ОБЪЕДИНЕННЫЙ
ИНСТИТУТ
ЯДЕРНЫХ
ИССЛЕДОВАНИЙ

ДУБНА

E2 - 9389

A.A.Slavnov

SPONTANEOUS BREAKING OF SUPERSYMMETRY IN NON-ABELIAN GAUGE THEORIES

1975

E2 - 9389

A.A.Slavnov*

SPONTANEOUS BREAKING OF SUPERSYMMETRY IN NON-ABELIAN GAUGE THEORIES

Submitted to ТМФ

* Steclov Mathematical Institute, Moscow.

The Higgs mechanism gives at present the only reliable method of spontaneous symmetry breaking in quantum field theory. However, straightforward application of this mechanism to supersymmetric theories meets certain difficulties due to specific form of the supersymmetric potential. Fayet and Illiopoulos /1/ succeeded to solve the problem for the case of the supersymmetric theory invariant with respect to Abelian gauge group (see also /2/). Unfortunately this method is not directly applicable to the case of semi-simple gauge group which is the most interesting from the practical point of view. In particular it does not allow one to construct a model simultaneously infra-red convergent and asymptotically free, for which the supersymmetric Yang-Mills theory is a natural candidate.

Other suggestions on the problem were given in the papers /3,4/. However the models considered there include an additional matter-matter interaction which enters with an independent coupling constant and destroys considerably the original beauty of the supersymmetric gauge theory. In addition these models contain residual massless scalars not absorbed by the Higgs mechanism.

In the present paper we propose a mechanism of spontaneous supersymmetry breaking, which is in some sense generalization of Fayet-Illiopoulos method but is applicable to the theories invariant with respect to semi-simple gauge group. More precisely the group of invariance of the model should be $G(local) \otimes U(1)$(global), where G can be semi-simple. Our main observation is that to produce supersymmetry breaking the Abelian subgroup $U(1)$ should not be realized dynamically. Consequently the method works in the case of supersymmetric Yang-Mills theory and in particular allows one to construct asymptotically free and infra-red convergent models.

The paper is organised as follows.
In Section 1 we discuss the renormalization procedure for spontaneously broken supersymmetric Abelian theory. Section 2 is devoted to the description of the supersymmetry breaking mechanizm for the semi-simple gauge groups. In Section 3 we give an example of a model, based on the proposed mechanism, which is simultaneously infra-red convergent and asymptotically free.

1. Renormalization of spontaneously broken supersymmetric Abelian theories

The Lagrangian of the supersymmetric quantum electrodynamics given firstly by Wess and Zumino [5] can be written in the form [6]

$$\mathcal{L}_S = \frac{1}{8}(\bar{D}D)^2\{\varphi_+^+ e^{g\Psi}\varphi_+ + \varphi_-^+ e^{-g\Psi}\varphi_-\} - \frac{M}{2}(\bar{D}D)\{\varphi_-^+\varphi_+ + \varphi_+^+\varphi_-\} + \mathcal{L}_\Psi^0, \quad (1)$$

where Ψ is a gauge supermultiplet with the components $\{A, \chi, \mathcal{F}, G, A_\nu, \lambda, \mathcal{D}\}$, and $\varphi_\pm$ are chiral superfields describing matter fields

$$\varphi_\pm(x,\theta) = \exp\{\mp\frac{1}{4}\bar{\theta}\hat{\partial}\gamma_5\theta\}\left[A_\pm(x) + \bar{\theta}\psi_\pm(x) + \frac{1}{4}\bar{\theta}(1\pm i\gamma_5)\theta\mathcal{F}_\pm(x)\right]. \quad (2)$$

$\mathcal{D}$ is a covariant derivative, $\mathcal{L}_\Psi^0$ is a free Lagrangian for the gauge fields

$$\mathcal{L}_\Psi^0 = -\frac{1}{4}(\partial_\mu A_\nu - \partial_\nu A_\mu)^2 + \frac{i}{2}(\bar{\lambda} - i\hat{\partial}\bar{\chi})\hat{\partial}(\lambda + i\hat{\partial}\chi) + \frac{1}{2}(\mathcal{D} + \Box A)^2. \quad (3)$$

The Lagrangian (1) is manifestly supersymmetric and invariant with respect to generalized gauge transformations

$$\varphi_\pm \to \Omega_\pm\varphi_\pm, \qquad e^{g\Psi} \to \Omega_- e^{g\Psi}\Omega_+^{-1}, \quad (4)$$

where $\Omega_\pm$ is an arbitrary chiral superfields satisfying the condition $\Omega_+^+ = (\Omega_-)^{-1}$.

To obtain a spontaneously broken solution one should add to the Lagrangian (1) supersymmetric and gauge invariant term $\xi g^{-1}\mathcal{D}$. This leads to a nonvanishing vacuum expectation value of $\mathcal{D}$ and performing canonical transformation

$$\mathcal{D} \to \mathcal{D} - \xi g^{-1} \quad (5)$$

we obtain the Lagrangian of spontaneously broken theory, which can be written (omitting total divergence) as follows

$$\mathcal{L} = \mathcal{L}_S - \frac{\xi}{2}\left(\varphi_+^+ e^{g\Psi}\varphi_+ - \varphi_-^+ e^{-g\Psi}\varphi_-\right)_A, \quad (6)$$

where $(\ )_A$ means A-component of a superfield. This Lagrangian is still invariant with respect to the gauge transformations (4) but the supersymmetry transformation is changed:

$$\delta A = \bar{\varepsilon}\gamma_5\chi$$

$$\delta\chi = \frac{1}{2}\left[\gamma_5\mathcal{F} + i\gamma_\mu A_\mu - G - i\gamma_5\hat{\partial}A\right]\varepsilon$$

$$
\begin{aligned}
\delta \mathcal{F} &= \frac{1}{2}\bar{\varepsilon}\gamma_5\lambda - \frac{i}{2}\bar{\varepsilon}\gamma_5\hat{\partial}\chi \\
\delta G &= -\frac{1}{2}\bar{\varepsilon}\lambda - \frac{i}{2}\bar{\varepsilon}\hat{\partial}\chi \\
\delta A_\nu &= \frac{1}{2}\bar{\varepsilon}\gamma_\mu\gamma_\nu\partial_\mu\chi - \frac{i}{2}\bar{\varepsilon}\gamma_\nu\lambda \\
\delta\lambda &= \frac{1}{2}\left[\gamma_5\mathcal{D} - i\gamma_5\hat{\partial}\mathcal{F} - i\hat{\partial}G - i\gamma_\nu i\hat{\partial}A_\nu - \xi g^{-1}\gamma_5\right]\varepsilon \\
\delta\mathcal{D} &= -i\bar{\varepsilon}\hat{\partial}\gamma_5\lambda
\end{aligned}
\tag{7}
$$

λ - component is shifted by a constant number, i.e. the supersymmetry is spontaneously broken.

In the Zumino-Wess gauge $A=\chi=\mathcal{F}=G=0$, the additional term in (6) is nothing but the mass term

$$
-\frac{\xi}{2}\left(A_+^+A_+ - A_-^+A_-\right). \tag{8}
$$

The masses of different members of the chiral supermultiplets are no longer equal.

Nevertheless, the renormalization procedure can be performed analogously to the symmetric case[7,8] and no new independent counterms are needed.

To show that we note first of all that the Lagrangian (6) can be quantized exactly in the same way as a symmetric one[7,8] because it possesses the same gauge invariance. Introducing the supersymmetric gauge fixing term

$$
\frac{1}{4\beta}\left(\bar{\mathcal{D}}\mathcal{D}\right)^2\Psi_+^* P\Psi_+ , \tag{9}
$$

where P is some polynomial over the d'Alambert operator $\Box$, one easily obtains the generalized Ward identities derived in our paper[7] (eq.22). These identities express all the diagrams with external unphysical gauge lines in terms of the diagrams without such lines.

The second part of the renormalization procedure, the supersymmetry relations, needs however more careful investigation. We shall show that the renormalized Green function generating functional

$$
Z = N^{-1}\int \exp\left\{i\int\left[\mathcal{L}_R(x) + \frac{1}{8}(\bar{\mathcal{D}}\mathcal{D})^2(J_\Psi\Psi) + \frac{1}{2}(\bar{\mathcal{D}}\mathcal{D})(J_{\varphi_\pm}\varphi_\pm)\right]dx\right\}d\mu, \tag{10}
$$

where

$$
\begin{aligned}
\mathcal{L}_R = {} & \frac{1}{8}(\bar{\mathcal{D}}\mathcal{D})^2 z_2\left\{\varphi_+^+ e^{g\Psi}\varphi_+ + \varphi_-^+ e^{-g\Psi}\varphi_-\right\} + \frac{1}{4\beta}(\bar{\mathcal{D}}\mathcal{D})^2\Psi_+^* P\Psi_+ - \\
& - \frac{z_2}{2}\xi\left\{\varphi_+^+ e^{g\Psi}\varphi_+ - \varphi_-^+ e^{-g\Psi}\varphi_-\right\}_A - \frac{M}{2}(\bar{\mathcal{D}}\mathcal{D})\left\{\varphi_-^+\varphi_+ + \varphi_+^+\varphi_-\right\} + z_3\mathcal{L}_\Psi^0
\end{aligned}
\tag{11}
$$

leads to the divergence-free S-matrix.

The identities associated with the transformations (7) are obtained as usual by making in the integral (10) the change of variables (7) with ε depending on x and putting

$$
dZ/d\varepsilon\,|_{\varepsilon=0} = 0 .
$$

We write down these identities in terms of one particle irreducible Green functions generated by the functional

$$
\Gamma(R_\Psi, R_\varphi) = W - \int\left\{\frac{1}{8}(\bar{\mathcal{D}}\mathcal{D})^2(J_\Psi R_\Psi) + \frac{1}{2}(\bar{\mathcal{D}}\mathcal{D})(J_{\varphi_\pm} R_{\varphi_\pm})\right\}dx , \tag{12}
$$

where

$$W(J_\psi, J_\varphi) = i \ln Z(J_\psi, J_\varphi); \quad R_{\psi,\varphi} = \frac{\delta W}{\delta J_{\psi,\varphi}}, \quad J_{\psi,\varphi} = -\frac{\delta \Gamma}{\delta R_{\psi,\varphi}} . \tag{13}$$

Their explicit form is

$$\int \Big\{ \Big[\frac{\delta \Gamma}{\delta R_{A_\nu}(x)} \frac{1}{2} \gamma_\nu + \frac{\delta \Gamma}{\delta R_D(x)} \partial \gamma_5 + i \xi g^{-1} \gamma_5 \Big] R_\lambda(x) + \frac{i}{2} \frac{\delta \Gamma}{\delta R_{\bar\lambda}(x)} \Big[\gamma_5 R_D(x) +$$

$$+ \hat\partial R_{A_\nu}(x) \gamma_\nu - \xi g^{-1} \gamma_5 \Big] - i \frac{\delta \Gamma}{\delta R_{A_\pm}(x)} R_{\psi_\pm}(x) + \frac{\delta \Gamma}{\delta R_{\bar{\mathcal{F}}_\pm}(x)} \hat\partial R_{\psi_\pm}(x) + \tag{14}$$

$$+ \frac{1 \mp i \gamma_5}{2} \big(R_{\mathcal{F}_\pm}(x) - i \hat\partial R_{A_\pm}(x) \big) \frac{\delta \Gamma}{\delta R_{\bar\psi_\pm}(x)} \Big\} dx = 0 .$$

(Index c means charge conjugation, we put sources of unphysical gauge components equal to 0).

Differentiating eq. (14) one obtains

$$\frac{i}{2} \frac{\delta^2 \Gamma}{\delta R_{\bar\lambda}(x) \delta R_\lambda(y)} - \hat\partial \frac{\delta^2 \Gamma}{\delta R_D(x) \delta R_D(y)} - \frac{i}{2} \xi g^{-1} \int \frac{\delta^3 \Gamma}{\delta R_{\bar\lambda}(z) \delta R_\lambda(y) \delta R_D(x)} dz = 0 . \tag{15}$$

The last term is absent in the corresponding identity for the symmetric case. But this term is superficially convergent, therefore if the constant Z_3 is fixed by demanding $\Gamma_{DD}(p)$ to be finite, the Green function $\Gamma_{\bar\lambda\lambda}(p)$ is also finite (note however that contrary to the symmetric case $\Gamma_{\bar\lambda\lambda}(p)$ contains not only a $\hat p\, \Gamma^{(1)}_{\bar\lambda\lambda}(p^2)$ term but also a finite term $\Gamma^{(2)}_{\bar\lambda\lambda}(p^2)$).

Analogously one can show that $\Gamma_{A_\nu A_\mu}$ is finite, and three-point vertices $\Gamma_{A_\pm \lambda \psi_\pm}$, $\Gamma_{A_\mu \psi_\pm \psi_\pm}$, etc., may differ only by finite terms. Therefore if the constant Z_2 is chosen to make $\Gamma_{A_\pm \lambda \psi_\pm}$ finite all other three-point functions are finite also. On the other hand electromagnetic three-point functions $\Gamma_{A_\mu \psi_\pm \psi_\pm}$, etc., are related to the matter field

Green functions by the usual Ward identities. By the usual arguments it follows that the corresponding wave function renormalizations are finite. This result is in fact quite expectable because the Lagrangian (6) differs from $\mathcal{L}_S$ essentially by the mass term. But there exists a general statement [10,11,9] that such terms do not affect logarithmic divergences.

The identities (14) allow one to make a stronger statement that also no independent mass renormalization arises.

Differentiating eq. (14) with respect to $A_\pm$, $\psi_\pm$ we obtain

$$-i \frac{\delta^2 \Gamma}{\delta R_{A_\pm}(x) \delta R_{A_\pm}(y)} + \frac{1 \mp i \gamma_5}{2} \Big(-i \hat\partial \frac{\delta^2 \Gamma}{\delta R_{\bar\psi_\pm}(x) \delta R_{\psi_\pm}(y)} \Big) +$$

$$+ \hat\partial \frac{\delta^2 \Gamma}{\delta R_{A_\pm}(x) \delta R_{\mathcal{F}_\pm}(y)} - \frac{i}{2} \xi g^{-1} \gamma_5 \int \frac{\delta^3 \Gamma}{\delta R_{\bar\lambda}(z) \delta R_{A_\pm}(x) \delta R_{\psi_\pm}(y)} dz = 0 . \tag{16}$$

The last term which is absent in the symmetric case is finite due to our choice of Z_2 . Therefore $\Gamma_{A_\pm A_\pm}(p^2) = $ $= \mathrm{const} < \infty$,etc. Finally the relations between $\Gamma_{\bar\psi_\pm \psi_\pm}$ and $\Gamma_{A_\pm \mathcal{F}_\pm}$ are affected also by finite terms, and no independent mass renormalization arises (the absence of mass renormalization in the supersymmetric case was shown in the paper [12])

This completes the renormalization program for the Abelian gauge theory with spontaneously broken supersymmetry.

2. Spontaneous breaking of supersymmetry in the case of semi-simple gauge group

In this section we shall show that the mechanism described above can be generalized to include the Yang-Mills theories which possess in addition to the local SU(N) invariance global $\mathcal{U}(1)$ invariance. The last one is not realized dynamically so in the theory there is only one independent coupling constant.

To illustrate our idea we note firstly that the Fayet-Illiopoulos trick works as well in the case of more complicated group containing Abelian subgroup. For example if the matter fields $\varphi_{\pm}$ are doublets, transforming according to the representation of $SU(2)\otimes\mathcal{U}(1)$ group

$$\varphi_{\pm} \to \Omega_{\pm}^{1}\Omega_{\pm}\varphi_{\pm}\,, \tag{17}$$

where

$$(\Omega_{+})^{+}=(\Omega_{-})^{-1};\quad (\Omega_{+}^{1})^{+}=(\Omega_{-}^{1})^{-1};\quad \det\Omega_{\pm}=1\,,$$

then the gauge invariant Lagrangian producing spontaneous breakdown of supersymmetry is

$$\mathcal{L}=\frac{1}{8}(\bar{D}D)^{2}\{\varphi_{+}^{+}e^{g_1\Psi_1+g\Psi}\varphi_{+}+\varphi_{-}^{+}e^{-g_1\Psi_1-g\Psi}\varphi_{-}\}+\xi g_1^{-1}\mathcal{D}_1+\dots \tag{18}$$

where Ψ_1 is an Abelian gauge superfield and Ψ is a matrix-superfield transforming as

$$e^{g_1\Psi_1}\to\Omega_{-}^{1}e^{g_1\Psi_1}(\Omega_{+}^{1})^{-1};\quad e^{g\Psi}\to\Omega_{-}e^{g\Psi}\Omega_{+}^{-1} \tag{19}$$

$\dots$ denotes the free Lagrangians for the fields Ψ and Ψ_1 and possible supersymmetric terms.

Performing the canonical transformation $\mathcal{D}_1\to\mathcal{D}_1-\xi g_1^{-1}$ one can proceed further in complete analogy with the discussion of the previous section.

The generalized Ward identities derived in[7,8] remain

valid and supersymmetry relations are changed only by finite terms.

The Lagrangian (18) written in terms of the shifted fields has a definite limit when $g_1\to 0$. The limiting Lagrangian is invariant under local SU(2) transformations and in fact is nothing but the Yang-Mills Lagrangian plus some additional renormalizable interactions of scalars and spinors. This Lagrangian possesses also global $\mathcal{U}(1)$ invariance, but the local $\mathcal{U}(1)$ invariance is lost.

Omitting nonessential additive constant one can write

$$\mathcal{L}=\frac{1}{8}(\bar{D}D)^{2}\{\varphi_{+}^{+}e^{g\Psi}\varphi_{+}+\varphi_{-}^{+}e^{-g\Psi}\varphi_{-}\}-\frac{\xi}{2}\{\varphi_{+}^{+}e^{g\Psi}\varphi_{+}-\varphi_{-}^{+}e^{-g\Psi}\varphi_{-}\}_{A}+\dots \tag{20}$$

The additional term is again nothing but the mass term for the scalar fields.

The supersymmetry identities (14) include explicitly the factor ξg_1^{-1} which tends to infinity when $g_1\to 0$. However this factor is multiplied by $\delta\Gamma/\delta R_\lambda$ which is proportional to g_1. Therefore the supersymmetry relations (14) also have well-defined limit.

One can consider the Lagrangian (18) as an intermediate regularization of the theory defined by the Lagrangian (20). All the discussions of the previous section concerning the renormalization constants are directly applicable to the Lagrangian (18). Therefore we conclude that in exact analogy with the Abelian case the symmetric counterterms are sufficient to remove all ultraviolet divergences. The renormalized Lagrangian can be written as follows

$$\mathcal{L}_R = \frac{1}{8}(\bar{D}D)^2 Z_2 \{\varphi_+^+ e^{\tilde{g}\Psi}\varphi_+ + \varphi_-^+ e^{-\tilde{g}\Psi}\varphi_-\} -$$

$$- \frac{Z_2 \xi}{2}\{\varphi_+^+ e^{\tilde{g}\Psi}\varphi_+ - \varphi_-^+ e^{-\tilde{g}\Psi}\varphi_-\}_A + \tilde{Z}_2 \mathcal{L}^{\circ}_{\Psi}(\tilde{g}) , \qquad (21)$$

where $\tilde{g} = Z_2^{-1} Z_1 g = \tilde{Z}_2 \tilde{Z}_1^{-1} g$, Z_1 is a three-point vertex renormalization constant and $\tilde{Z}_2 , \tilde{Z}_1$ are the corresponding constants for the Yang-Mills field.

Summarizing we can say that if one uses for the regularization of the Lagrangian (21) the supersymmetric expression (18), one has a theory which is invariant under the "broken supersymmetry" transformation (7) for any $g_1 \neq 0$ and satisfies the "broken supersymmetry relations" (14) for any g_1 (including $g_1 = 0$). However there does not exist the limiting Lagrangian invariant with respect to limiting transformations (7) (in terms of unshifted fields the Lagrangian becomes singular, and in terms of shifted fields the transformation (7) becomes singular).

This situation is quite common for the quantum theory, where the renormalized Lagrangian simply does not exist and the symmetry manifests itself in the existence of generalized Ward identities or in the invariance of the regularized Lagrangian. In our case the limiting invariant Lagrangian does not exist even at the classical level. The last point has no physical significance and is in fact due to the special formulation of the model we used above. To show that we present an alternative formulation which does not use the limiting procedure and allows to write down the invariant classical Lagrangian.

Such a Lagrangian can be written in the form

$$\mathcal{L} = \frac{1}{8}(\bar{D}D)^2\{\varphi_+^+ e^{g\Psi+\Psi_1}\varphi_+ + \varphi_-^+ e^{-g\Psi-\Psi_1}\varphi_-\} -$$

$$- \frac{1}{4}(\partial_\mu A_\nu^1 - \partial_\nu A_\mu^1)(\partial_\mu \tilde{A}_\nu^1 - \partial_\nu \tilde{A}_\mu^1) + \frac{i}{2}(\bar{\lambda}_1 - i\hat{\partial}\bar{\chi}_1)\hat{\partial}(\tilde{\lambda}_1 + i\hat{\partial}\tilde{\chi}_1) +$$

$$+ \frac{1}{2}(D_1 + \Box A_1)(\tilde{D}_1 + \Box\tilde{A}_1) + \xi(\tilde{D}_1 + \Box\tilde{A}_1) \qquad (22)$$

... means the free Lagrangian for the Yang-Mills field Ψ and also possible mass terms. Ψ_1 and $\tilde{\Psi}_1$ are Abelian superfields, transforming as

$$e^{\Psi_1} \to \Omega_-^1 e^{\Psi_1}(\Omega_+^1)^{-1} ; \quad e^{\tilde{\Psi}_1} \to \Omega_-^1 e^{\tilde{\Psi}_1}(\Omega_+^1)^{-1} \qquad (23)$$

The fields Ψ_1 and $\tilde{\Psi}_1$ are auxiliary fields which can be eliminated from the Lagrangian (22) providing the Lagrangian of the Yang-Mills theory. The field $\tilde{\Psi}_1$ is in fact a Lagrange multiplier: the equations arising after variation over $\tilde{\Psi}_1$ are the free field equation for the field Ψ_1 .

The Lagrangian (22) is manifestly supersymmetric and gauge invariant. The supersymmetry is spontaneously broken: canonical transformation $D_1 \to D_1 - \xi$ eliminates linear term and produces a mass form

$$\mathcal{L}_m = -\frac{\xi}{2}\{\varphi_+^+ e^{g\Psi+\Psi_1}\varphi_+ - \varphi_-^+ e^{-g\Psi-\Psi_1}\varphi_-\}_A \ . \qquad (24)$$

The renormalized Lagrangian in terms of the shifted fields is

$$\mathcal{L}_R = \frac{1}{8}(\bar{D}D)^2 Z_2\{\varphi_+^+ e^{\tilde{g}\Psi+\Psi_1}\varphi_+ + \varphi_-^+ e^{-\tilde{g}\Psi+\Psi_1}\varphi_-\} -$$

$$- \frac{\xi}{2} Z_2\{\varphi_+^+ e^{\tilde{g}\Psi+\Psi_1}\varphi_+ - \varphi_-^+ e^{-\tilde{g}\Psi-\Psi_1}\varphi_-\}_A + \ldots \qquad (25)$$

We leave to the reader to repeat the arguments given above to show that the corresponding Green functions satisfy generalized Ward identities and "broken supersymmetry relations" analogous to eq. (14).

3. Asymptotically free and infra-red convergent model.

In this section we illustrate the possibilities opened by the mechanism described in Sec.2 for constructing the model which is simultineously infra-red convergent and asymptotically free. This model certainly does not pretend to any relation to the experiment and is just a methodical example.

The model is fixed by the Lagrangian

$$\mathcal{L}=\frac{1}{16}(\bar{D}D)^2 \operatorname{Tr}\left\{\Omega_+^+ e^{g\Psi} \quad \Omega_+(1+\tau_3)\right\}+\frac{\xi}{2}\operatorname{Tr}\left\{\Omega_+^+ e^{g\Psi} \quad \Omega_+(1+\tau_3)\right\}_{A}+\dots \tag{26}$$

Here we find it more convenient to use a matrix chiral superfield

$$\Omega_+=\exp\left\{-\frac{1}{4}\bar{\theta}\hat{\partial}\gamma_5\theta\right\}\left[A_+ +\vec{\tau}\vec{A}_+ +\bar{\theta}(\Psi_+ +\vec{\tau}\vec{\Psi}_+)+\frac{1}{4}\bar{\theta}(1+i\gamma_5)\theta(F_+ +\vec{\tau}\vec{F}_+)\right], \tag{27}$$

other notations are the same as before.

Due to the presence of the projection operator $(1+\tau_3)/2$ Ω_+ is equivalent to the complex doublet φ_+ with the components

$$\tilde{A}_+ = A_+ + A_+^3, \quad \tilde{A}_{1+} = A_+^1 - i A_+^2, \quad \text{etc.}$$

The physical contents of the model is most easily analysed in the Zumino-Wess gauge after elimination of auxiliary fields. In this gauge the Lagrangian can be written as follows

$$\begin{aligned}\mathcal{L}=&\partial_\mu \tilde{A}_+^+ \partial_\mu \tilde{A}_+ +\partial_\mu \tilde{A}_{1+}^+ \partial_\mu \tilde{A}_{1+} + i\bar{\tilde{\Psi}}_+ \hat{\partial}\tilde{\Psi}_+ + i\bar{\tilde{\Psi}}_{1+}\hat{\partial}\tilde{\Psi}_{1+} + \tilde{F}_+^+\tilde{F}_+ + \tilde{F}_{1+}^+\tilde{F}_{1+} +\\ &+g^2\vec{A}_\mu^2\left(\tilde{A}_+^+\tilde{A}_+ + \tilde{A}_{1+}^+\tilde{A}_{1+}\right)-\frac{g^2}{2}\left(\tilde{A}_+^+\tilde{A}_+ - \tilde{A}_{1+}^+\tilde{A}_{1+}\right)^2-2g^2\tilde{A}_+^+\tilde{A}_+\tilde{A}_{1+}^+\tilde{A}_{1+}+\\ &+\xi\left(\tilde{A}_+^+\tilde{A}_+ + \tilde{A}_{1+}^+\tilde{A}_{1+}\right)+\frac{i}{2}\bar{\lambda}^a\hat{\partial}\lambda^a-\bar{\lambda}_3\left(\tilde{A}_+^+\tilde{\Psi}_+ + \tilde{\Psi}_+^c A_+\right)-\\ &-(\bar{\lambda}_1+i\bar{\lambda}_2)\left(\tilde{\Psi}_{1+}\tilde{A}_+^+ + \tilde{\Psi}_{1+}^c A_+\right) + \dots\end{aligned} \tag{28}$$

... denotes interaction terms which are irrelevant for the present discussion.

For positive ξ the Lagrangian (28) evidently corresponds to the unstable theory, due to the wrong sign of the mass term. That means the gauge invariance is also spontaneously broken and one should translate the fields $\tilde{A}$. Substituting $\tilde{A}$ by $\tilde{A}+\alpha$ we obtain the stability condition

$$-g^2\alpha^3+\xi\alpha=0. \tag{29}$$

Now the spectrum of the model contains massive vector triplet $\vec{A}_\mu$, two massive complex spinors $\lambda_+^3+\tilde{\Psi}_+$ and $\lambda_-^1+i\lambda_-^2+\tilde{\Psi}_+^1$, and massive Hermitian scalar $\tilde{A}_+^+ + \tilde{A}_+$, all with the mass ξ. In addition there are two-component massless spinor $\lambda_+^1+i\lambda_+^2$ and three Goldstone scalars which are absorbed into the longitudinal part of the vector field A_μ.

The model is infrared convergent. On the other hand the only independent coupling constant is the Yang-Mills constant g. The theory is known to be asymptotically free.

It is not difficult to construct other asymptotically free models introducing for example also interaction with the left-handed matter fields and additional supersymmetric mass terms. At present, however, we do not succeed to invent parity conserving asymptotically free model.

References.

1. P.Fayet, J.Illiopoulos, Phys. Let. 51B, 461, 1974.
2. P.Fayet, Nuclear Phys. B90, 104, 1975, Preprint de l'Ecole Normale Superieure. PTENS 7515.
3. S.Browne, L.O'Raifeartaigh, T.Sherry, Preprint Dublin Institute for Advanced Studies DIAS-TP-75-16.
4. P.Fayet, Preprint de l'Ecole Normale Superieure PTENS 75/1.
5. J.Wess, B.Zumino, Nuclear Phys. B78, 1, 1974.
6. A.Salam, J.Strathdee, Preprint ICTP IC/74/42.
7. A.A.Slavnov. Preprint E-2-8308, JINR, 1974 and Theor. and Math. Phys. 23, 3, 1975.
8. A.A.Slavnov, Preprint E2-8443, JINR, 1974.
9. S.Weinberg, Phys.Rev. D, 8, 3497, 1973.
10. J.C.Collins, A.J.Macfarlane, Phys. Rev. D, 10, 1201, 1974.
11. I.V.Tiutin, B.L.Voronov, JETP Let. 21, 396, 1975.
12. S.Ferrara, O.Piguet, Preprint TH 1995-CERN, 1975.

Received by Publishing Department
on December 16, 1975