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1. SPECTRAL PROPERTIES OF GROUP EQUATION SOLUTION

The ultraviolet asymptotic behaviour of any renormalized field model with one coupling constant g is governed by the behaviour of asymptotic ICC $\bar{g}(x,g)$ at large g. Generally $\bar{g}$ can be represented as a product of small number of dimensionless propagators d_i and symmetrical vertices Γ_s. A simplified case is provided by spinor electrodynamics where due to the Ward identity the ICC is proportional to photon propagator d:

$$\bar{a}(x,a) = a d(x,a), \quad a = e^2/4\pi$$

satisfying the well known spectral representation. Hence

$$\bar{a}(x,a) = a + \frac{x-1}{\pi} \int_{\infty}^{0} \frac{\sigma(x',a)\, dx'}{(x'-1)(x'-x)} \qquad (1)$$

(Here we took into account that normalization parameter λ is negative). In a more general case the representations similar to (1) are valid for d_i and can be conjectured (see the Appendix to paper[1]) for symmetrical vertex Γ_s. As a consequence we assume that it is valid for $\bar{g}(x,g)$. In other words we assume that the ICC $\bar{g}(x,g)$ is an analytic function of x variable in the plane cut along the negative part of the real axis with a pole of finite order at infinity.

We call this conjecture "the spectrality hypothesis" (SH). It will be used below within the framework of RG formalism.

Now let us use the well known fact that the general solution of RG functional equation for ICC

$$\bar{g}(x,g) = \bar{g}\left(\frac{x}{t}, \bar{g}(t,g)\right) \tag{2}$$

can be written in the form

$$\Psi[\bar{g}(x,g)] - \Psi(g) = \ln x$$

or

$$\bar{g}(x,g) = \Phi(x\,\Phi^{-1}(g)), \quad \Phi^{-1} - \exp\Psi . \tag{3}$$

From (3) it follows that the function $\Phi(z)$ should have analytic properties in z analogous to those of $\bar{g}(x,g)$ with respect to x.

On the basis of SH we conclude that $\Phi(z)$ cannot have singularities in the finite part of z-plane outside the negative real axis, and cannot grow faster than a polynomial as z tends to infinity.

This property considerably limits the class of functions Ψ and Φ which in the framework of usual RG formalism are arbitrary single-valued reversible functions of one variable.

To illustrate the type of limitation imposed by SH consider the example based on the perturbation theory for logarithmic renormalized models.

In the one-loop approximation we have

$$\Psi(g) = -\frac{a}{g} . \tag{4}$$

The constant a is positive for the "normal" models which correspond to the transition out of the region of weak coupling

(to the ghost-trouble case). In spinor electrodynamics, e.g., $a=3\pi$. Negative a corresponds to the asymptotic freedom.

For the Φ from (4) we get

$$\Phi(z) = -\frac{a}{\ln z} . \tag{5}$$

For $a>0$ small g values in (4) correspond to small positive z in (5). The r.h.s. of eq.(5) evidently contradicts SH. It has a first order pole at $z=1$. The account of two-loop corrections

$$\Phi(z) = -\frac{a}{\ln z} + \frac{b}{\ln^2 z}\ln\ln\left(\frac{1}{z}\right) \simeq \frac{-a}{\ln z + (b/a)\ln\ln z} \tag{6}$$

does not change the situation qualitatively.

Below we consider the possibility of "improving" of approximations for $\Phi(z)$ obtained from perturbation theory. This possibility is based on the imperative of spectrality. It will be shown that the analytical properties of ICC $\bar{g}$ with respect to g variable in the vicinity of $g=0$ due to Eq. (3) and information from perturbation theory are determined by analytical properties of $\bar{g}$ with respect to x.

To avoid possible semantic confusion in the following we shall call the analyticity with respect to momentum squared x as spectrality (or causality), and reserve the term analyticity for the properties of $\bar{g}(x,g)$ in the (complex) g variable in the vicinity of $g=0$.

2. ANALYTICITY IN THE COUPLING CONSTANT

Consider now the possible way of transforming the expressions of Eqs. (5), (6) type to the correspondence with SH. The problem can be formulated as follows:

To construct the function $\Phi(z)$ of complex variable z which

|A| for small z behaves as (5),

|B| satisfies SH, i.e.,

is analytic in the cut along $(-\infty, 0)$ z-plane and grows as a polynomial for $z \to \infty$.

Equation (5) gives the order of vanishing of $\Phi(z)$ as $z \to 0$, as well as the type of branching at $g=0$. From the two-loop expression (6) it follows that to the logarithmic branching there can be added more "weak" singularities like $\ln \ln z$, etc. Hence the type of branching is the logarithmic one.

The most general form of $\Phi(z)$ corresponding to the |A| and |B| conditions is

$$\Phi(z) = M(z, \ln z), \tag{7}$$

where $M(z_1, z_2)$ is the ratio of polynomials in z_1 and z_2. The poles of M in z_1 and z_2 variables are of such a special structure that the function (7) has no poles outside the negative real semi-axis and infinite point.

Besides, due to |A| for $z=0$ one has

$$M(z, \ln z)\Big|_{z=0} \cong -\frac{a}{\ln z}.$$

Here it is worthwhile to note that the expression for the photon propagator obtained in paper [2] first by summation the main logarithm contributions under the sign of

spectral representation and by subsequent use of RG functional equations exactly corresponds (see eq. (4.2) from [2]) to the "simplest" possibility for the M function

$$M(z, \ln z) = -\frac{a}{\ln z} + \frac{a}{z-1}.$$

This choice does not add any new parameters and does not change the asymptotic behaviour at $z \to \infty$. Unfortunately even if we fix the degree of growth $\Phi(z)$ at infinity, the choice of $M(z, \ln z)$ still remains non-unique (in this connection see [3]).

Nevertheless, the given analysis allows one to make an important conclusion about the type of nonanalyticity of $\bar{g}$ at small g. For this aim one should take into account that due to Eq. (5) for small g.

$$\Phi^{-1}(g) = \exp\left(-\frac{a}{g}\right). \tag{8}$$

Hence the arguments $z = x\exp(-a/g)$, $\ln z = \ln x - a/g$ being introduced into r.h.s. of Eq. (7) yield the rational dependence of g (via $\ln z$) and exponential (via z). Thus, the type of nonanalyticity of ICC at $g=0$ is completely defined by essential singularity (8). Here naturally we take as granted that perturbation theory correctly reflects that part of functional dependence which can be expanded into power series.

In spinor electrodynamics this assumption is justified by its correspondence with experiment. As far as SH here also is equivalent to well known spectral representation for photon propagator, we can conclude that the obtained result is valid for spinor electrodynamics.

The analysis performed can be repeated for quantum field models with dimensional coupling constant G. The main assumption here is the validity of RC use. As a result, we get that the nonanalyticity in G is of the logarithmic type. This is in agreement with conclusions of papers [2,4] obtained also with the help of RG motivations. The details of analysis for the models with dimensional coupling constants will be published elsewhere.

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