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QUARK-LEPTON ANALOGY AND NEUTRINO OSCILLATIONS

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Neutrino mixing and neutrino oscillations were considered a long time ago[1]. A theory of neutrino oscillations was presented in ref. [2]. Possible applications to neutrino astronomy were also considered[2-5].

In ref.[2] the starting point was the assumption on the existence of four neutral lepton states in all (two Majorana neutrinos with masses). In such a theory the two neutrinos have a special place among the fundamental fermions inasmuch as all the other leptons and quarks are described by four-component spinors. This is in principle a three parameter theory (two masses and the mixing angle). However, there were given some arguments[2] in favour of maximum mixing.

In these note we consider neutrino mixing starting from a different point of view suggested by an analogy between leptons and quarks. We assume that each neutrino is described by a four component spinor (that is there are eight neutral lepton states in all). It will be seen that the results of our theory and of the theory presented in ref.[2] are practically identical, although from a theoretical point of view there are differences.

2. As is well known[6], the hadron weak interaction for the case of four quarks can be written in such a way that the stran-

geness changing neutral current does not appear in the first order but does appear (as needed) in higher orders. This was obtained[6] by introducing besides the Cabibbo combinations of n and λ quarks

$$n_c = n\cos\theta_c + \lambda\sin\theta_c \qquad (1)$$

the combination orthogonal to it

$$\lambda_c = -n\sin\theta_c + \lambda\cos\theta_c . \qquad (2)$$

In order to forbid processes of type $\mu \to e + \gamma$ and similar ones in the first order, we might introduce (in analogy with quarks) the two orthogonal combinations of neutrino fields

$$\begin{aligned} \nu_e^{\theta} &= \nu\cos\theta + \nu'\sin\theta , \\ \nu_\mu^{\theta} &= -\nu\sin\theta + \nu'\cos\theta . \end{aligned} \qquad (3)$$

This neutrino mixing was adopted in refs.[7] where there were discussed various effects by considering θ as a parameter and also by us[8]. However we should take into account a profound difference between leptons and quarks: in addition to the weak interaction quarks undergo the strong interaction, where all the charges (strangeness...) are conserved. The lepton-quark analogy at issue is a "weak interaction analogy". If we introduce neutrino mixing, we loose the muon charge conservation so that for the lepton case there are no quantum numbers analogous to strangeness and/or charm. Consequently a parameter analogous to the Cabibbo angle which would characterize the degree of the muon charge violation has no meaning at this stage. And yet just because of such analogues equations (1) with θ as a free parameter were suggested[7,8]. Thus we feel now that $\theta = \frac{\pi}{4}$ is the only value of physical interest in a mixing scheme (other assumptions seem to us premature). Thus we assume that neutrino fields participate in the interaction as the combinations

$$\begin{aligned} ,,\nu_e'' &= \frac{1}{\sqrt{2}}(\nu + \nu') , \\ ,,\nu_\mu'' &= \frac{1}{\sqrt{2}}(-\nu + \nu') . \end{aligned} \qquad (4)$$

Obviously the very notion of muon charge has disappeared. There arises then the question about the difference between ν and ν'. It is clear that ν and ν' must have different masses (for which we use the notations m and m′). The charged lepton current in our scheme has the form:

$$j_\alpha = (\bar{e}\gamma_\alpha(1+\gamma_5),,\nu_e'') + (\bar{\mu}\gamma_\alpha(1+\gamma_5),,\nu_\mu'') . \qquad (5)$$

We are left with a two parameter (m and m′) theory* and it is just this circumstance which allows us to make definite conclusions

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*It may be asked how can we go to the limiting case of exact muon charge conservation if $\theta = \frac{\pi}{4}$? The answer is by having $m' \to m$. Of course, our choice $\theta = \frac{\pi}{4}$ is speculative, but at this stage it is the only fruitfull approach in our opinion. Let us remark also that if the interaction, (if any), responsible for the mass difference of ν and ν' is also related to the $\mu - e$ - mass difference, the $\mu - e$ symmetry tells us that $\theta = \frac{\pi}{4}$.

of physical interest. As it should be, the neutral current arising say, in a Salam-Weinberg class of the theories is symmetrical, while asymmetrical neutral cirrent effects ($\mu \to e + \gamma$, etc.) may appear only in higher order.

3. The mass of the antineutrino emitted together with the electron in the β-decay of ^{3}H is known to be < 60 eV. Since in the relevant experiments ν and ν' are not distinquished, we get*

$$m < 60 \text{ eV}$$
$$m' < 60 \text{ eV}. \qquad (6)$$

This conclusion applies to all theories without muon charge conservation and thus is relevant for the masses of both Majorana neutrino of ref. /2/. It is possible to calculate the probability of the process $\mu \to e + \gamma$. Using (6) we find, that $\frac{\Gamma_{\mu \to e\gamma}}{\Gamma_\mu} \lessapprox 10^{-25}$, which is incomparably smaller than the experimental upper limit.

Clearly we need more sensitive types of experiments to reveal neutrino mixing. Such are those in which effects connected with neutrino oscillations might be observed, because in such experiments there are measured amplitudes instead of amplitudes squared.

The neutrino beams are no longer described by stationary states. There will be such effects as the appearance of electrons in collisions with nuclei of neutrino from $\pi - \mu$ decay. Similarly there will appear "sterile" neutrinos in the neutrino beams from the reactor, from the Sun, etc.

For the ratio of the numbers of electrons and muons appearing in nuclear collisions of high energy neutrinos from $\pi - \mu$ decay we have

$$\left(\frac{N_e}{N_\mu}\right)_L = \frac{1 - \cos 2\frac{L}{L_0}}{1 + \cos 2\frac{L}{L_0}}, \qquad (7)$$

where L is the distance from the neutrino source to the detector $L_0 = \frac{2p}{|m'^2 - m^2|}$ is the oscillations length, p is the neutrino momentum in the lab. sys. From CERN results /9/ we get

$$|m' - m| \lessapprox 1 \text{ eV}.$$

A much smaller upper limit of $|m' - m|$ can be obtained from reactor experiments, by comparing the measured yield of positrons in the reaction

$$„\bar{\nu}_e" + p \to e^+ + n$$

with the value expected under the assumption that there are no oscillations. From the results of ref. /10/ we get

$$|m' - m| \lessapprox 10^{-1} \text{ eV}.$$

Let us turn now to solar neutrino astronomy. The intensity of detectable solar neutrinos („ν_e") with momentum p at the Earth is given by*

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*This conclusion can be made as it's shown later $|m' - m| \lessapprox 1$eV.

*In our scheme the heavier neutrino will decay into the light one with emission of a photon. If limits (6) are used, it can be shown that the neutrino during its life time will travel a way several orders of magnitude larger than the Earth-Sun distance.

$$I(R,p) = \frac{1}{2} I_0(R,p)\left(1 + \cos 2\frac{R}{L_0}\right), \qquad (8)$$

where $I_0(R,p)$ is the intensity of detectable neutrino (ν_e) expected if oscillations were absent, R is the Sun-Earth distance. The effects connected with oscillations can in principle be observed only if $L_0 \lesssim R$ that is if

$$|m' - m| \gtrsim 10^{-6}\ \mathrm{eV}.$$

In fact the term $\cos 2\frac{R}{L_0}$ in (8) disappears because of averaging and

$$I = \frac{1}{2} I_0 . \qquad (9)$$

From the point of view of a realistic test of this relation, the most promising perspective has arisen recently in connection with the development of new neutrino detectors based on the Ga-Ge radiochemical methods [11-14].

4. Let us make a few concluding remarks. As is well known, if the two neutrino masses are identically equal to zero, there are two equivalent theories, in terms of

i) two two-component neutrinos,
ii) two Majorana neutrinos.

When we consider theories with mixing of the two neutrinos, we must have orthogonal combinations in order to suppress extremely unlikely the processes such as $\mu \to e + \gamma$ and others. Now the two neutrinos must have different masses (by the way if they had equal masses there would be no mixing). Let us stress here that in our opinion maximum mixing ($\theta = \frac{\pi}{4}$) is by far the most plausible and fruitful assumption.

The theory proposed in ref. [2] is a generalization of the case ii) and consequently is a theory where the two neutrinos with mass have each two states.

The theory considered here starts from i) and consequently is a theory, where each neutrino (with mass) is described by four component spinor. In this sense in our scheme ν and ν' are just as other lepton or quark (which, may be, is an attractive feature), while in the theory given in ref. [2] the two neutrino have a special position among the other fundamental particles.

Practically the two schemes have identical consequences. Note, nevertheless, that in the theory given in ref. [2] neutrinoless double β-decay and similar processes are possible in high orders (or course they are extremely unlikely [15]) while in the scheme considered here such processes are strictly forbidden (see eq. (5)).

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