetric (under crossing) amplitudes the noticeable contributions of pole terms is suppressed [2].

The forward dispersion relations containing an unphysical cut (the inequality $\omega_0 < \omega_{el}$ holds in this case) represent the second family. The low energy behaviour of a real part of amplitude evaluated by means of such dispersion relations depends to some extent on models used for imaginary part in unphysical region.

There is another characteristic feature being in a relationship with the appearance of an unphysical cut which is inherent for binary processes belonging to the same family. We have in mind the singular nature of total cross section in elastic threshold (known in nuclear physics as $1/v$ law [3] , v is the velocity of an incident particle in laboratory system) in the case $\omega_0 < \omega_{e\ell}$. If no unphysical cut is present (i.e., $\omega_0 > \omega_{e\ell}$) the total cross section takes nonnegative finite value. This feature can be simply understood as a direct consequence of the optical theorem, the parametrization of partial wave amplitudes and their threshold behaviour. Really, for all processes belonging to the first family, the imaginary part of forward scattering amplitude at the elastic threshold is zero. This will be fulfilled by taking the limit $k_L \to 0$ (k_L is an incident laboratory system momentum) in the optical theorem for any nonnegative finite value of total cross section. On the other hand the imaginary part of forward scattering amplitude of processes from the second family is equal to the imaginary part of the complex s-wave scattering length which is always a positive real number. Any finite real positive number will be reached by taking the limit $k_L \to 0$ in the optical theorem only if $\sigma_{tot} \to +\infty$ at elastic threshold.

In this paper we shall be interested in the forward π^4He scattering which belongs to the second family. The corresponding forward dispersion relation will contain the contribution from an unphysical cut because there is open the following channel $\pi^4He \to TN$ (T means ^3He or ^3H and N represents the nucleon) which is responsible for the lowest of all existing branch points in π^4He scattering process.

The correct evaluation of π^4He forward dispersion relation and a comparison of obtained results with experimental data, strictly speaking, can be carried out only if experimental data on σ_{tot} and $Re f(\omega)$ for both positive and negative pions at the same energies and in a sufficiently wide energy interval will be available. At present we have the situation when the aforementioned quantities were measured at only very few points for both charged pions simultaneously and the remainder of experimental data exists either for positive or negative pions only. Despite of this it seems to us that some interesting results can be obtained also under the present experimental situation. In recent past it was already confirmed [2,4,5] to some extent.

The first results on real part of forward π^4He scattering amplitude by a dispersion method were obtained by Ericson and Locher [2]. At that time only four experimental points on real part were available and nothing could be said about the agreement between experimental and theoretical results.

The authors of paper [6] were discussing the inconsistency of the dispersion relation prediction [2] with new experimental data. This incited one of us (S.D.) to analyze in detail the calculation of the real part [4] and to look for the most probable cause why it was impossible to get the consistency between the experimental data on real part and total cross section through forward dispersion relation.

The result of paper [4] resides in the following. To get a better agreement of a calculated real part with experiment, one has to shift the maximum of total cross section in resonant region to lower energies and to higher values. This prediction was later confirmed experimentally by Wilkin et al. [7] and by inclusion of the last data the agreement with experimental values of real part was improved [5].

Recently again new experimental data (four points) on real part have appeared [8] and three of them at higher energies seem to be inconsistent also with the newest dispersion predictions [5].

The aim of the present paper is to investigate to what extent one can expect that the imaginary part of forward π^4He scattering amplitude in unphysical region can improve the situation. Because, as it will be seen later, neither physical (here we have experimental data on total cross section) nor asymptotical regions can be responsible for such changed behaviour of real part.

2. EVALUATION OF DISPERSION INTEGRALS

The once-subtracted forward dispersion relation for symmetric elastic π^4He scattering amplitude in laboratory system takes the form [2,4,5]

$$\mathrm{Re}\, f(\omega) = \mathrm{Re}\, f(\omega_s) + \frac{2(\omega^2-\omega_s^2)}{\pi} P\{ \int_{\omega_{TN}}^{1} + \int_{1}^{17.14} + \int_{17.14}^{\infty} \} \times \qquad (1)$$

$$\times \frac{\omega' \,\mathrm{Im}\, f(\omega')}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)} d\omega',$$

where units $h = c = \mu = 1$ are used, μ is the mass of pion, $\mathrm{Re}\, f(\omega_s)$ is a subtraction constant and $\omega_{TN} \approx 0.143\ [\mu]$.

It will be seen in section 3 that one has to choose the subtraction constant at the value ω_s from the physical region unlike the papers /2,4,5/. Assuming that $\mathrm{Re}\, f(\omega)$ near the elastic threshold will change its behaviour slowly we shall choose $\omega_s = 1.007\ [\mu]$ (this corresponds to the shift of 1 *MeV* into the physical region) and the value of real part will be kept equal to $\mathrm{Re}\, f(\omega_s) = -0.087 \pm 0.002\ [\mu^{-1}]$ as it was determined by analysis of $\pi^-\ {}^4\mathrm{He}$ mesic atom data /5/.

The integrals in (1), which we denote by $J_1(\omega)$, $J_2(\omega)$, $J_3(\omega)$ are contributions to $\mathrm{Re}\, f(\omega)$ from the unphysical, physical and asymptotical regions, respectively. Such a decomposition is appropriate since the main sourse of our information about the behaviour of $\mathrm{Re}\, f(\omega)$ is in the total cross section and this is known only for $1 < \omega \leq 17.14\ [\mu]$. At present time, there are no experimental data on σ_{tot} for $\omega > 17.14\ [\mu]$ and no information on $\mathrm{Im}\, f(\omega)$ can be obtained from direct measurements in unphysical region.

To evaluate the integral

$$J_2(\omega) = \frac{2(\omega^2-\omega_s^2)}{\pi} \int_{1}^{17.14} \frac{\omega' \,\mathrm{Im}\, f(\omega')}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)} d\omega' \qquad (2)$$

one must choose a suitable function (dependeing on a few free parameters) for $\mathrm{Im}\, f(\omega)$ by means of which one gets a good fit of existing experimental data.

The authors of paper /2,5/ did not show it explicitly and they confine themselves to mentioning only that a smooth polynomial fit through the imaginary part has

been carried out. We have tried to repeat the fitting procedure with the polynomial

$$\mathrm{Im}\, f(\omega) = \sum_n A_n \omega^n \qquad (3)$$

but without any success. The minimum was reached at the value $\chi^2 \approx 84$ on 14 degrees of freedom what is clearly unsatisfactory result.

In paper /4/ for that reason the physical region interval $1 < \omega < 17.14\ [\mu]$ was decomposed into three parts each of which has been interpolated by means of different function for σ_{tot} . The singular nature of total cross section at the elastic threshold was not taken into account because the analytic continuation of zero-effective-range amplitude consisting of two first partial waves has been used to calculate the imaginary part for $1 < \omega < 1.16\ [\mu]$.

In the present paper we succeeded in finding the following formula

$$\sigma_{tot} = \frac{A_1 \omega}{(\omega - A_2)^2 + A_3} + \frac{P_5(\omega)}{\sqrt{\omega^2 - 1}} \qquad (4)$$

for total cross section where $P_5(\omega)$ means a polynomial of the fifth order in energy. Expression (4) possesses all the desired properties:

a) it is one smooth function for all known experimental region,

b) it diverges at the elastic threshold,

c) by means of it one gets an excellent fit of existing experimental data.

The fitting procedure was carried out through the optical theorem

$$\mathrm{Im}\, f(\omega) = \frac{\sqrt{\omega^2 - 1}}{4\pi} \sigma_{tot} \qquad (5)$$

and the result for $\chi^2 \approx 10$ on 14 degrees of freedom is graphically shown in *fig. 1*. The shape of the imaginary part of forward scattering amplitude following from (5) and its comparison with experimental data is pictured in *fig. 2*.

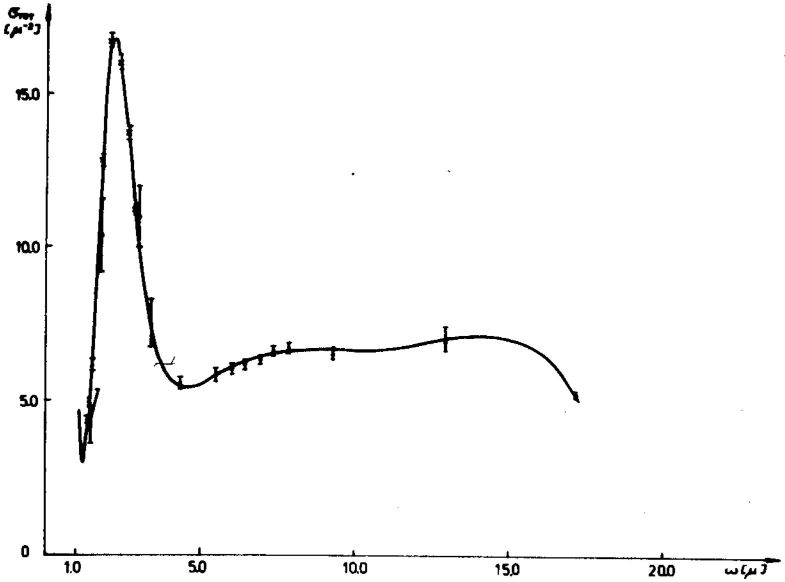

Fig. 1. The result of the fit of experimental data on σ_{tot} *by (4).*

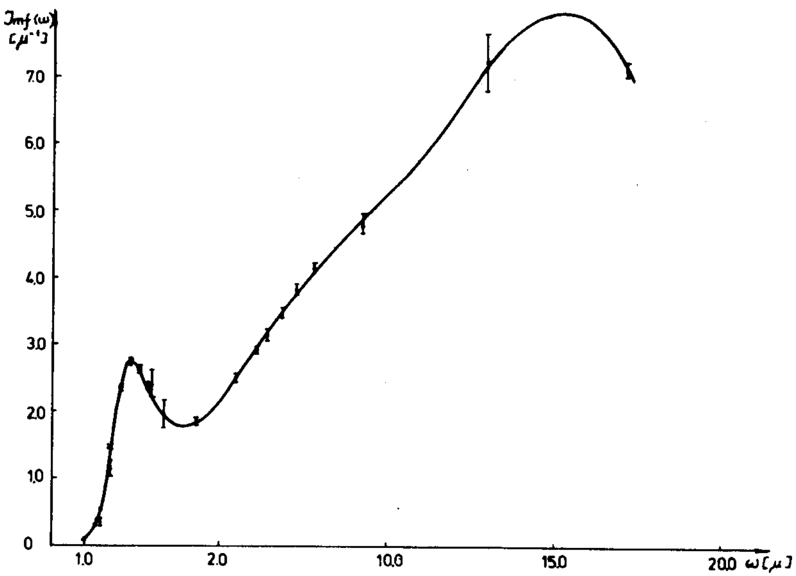

Fig. 2. The shape of $\mathrm{Im}\,f(\omega)$ *in unphysical region found through optical theorem (5) and eq. (4).*

Now, combining eq. (5) with (4) and replacing the imaginary part of the amplitude under the integral in relation (2) one can calculate the physical region contribution into the real part of the forward scattering amplitude which is graphically presented in *fig. 3* by dashed line 2.

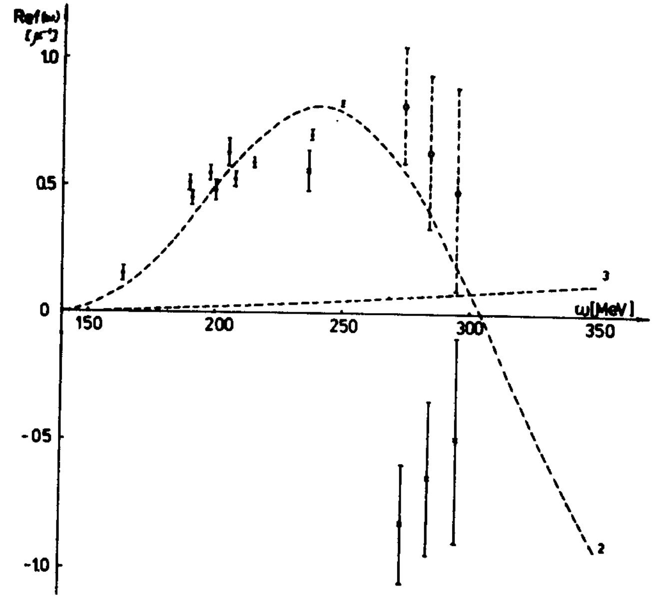

Fig. 3. The comparison of the recalculated behaviours of $J_2(\omega)$ *and* $J_{30}(\omega)$ *with experimental data on* $\mathrm{Re}\,f(\omega)$. *The new experimental data* [8] *are denoted by crosses.*

The contribution from the asymptotical region $\omega > 17.14\ [\mu]$ (in which we have no information on σ_{tot}) is represented by the following integral

$$J_3(\omega) = \frac{2(\omega^2 - \omega_s^2)}{\pi} \int_{17.14}^{\infty} \frac{\omega' \,\mathrm{Im}\,f(\omega')}{(\omega'^2 - \omega_s^2)(\omega'^2 - \omega^2)}\, d\omega'. \qquad (6)$$

To find some approximate behaviour of σ_{tot} in this energy region one can proceed, for instance, from the following considerations. It is known [2] that the symmetrical total cross section of light nuclei in physical region can be empirically obtained from the symmetrical πN total cross section by a scaling factor A^n with $n \approx \approx 0.83$, where A means a number of nucleons in the nucleus. One can expect that this experimental similarity in shape will extend the validity also to higher energies. Then, taking into account the behaviour of total cross section, $\pi\,{}^4He$ total cross section is expecting to decrease nearly up to energy $\omega \approx 286\ [\mu]$ and then will start to grow, most probably, logarithmically.

Here we shall be a little more conservative and for $17.14 < \omega < 286\ [\mu]$ we take the constant value of total cross section equal to the last experimental point at $\omega = 17.14\ [\mu]$. Then the contribution

$$J_{30}(\omega) = \frac{(\omega^2-\omega_s^2)}{2\pi^2}\, 5.24 \int_{17.14}^{286} \frac{\omega'\sqrt{\omega'^2-1}}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)}\, d\omega' \tag{7}$$

can be evaluated explicitly and takes the following form

$$J_{30}(\omega) = \frac{5.24}{2\pi^2}\{\sqrt{\omega^2-1} - \sqrt{\omega_s^2-1}\}\times$$

$$\times\ \ln\left|\frac{(t_a+t_1)(t_a-t_2)(t_b-t_1)(t_b+t_2)}{(t_a-t_1)(t_a+t_2)(t_b+t_1)(t_b-t_2)}\right|, \tag{8}$$

where

$$t_a = 17.14 + \sqrt{(17.14)^2-1} \qquad t_b = 286 + \sqrt{(286)^2-1}$$

and

$$t_1 = \omega + \sqrt{\omega^2-1} \qquad t_2 = \omega - \sqrt{\omega^2-1}\ .$$

One can see from *fig. 3* (dashed line 3) that expression (8) at those energies at which we have now an experimental information on $\mathrm{Re} f(\omega)$ gives a small contribution.

The remainder of the integral (6) is on the whole negligible in low energy region despite of the fact that the total cross section was assumed to rise logarithmically for $286\ [\mu] < \omega < \infty$.

Now, one can see immediately from *fig.3* that the sum of physical and asymptotical region contributions are unable to describe the new experimental data [8] on $\mathrm{Re}\, f(\omega)$ (denoted by crosses in *fig.3*) and for an eventual disagreement only $\mathrm{Im}\, f(\omega)$ in unphysical region can be responsible.

3. IMAGINARY PART IN UNPHYSICAL REGION

In this section we shall investigate the question to what extent one can expect that the imaginary part of forward $\pi\,{}^4He$ scattering amplitude in unphysical region really has a behaviour such that the contribution of unphysical cut

$$J_I(\omega) = \frac{2(\omega^2-\omega_s^2)}{\pi}\int_{\omega_{TN}}^{1} \frac{\omega'\,\mathrm{Im} f(\omega')}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)}\, d\omega' \tag{9}$$

will secure the agreement between the dispersion prediction for $\mathrm{Re} f(\omega)$ and its new experimental data [8].

In papers [2,5] the imaginary part in unphysical region was evaluated (see dashed line *fig. 4*) simply by using a scattering length expansion

$$\mathrm{Im} f(\omega) = \mathrm{Im} A_0 - 2\,\mathrm{Re} A_0\, \mathrm{Im} A_0 |k| - 3\,\mathrm{Im} A_1\, |k|^2, \tag{10}$$

where A_0 and A_1 are s and p-wave complex scattering lengths.

In paper [4] more complicated form for imaginary part in unphysical region was used (the behaviour of it is shown in *fig. 4* by full line) which has been found by

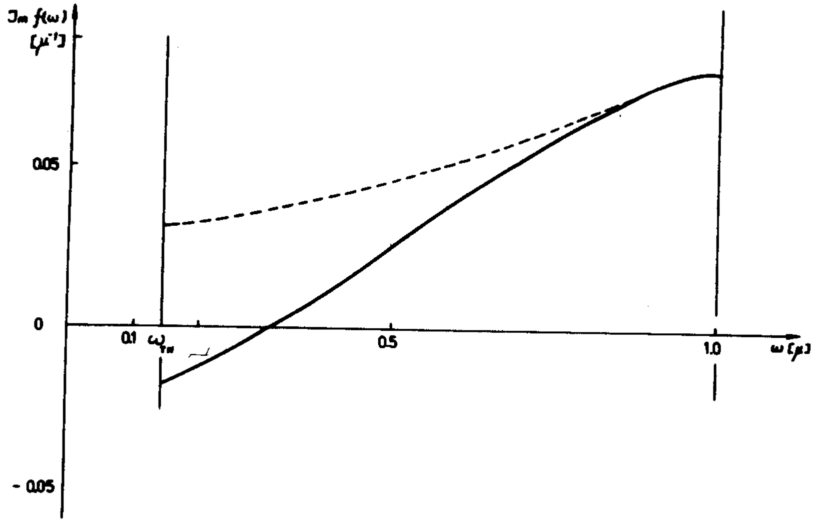

Fig. 4. The model-dependent behaviours of $\mathrm{Im}\, f(\omega)$ *in unphysical region following from (10) (dashed line) and from the analytic continuation of (11) (full line).*

analytic continuation of the zero-effective-range amplitude

$$f(\omega) \approx \frac{A_0}{1 - iA_0 k} + 3\,\frac{A_1 k^2}{1 - i\,A_1 k^3}\,. \tag{11}$$

keeping only two first terms in it.

Both parametrizations were derived under the assumption that the expansion of the function $k^{2\ell+1}\mathrm{cotg}\,\delta_\ell$ (δ_ℓ is a complex $\pi\,{}^4$He phase shift) into the Taylor series

$$k^{2\ell+1}\,\mathrm{cotg}\,\delta_\ell = \frac{1}{A_\ell} + B_\ell k^2 + \ldots \tag{12}$$

is convergent up to the branch point ω_0 . This must not be the case because any nearest threshold or zero of a partial wave amplitude (this generates the pole in function (12)) to $\omega_{e\ell}$ plays the role of the first singularity to which expansion (12) is only convergent.

So, there are indications that the unphysical region parametrizations for $\mathrm{Im}\, f(\omega)$ used in papers [2,4,5] may be doubtful.

To get a model-independent information about the behaviour of $\mathrm{Im}\, f(\omega)$ in unphysical region we shall start with confidence in experimental data on $\mathrm{Re}\, f(\omega)$ and the utilization (to the integral (9)) of the generalized mean value theorem which says:

Let

1. $f(x), g(x)$ are integrable in $[a,b]$
2. $f(x)$ is bounded in $[a,b]$
3. $g(x)$ does not change the sign in $[a,b]$

then

$$\int_a^b f(x)\,g(x)\,dx = f(X_i)\int_a^b g(x)\,dx\,, \text{ where } X_i \in [a,b].$$

In the case of integral (9) we shall identify

$$f(x) \to \mathrm{Im}\, f(\omega')\,, \qquad g(x,\omega) \to \frac{\omega'}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)}$$

where ω takes the physical region values.

Then all conditions of the generalized mean value theorem are fulfilled and we can write for averaged value of $\mathrm{Im}\, f(\omega)$ in unphysical region the following equation

$$\mathrm{Im}\, f(X_i) = \frac{\mathrm{Re}\, f(\omega) - \mathrm{Re}\, f(\omega_s) - J_2(\omega) - J_{30}(\omega)}{\dfrac{2(\omega^2-\omega_s^2)}{\pi}\displaystyle\int_{\omega_{TN}}^{1} \frac{\omega'}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)}\,d\omega'}\,. \tag{13}$$

The denominator of (13) can be calculated explicitly and the last relation takes the form

$$\mathrm{Im}\, f(X_i) = \frac{\mathrm{Re}\, f(\omega) - \mathrm{Re}\, f(\omega_s) - J_2(\omega) - J_{30}(\omega)}{\dfrac{1}{\pi}\,\ell n \left| \dfrac{(\omega^2-1)(\omega_s^2-\omega_{TN}^2)}{(\omega^2-\omega_{TN}^2)(\omega_s^2-1)} \right|} \tag{14}$$

from which one can see immediately why we have chosen the subtraction constant at $\omega_s \neq 1$ unlike papers /2,4,5/.

Now taking the experimental values on $\mathrm{Re}\,f(\omega)$ at different energies, the subtraction constant equal to $\mathrm{Re}\,f(\omega_s) = -0.087 \pm 0.002\ [\mu^{-1}]$ /5/ and $J_2(\omega)$, $J_{30}(\omega)$ as they are presented in *fig. 3* we get from (14) a set of values of $\mathrm{Im}\,f(\omega)$ in unphysical region (see table). Unfortunately we do not know to what values of energies in unphysical region they correspond. It does not follow from the generalized mean value theorem.

To get at least some image about the width of an energy interval which can be reached by changing the parameter ω in

$$\mathrm{Im}\,f(X_i) = \frac{\dfrac{2(\omega^2-\omega_s^2)}{\pi}\displaystyle\int_{\omega_{TN}}^{1} \frac{\omega'\,\mathrm{Im}\,f(\omega')}{(\omega'^2-\omega_s^2)(\omega'^2-\omega^2)}\,d\omega'}{\dfrac{1}{\pi}\ln\left|\dfrac{(\omega^2-1)(\omega_s^2-\omega_{TN}^2)}{(\omega^2-\omega_{TN}^2)(\omega_s^2-1)}\right|} \qquad (15)$$

we have replaced in the last equation $\mathrm{Im}\,f(\omega')$ by our parametrization (see full line *fig. 4*) and calculated the corresponding values of $\mathrm{Im}\,f(X_i)$ in unphysical region. They are also given in the table.

Comparing the obtained results in the table and the fact that $\mathrm{Im}\,f(\omega)$ in $\omega_{e\ell}$ is given by small value of complex part of s-wave scattering length ($b_0 = 0.081 \pm 0.006\ [\mu^{-1}]$) one can draw a conclusion that the new data on $\mathrm{Re}\,f(\omega)$ /8/ may be consistent with dispersion prediction only if $\mathrm{Im}\,f(\omega)$ in unphysical region has a drastic behaviour. The last effect is not very understable from the mathematical point of view because we do not find any strong singularity which could be responsible for the strange behaviour of $\mathrm{Im}\,f(\omega)$ in unphysical region.

It remains only the question to what extent one can confide in new experimental data on $\mathrm{Re}\,f(\omega)$. We would like to remind that they have been obtained by means of a phase shift analysis which suffers from well known ambiguities.

Table

The values of $\mathrm{Im}\,f(\omega)$ *in unphysical region calculated by an application of the generalized mean value theorem to the dispersion integral* $J_1(\omega)$.

ω [MeV]	$\mathrm{Im}\,f(X_i) \pm \Delta\mathrm{Im}\,f(X_i)$	$\overline{\mathrm{Im}\,f(X_i)}$
164	0.0565 ± 0.0299	0.0692
190	0.1926 ± 0.0300	0.0680
191	0.1233 ± 0.0227	0.0680
198	0.1237 ± 0.0257	0.0676
200	0.0415 ± 0.0334	0.0676
205	0.1099 ± 0.0486	0.0675
208	-0.0110 ± 0.0242	0.0675
215	-0.0224 ± 0.0179	0.0673
237	-0.1666 ± 0.0659	0.0669
238	-0.0528 ± 0.0173	0.0669
250	0.0590 ± 0.0083	0.0667
275	-1.0705 (0.2359) ± 0.1832	0.0663
285	-0.7949 (0.2185) ± 0.2375	0.0663
296	-0.5106 (0.2610) ± 0.3149	0.0662

If the phase shift analysis allows also the solution which gives the reflected values $\mathrm{Re}\, f(\omega)$ (see data denoted by circles and dashed errors in *fig.3*) then these should be chosen as a right solution and the discussed disagreement relation prediction will be removed. The last three positive (in brackets) values of $\mathrm{Im}\, f(X_i)$ in the table which were calculated under this assumption, confirm this.

At the end of this section we would like to mention that the choice of the right solution in an ambiguous phase shift analysis by means of the dispersion relation is a well known method. In the case of πN scattering it was used already 20 years ago [9].

4. CONCLUSIONS

The new experimental data on $\mathrm{Re}\, f(\omega)$ of forward π^4He scattering process were analyzed by the dispersion relation approach. The corresponding contributions of dispersion integrals were recalculated more carefully and the question whether one can expect that the behaviour of the imaginary part of forward π^4He scattering amplitude in unphysical region can remove the disagreement of dispersion relation predictions with the new experimental points on $\mathrm{Re}\, f(\omega)$ was discussed in detail.

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