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ISOSPIN CONSTRAINTS
FOR SPIN DENSITY MATRICES
OF ($0^- 1/2^+ \rightarrow 0^- 3/2^+$)
QUASI-TWO-BODY REACTIONS

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**ISOSPIN CONSTRAINTS
FOR SPIN DENSITY MATRICES
OF ($0^- 1/2^+ \rightarrow 0^- 3/2^+$)
QUASI-TWO-BODY REACTIONS**

Submitted to ЖЭТФ

Объединенный институт
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Изоспиновые ограничения на матрицы плотности спина
трех $(0^-1/2^+ \rightarrow 0^-3/2^+)$ реакций

Для $(0^-1/2^+ \rightarrow 0^-3/2^+)$ квазидвухчастичных реакций (с поляризованной мишенью) получены изоспиновые ограничения (равенства и границы) на матрицы плотности спина.

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Isospin Constraints for Spin Density Matrices
of $(0^-1/2^+ \rightarrow 0^-3/2^+)$ Quasi-Two-Body Reactions

In this paper, the isospin constraints (equalities and bounds) on the spin density matrix elements of the $(0^-1/2^+ \rightarrow 0^-3/2^+)$ quasi-two-body reactions (with polarized target) are obtained.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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Recently, Doncel et al.^{/1,2/} have presented a detailed procedure for measuring the multipole parameters of the usual quasi-two-body reactions in order to reconstruct all the amplitudes of these reactions in the transversity quantization. Then, it was shown that the total /even and odd multipole/ polarization parameters of the final baryon can be measured for baryonic resonances with a sequential weak decay [e.g., $\Sigma^* \rightarrow \Sigma + \pi$, $\Sigma \rightarrow N + \pi$; $\Xi^* \rightarrow \Xi + \pi$, $\Xi \rightarrow \Sigma(\Lambda) + \pi$]. Therefore, it is of great interest to use this formalism /1,2/ for the investigation of the isospin constraints [equalities and bounds] on spin density matrix elements of the quasi-two-body reactions related by isospin invariance via two channels. We focus our attention on $(0^-1/2^+ \rightarrow 0^-3/2^+)$ quasi-two-body reactions

$$\pi N \rightarrow K\Sigma^*; \quad \bar{K}N \rightarrow \pi\Sigma^*; \quad \bar{K}N \rightarrow K\Xi^*. \quad (1)$$

The isospin triangular relationships for the amplitudes of these types of reactions are given in table I, Therefore, we shall assume that the transversity amplitudes $\{f_{\ell}^{(a)}\}$, $a=1,2,3,4$, of three $(0^-1/2^+ \rightarrow 0^-3/2^+)$ reactions satisfy the triangular relationships

Table I
The isospin triangular relationships for 22 quasi-two-body
[$0^{-1/2^+} \rightarrow 0^{-3/2^+}$] reactions^{1/}

$\pi\pi \rightarrow$	$K \Sigma^*$	$f_1^{(\alpha)}$	$\bar{K}N \rightarrow$	$\pi \Sigma^*$	$f_1^{(\alpha)}$	$\bar{K}N \rightarrow$	$K \Sigma^*$	$f_2^{(\alpha)}$	$\bar{K}N \rightarrow$	$K \Sigma^*$	$f_2^{(\alpha)}$
$\pi^+\pi^+$	+	$f_1^{(\alpha)}$	$K^-\bar{P}$	+	$f_1^{(\alpha)}$	$K^-\bar{P}$	+	$f_1^{(\alpha)}$	$K^-\bar{P}$	+	$f_1^{(\alpha)}$
$\pi^-\pi^+$	-	$f_2^{(\alpha)}$		0	$f_2^{(\alpha)}$		0	$f_2^{(\alpha)}$			
$\pi^+\pi^0$	0	$f_3^{(\alpha)}$	$K^-\bar{N}$	0	$f_3^{(\alpha)}$	$K^-\bar{N}$	0	$f_3^{(\alpha)}$	$K^-\bar{N}$	0	$f_3^{(\alpha)}$
	0	$f_4^{(\alpha)}$		-	$f_4^{(\alpha)}$		-	$f_4^{(\alpha)}$			
$\pi^0\pi^0$	+	$f_3^{(\alpha)}$	$\bar{K}^0\bar{P}$	+	$f_3^{(\alpha)}$	$\bar{K}^0\bar{P}$	+	$f_3^{(\alpha)}$	$\bar{K}^0\bar{P}$	+	$f_3^{(\alpha)}$
	0	$f_4^{(\alpha)}$		0	$f_4^{(\alpha)}$		0	$f_4^{(\alpha)}$			
$\pi^-\pi^0$	0	$f_2^{(\alpha)}$	$\bar{K}^0\bar{N}$	0	$f_2^{(\alpha)}$	$\bar{K}^0\bar{N}$	0	$f_2^{(\alpha)}$	$\bar{K}^0\bar{N}$	0	$f_2^{(\alpha)}$
	0	$f_1^{(\alpha)}$		-	$f_1^{(\alpha)}$		-	$f_1^{(\alpha)}$			
$f_1^{(\alpha)} = f_2^{(\alpha)} + \sqrt{2} f_3^{(\alpha)}$, all α $f_1^{(\alpha)} = f_2^{(\alpha)} + \sqrt{2} f_3^{(\alpha)}$, all α $\sqrt{2} f_1^{(\alpha)} = 2[f_2^{(\alpha)} + f_3^{(\alpha)}] = f_2^{(\alpha)} - f_4^{(\alpha)}$ $= -2[f_3^{(\alpha)} + f_4^{(\alpha)}]$, all α											

See ref. ^{1/}.

$$\sum_{\ell=1}^3 c_{\ell} f_{\ell}^{(\alpha)} = 0 \quad (2)$$

for each $\alpha = 1, 2, 3, 4$ due to the isospin invariance, where c_{ℓ} are real numbers [e.g., see table I].

Next, for each pair $[f^{(a_p)}, f^{(a_r)}] \equiv [a, b]$, $[a', b']$, $[a, a']$, $[b, b']$, $[a, b']$, $[b, a']$, of the transversity amplitudes^{1/} we define the observables σ_{β} and $\xi_{\beta} \equiv [A_{\beta}, P_{\beta}, R_{\beta}]$, $\beta \equiv (a_p, a_r)$, by analogy with the differential observables of the $[0^{-1/2^+} \rightarrow 0^{-3/2^+}]$ reactions. Then, using the results of table 6 from ref. ^{1/}, we obtain the expressions of σ_{β} , A_{β} , P_{β} and R_{β} , in terms of the spin density matrix elements. These expressions are presented in table II. We note that x , y , z are the target polarization components: (x) normal to the beam direction in the reaction plane, (y) along the beam direction and (z) along the "Basel normal" to the reaction plane, respectively [see fig. 1 from ref. ^{1/}] while the σ_{ℓ} are the polarized differential cross sections [see eq. (3.12) from ref. ^{1/}]. Therefore, the introduction of the observables σ_{β} and ξ_{β} makes it possible to discuss the isospin constraints [equalities and bounds] on spin density matrix elements of $[0^{-1/2^+} \rightarrow 0^{-3/2^+}]$ reactions by analogy with the $[0^{-1/2^+} \rightarrow 0^{-1/2^+}]$ scattering case^{3,4,5/}. Hence, if we define the bilinear forms:

$$Y_{\beta ij} \equiv f_i^{(a_p)} f_j^{(a_r)} - f_i^{(a_r)} f_j^{(a_p)}, \quad \beta \equiv (a_p, a_r) \quad (3a)$$

with the properties

Table II

The observables σ_{β} and $\xi_{\beta} = [A_{\beta}, P_{\beta}, R_{\beta}]$ expressed in terms of density matrix elements $f_{\ell}^{(a)}$ in the transversity quantization for $[0^{-}1/2^{+} \rightarrow 0^{-}3/2^{+}]$ reactions with polarized target

β	$\sigma_{\beta} = f_{\ell}^{(a)} ^2 + f_{\ell}^{(a')} ^2$	$\xi_{\beta} [A_{\beta} + iP_{\beta}] = 2f_{\ell}^{(a)} f_{\ell}^{(a')}$	$R_{\beta} \sigma_{\beta} = f_{\ell}^{(a)} ^2 - f_{\ell}^{(a')} ^2$
1	$\frac{2\sigma}{1+\tau} [\rho_{11} + \rho_{3-3}]$	$4\rho_{1-3} \sigma / (1+\tau)$	$\frac{2\sigma}{1+\tau} [\rho_{11} - \rho_{3-3}]$
2	$\frac{2\sigma}{1-\tau} [\rho_{1-1} + \rho_{33}]$	$4\rho_{3-1} \sigma / (1-\tau)$	$\frac{2\sigma}{1-\tau} [\rho_{1-1} - \rho_{33}]$
3	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{11} + (\tau+2)\rho_{33}]$	$4 \frac{\tau+i\tau^2}{\tau^2+\tau^3} \rho_{1-1} \sigma$	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{11} - (\tau+2)\rho_{3-3}]$
4	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{1-3} + (\tau+2)\rho_{33}]$	$4 \frac{\tau+i\tau^2}{\tau^2+\tau^3} \rho_{3-3} \sigma$	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{3-3} - (\tau+2)\rho_{33}]$
5	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{11} + (\tau+2)\rho_{33}]$	$4 \frac{\tau+i\tau^2}{\tau^2+\tau^3} \rho_{31} \sigma$	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{11} - (\tau+2)\rho_{33}]$
6	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{3-3} + (\tau+2)\rho_{1-1}]$	$4 \frac{\tau+i\tau^2}{\tau^2+\tau^3} \rho_{1-3} \sigma$	$\frac{2\sigma}{1-\tau^2} [(\tau-2)\rho_{3-3} - (\tau+2)\rho_{1-1}]$

$$|Y_{\beta ij}|^2 = \frac{1}{2} [1 - \vec{\xi}_{\beta i} \cdot \vec{\xi}_{\beta j}] \sigma_{\beta i} \sigma_{\beta j} \equiv H_{\beta ij} \geq 0, \quad (3b)$$

then, using the $\sigma_{\beta\ell}$ and $\vec{\xi}_{\beta\ell} = [A_{\beta\ell}, P_{\beta\ell}, R_{\beta\ell}]$, from table II, we get the expressions presented in table III for $H_{\beta ij}$ in terms of spin density matrix elements. Next, since the sum rules (2) imply the equalities

$$c_1 c_2 Y_{\beta 12} = c_2 c_3 Y_{\beta 23} = c_3 c_1 Y_{\beta 31}, \quad (3c)$$

for each of $[f_{\ell}^{(a_p)}, f_{\ell}^{(a_r)}]$, then we obtain the following interesting results.

Constraints 1. The spin density matrix elements of three $[0^{-}1/2^{+} \rightarrow 0^{-}3/2^{+}]$ reactions, related by the isospin invariance via two channels, must obey the equalities:

$$\begin{aligned} & c_1^2 c_2^2 \sigma_1 \sigma_2 \{ \rho_{mm}^{(1)} \rho_{nn}^{(2)} + \rho_{mm}^{(2)} \rho_{nn}^{(1)} - 2\eta_{mn} \operatorname{Re}[\rho_{mn}^{*(1)} \rho_{mn}^{(2)}] \} = \\ & = c_2^2 c_3^2 \sigma_2 \sigma_3 \{ \rho_{mm}^{(2)} \rho_{nn}^{(3)} + \rho_{mm}^{(3)} \rho_{nn}^{(2)} - 2\eta_{mn} \operatorname{Re}[\rho_{mn}^{*(2)} \rho_{mn}^{(3)}] \} = \\ & = c_3^2 c_1^2 \sigma_3 \sigma_1 \{ \rho_{mm}^{(3)} \rho_{nn}^{(1)} + \rho_{mm}^{(1)} \rho_{nn}^{(3)} - 2\eta_{mn} \operatorname{Re}[\rho_{mn}^{*(3)} \rho_{mn}^{(1)}] \} \equiv H_{mn} \end{aligned} \quad (4a)$$

and the inequalities

$$2\eta_{mn} \operatorname{Re}[\rho_{mn}^{*(i)} \rho_{mn}^{(j)}] \leq \rho_{mm}^{(i)} \rho_{nn}^{(j)} + \rho_{nn}^{(i)} \rho_{mm}^{(j)}, \quad (4b)$$

with

Table III

The test quantities $H_{\beta ij}$ and the constraints $\xi_{\beta 0}^2 = 1$, in terms of density matrix elements for $[0^{-1/2^+} \rightarrow 0^{-3/2^+}]$ reactions with polarized target

β	$\frac{1}{4} H_{\beta ij}$	$\xi_{\beta 0}^2 = 1$ imply:
1	$\frac{\sigma_{\beta 0}}{(1+z^2)^{1/2}} \{ \rho_{11}^{(i)} \rho_{33}^{(j)} + \rho_{11}^{(j)} \rho_{33}^{(i)} - 2 \operatorname{Re} [\rho_{13}^{*(i)} \rho_{13}^{(j)}] \}$	$ \rho_{13}^{(i)} ^2 = \rho_{11}^{(i)} \rho_{33}^{(i)}$
2	$\frac{\sigma_{\beta 0}}{(1-z^2)^{1/2}} \{ \rho_{11}^{(i)} \rho_{33}^{(j)} + \rho_{11}^{(j)} \rho_{33}^{(i)} - 2 \operatorname{Re} [\rho_{13}^{*(i)} \rho_{13}^{(j)}] \}$	$ \rho_{33}^{(i)} ^2 = \rho_{33}^{(i)} \rho_{11}^{(i)}$
3	$\frac{\sigma_{\beta 0}}{1-z^2} \{ \rho_{11}^{(i)} \rho_{1-1}^{(j)} + \rho_{11}^{(j)} \rho_{1-1}^{(i)} - 2 \frac{(1-z^2)}{x^2+y^2} \operatorname{Re} [\rho_{1-1}^{*(i)} \rho_{1-1}^{(j)}] \}$	$\frac{1-z^2}{x^2+y^2} \rho_{1-1}^{(i)} ^2 = \rho_{11}^{(i)} \rho_{1-1}^{(i)}$
4	$\frac{\sigma_{\beta 0}}{1-z^2} \{ \rho_{33}^{(i)} \rho_{3-3}^{(j)} + \rho_{33}^{(j)} \rho_{3-3}^{(i)} - 2 \frac{(1-z^2)}{x^2+y^2} \operatorname{Re} [\rho_{3-3}^{*(i)} \rho_{3-3}^{(j)}] \}$	$\frac{1-z^2}{x^2+y^2} \rho_{3-3}^{(i)} ^2 = \rho_{33}^{(i)} \rho_{3-3}^{(i)}$
5	$\frac{\sigma_{\beta 0}}{1-z^2} \{ \rho_{11}^{(i)} \rho_{33}^{(j)} + \rho_{11}^{(j)} \rho_{33}^{(i)} - 2 \frac{(1-z^2)}{x^2+y^2} \operatorname{Re} [\rho_{13}^{*(i)} \rho_{33}^{(j)}] \}$	$\frac{1-z^2}{x^2+y^2} \rho_{33}^{(i)} ^2 = \rho_{33}^{(i)} \rho_{11}^{(i)}$
6	$\frac{\sigma_{\beta 0}}{1-z^2} \{ \rho_{1-1}^{(i)} \rho_{3-3}^{(j)} + \rho_{1-1}^{(j)} \rho_{3-3}^{(i)} - 2 \frac{(1-z^2)}{x^2+y^2} \operatorname{Re} [\rho_{1-3}^{*(i)} \rho_{3-3}^{(j)}] \}$	$\frac{1-z^2}{x^2+y^2} \rho_{1-3}^{(i)} ^2 = \rho_{1-1}^{(i)} \rho_{3-3}^{(i)}$

$$\eta_{mn} = \begin{cases} 1 & \text{for } [m,n] \equiv [1,-3], [3,-1], \\ \frac{1-z^2}{x^2+y^2} & \text{for } [m,n] \equiv [1,-1], [3,-3], [3,1], [-1,-3] \end{cases} \quad (4c)$$

for any kinematical variables in the physical region, for any $[m,n] \equiv [1,-3], [3,-1], [1,-1], [3,-3], [3,1], [-1,-3]$ and any $i,j=1,2,3$. The sign of equality in (4b) holds if and only if

$$\rho_{mm}^{(i)} = \rho_{mm}^{(j)}, \quad \rho_{nn}^{(i)} = \rho_{nn}^{(j)}, \quad \rho_{mn}^{(i)} = \rho_{mn}^{(j)}, \quad (4d)$$

for any $i,j=1,2,3$ [see eqs. (4a) and the rank constraints listed in table II]. All these results are derived by using the results of table III, the equalities implied by eqs. (3b,c) [see eqs. (6b)] and the positivity conditions (3b) respectively. We remark that if the initial polarization vector lies in the reaction plane and if the degree of polarization is 100%, then $\eta_{mn} = 1$ for all $[m,n]$ and $\sigma_{\ell} = \sigma_{0\ell}$ [unpolarized differential cross sections]. Next, let us introduce the functions

$$F_{\beta \ell}^{(+\kappa)} = \frac{\sqrt{2}}{[1+|w|^2]^{1/2}} [f_{\ell}^{(a_p)} + w f_{\ell}^{(a_r)}], \quad F_{\beta \ell}^{(-\kappa)} = \frac{\sqrt{2}}{[1+|w|^2]^{1/2}} [-w^* f_{\ell}^{(a_p)} + f_{\ell}^{(a_r)}], \quad (5a)$$

which have the properties

$$|F_{\beta \ell}^{(\pm\kappa)}|^2 = [1 \pm \vec{\kappa} \cdot \vec{\xi}_{\beta \ell}] \sigma_{\beta \ell}, \quad (5b)$$

where w is an arbitrary complex number and $\vec{\kappa}$ is an arbitrary unit vector expressed in terms of w by the relation

$$\vec{\kappa} \equiv [2\text{Re}w, 2\text{Im}w, 1 - |w|^2] / [1 + |w|^2], \quad (5c)$$

when $\vec{\xi}_{\beta\ell}$ is defined as $\vec{\xi}_{\beta\ell} \equiv [A_{\beta\ell}, P_{\beta\ell}, R_{\beta\ell}]$. Then it is easy to see that, using the bilinear forms (5a,b,c) from ref.^{3/} for the functions $F_{\beta\ell}^{(\pm\kappa)}$ and also the results similar to eqs. (7b,c,d,f) [ref.^{3/}], we obtain the following isospin constraints:

Constraints 2. The isospin sum rules (2) alone imply that the spin density matrix elements must obey the constraints [see tables II and III] :

$$0 \leq -\lambda_{\beta\kappa}^{(\pm)} \equiv -\lambda [c_1^2 \sigma_{\beta 1} (1 \pm \vec{\kappa} \cdot \vec{\xi}_{\beta 1}), c_2^2 \sigma_{\beta 2} (1 \pm \vec{\kappa} \cdot \vec{\xi}_{\beta 2}), c_3^2 \sigma_{\beta 3} (1 \pm \vec{\kappa} \cdot \vec{\xi}_{\beta 3})], \quad (6a)$$

$$H_{\beta} \equiv c_1^2 c_2^2 H_{\beta 12} = c_2^2 c_3^2 H_{\beta 23} = c_3^2 c_1^2 H_{\beta 31}, \quad (6b)$$

$$4H_{\beta} \geq \lambda [\vec{\kappa} \cdot \vec{\xi}_{\beta} \sigma_{\beta}] \equiv \lambda [c_1^2 \sigma_{\beta 1} \vec{\kappa} \cdot \vec{\xi}_{\beta 1}, c_2^2 \sigma_{\beta 2} \vec{\kappa} \cdot \vec{\xi}_{\beta 2}, c_3^2 \sigma_{\beta 3} \vec{\kappa} \cdot \vec{\xi}_{\beta 3}], \quad (6c)$$

$$4H_{\beta} \leq -\lambda [\sigma_{\beta}] \equiv -\lambda [c_1^2 \sigma_{\beta 1}, c_2^2 \sigma_{\beta 2}, c_3^2 \sigma_{\beta 3}], \quad (6d)$$

$$|8H_{\beta} + \lambda [\sigma_{\beta}] - \lambda [\vec{\kappa} \cdot \vec{\xi}_{\beta} \sigma_{\beta}]| = [-\lambda_{\beta\kappa}^{(+)}]^{1/2} [-\lambda_{\beta\kappa}^{(-)}]^{1/2}, \quad (6e)$$

$$\lambda [x_1, x_2, x_3] \equiv x_1^2 + x_2^2 + x_3^2 - 2(x_1 x_2 + x_2 x_3 + x_3 x_1) \quad (6f)$$

for any kinematical variables in the physical region, any unit vector $\vec{\kappa}$ and all β .

An analysis of eqs.(4a,b,c) and (6a,b,c,d,e) in terms of the spin density matrix ele-

ments [based on the results listed in table II and III] suggests the following comments:

(i) The spin density matrix elements $\rho_{[m,m]}^{(\ell)}$, $[m,m] \equiv [1,1], [-1,-1], [3,3], [-3,-3]$, $\ell = 1, 2, 3$ must satisfy the inequalities

$$0 \leq -\lambda [c_1^2 \sigma_1 \rho_{mm}^{(1)}, c_2^2 \sigma_2 \rho_{mm}^{(2)}, c_3^2 \sigma_3 \rho_{mm}^{(3)}], \quad (7a)$$

which are equivalent to the usual triangle inequalities^{6/}:

$$|c_k |[\sigma_k \rho_{mm}^{(k)}]^{1/2}| \leq |c_i |[\sigma_i \rho_{mm}^{(i)}]^{1/2}| + |c_j |[\sigma_j \rho_{mm}^{(j)}]^{1/2}|. \quad (7b)$$

(ii) The isospin constraints (6c,d,e) for $\vec{\kappa} \equiv (0,0,1)$ and $[m,n] \equiv [1,-3], [3,-1]$ can be written in the following simple forms

$$\lambda [\sigma (\rho_{mm} - \rho_{nn})] \leq 4H_{mn} \leq -\lambda [\sigma (\rho_{mm} + \rho_{nn})], \quad (7c)$$

and

$$|8H_{mn} + \lambda [\sigma (\rho_{mm} + \rho_{nn})] - \lambda [\sigma (\rho_{mm} - \rho_{nn})]| = 4[-\lambda [\sigma \rho_{mm}]]^{1/2} [-\lambda [\sigma \rho_{nn}]]^{1/2},$$

respectively, where H_{mn} is defined by eqs. (4a,c). Hence, the test quantities H_{mn} up to a two-fold ambiguity can be expressed in terms of $\sigma_{\ell} \rho_{mm}^{(\ell)}$ and $\sigma_{\ell} \rho_{nn}^{(\ell)}$, $\ell = 1, 2, 3$. It is important to note that the isospin constraints: (4a,b,c) for $[m,n] \equiv [1,-3], [3,-1]$ and eqs. (7b,c,d) can be tested in the experiments with unpolarized target. If the initial polarization vector lies in the reaction plane ($z = 0$), then the constraints (7c,d)

hold also for $[m, n] \equiv [1, -1], [3, -3], [3, 1]$ and $[-1, -3]$ respectively.

Next, we remark that the equalities (4a,b,c) and (7d) and those similar to them are of great interest for a detailed test of isospin invariance in the quasi-two-body reactions (1) when complete and accurate experimental data will be available. These constraints are also important to test the treatments of experimental data such as the extraction of the quasi-two-body channels from the many body final states or the Λ^0 - Σ^0 separations, etc.

Finally, we note that each of the isospin constraints (4a,b,c) and (6a,b,c,d) has an integrated analog. This consequence of the isospin sum rule (2) can be derived by using the method of the $I_p[F]$ -integrals^{/3/} and the results of ref.^{/7/} for each of the functions $F_\beta^{(\pm\kappa)}$ defined by eqs. (5a,b).

The constraints (4a,b,c) and (6a,b,c,d,e) improve in the most general form the isospin bounds on spin density matrix elements previously obtained^{/6, 8, 9/}. We note, of course, that eqs. (4a,c) and (7c) represent a complete extension of the equalities (10) and bound (12) from ref.^{/5/} to the spin density matrices of $[0^- 1/2^+ \rightarrow 0^- 3/2^+]$ reactions.

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