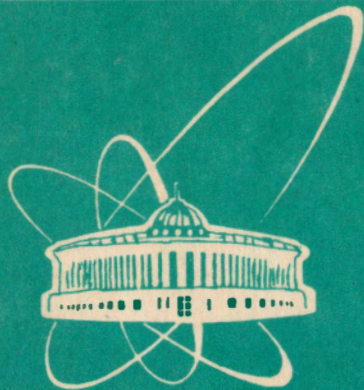


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СООБЩЕНИЯ  
ОБЪЕДИНЕННОГО  
ИНСТИТУТА  
ЯДЕРНЫХ  
ИССЛЕДОВАНИЙ  
ДУБНА

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MICROSCOPIC CALCULATIONS  
OF NUCLEON-NUCLEUS SCATTERING  
IN GLAUBER THEORY

1993

1. The celebrated Glauber multiple scattering theory [1] is a widely accepted theory of high-energy hadron-nucleus and nucleus-nucleus diffraction scattering.

In generally calculating the hadron-nucleon scattering one must take into account quark-clusterization effects. For this purpose we consider the matrix element of the profile operator calculation method in the nuclear shell model.

It is easily shown that the result of using Slater determinant wave functions between single particle states of the nuclear-shell model [2] is

$$F_{fi} = \frac{iR}{2\pi} \int e^{iqb} \left[ \delta_{fi} - \text{Det} \left| A_{n'l'm'}^{nlm} \right| \right]. \quad (1)$$

The matrix element  $A_{n'l'm'}^{nlm}$  is a linear combination of integrals of the form:

$$\int d^3r R_{nl}(r) R_{n'l'}(r) Y_{ln}^*(\hat{r}) Y_{l'm'}(\hat{r}) \Gamma(b-s), \quad (2)$$

where  $R_{nl}$  are harmonic oscillator radial wave functions,

$$\Gamma(b-s) = \gamma_0 e^{-(b-s)^2/2B}.$$

These integrals are evaluated analytically [2]:

$$\begin{aligned} M_{nem}^{n'e'm'} &= \sum_{k=0}^{n+n'} \phi(k) e^{i(m-m')\frac{\pi}{2}} \int bdbJ_{|m-m'|}(qb) \times \\ & \times \frac{e^{-\frac{\alpha^2 b^2}{\alpha^2+a}} (-2jab)^{|m-m'|} \Gamma\left(\frac{2k+|m|+|m'|+|m-m'|+2}{2}\right)}{2^{|m-m'|+1} (a+\alpha^2)^{2k+|m|+|m'|+|m-m'|+2} \Gamma(|m-m'|+1)} \times \\ & \times {}_1E_1\left(\frac{|m-m'|-2k-|m|-|m'|}{2}, |m'-m|+1, \frac{\alpha^2 b^2}{\alpha^2+a}\right), \quad (3) \end{aligned}$$

where

$$\phi(k) = \left[ 2(n+n') - 2k + l + l' - |m| - |m'| - 1 \right] !! \quad (4)$$

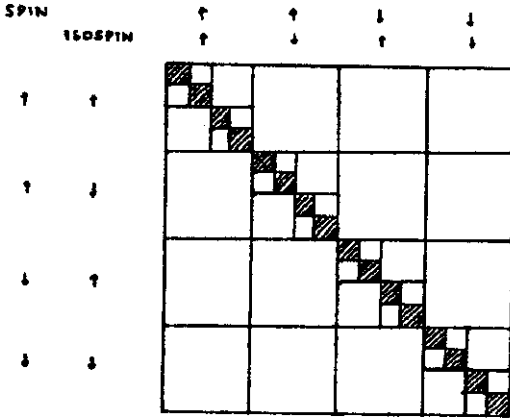


Fig.1

When  $m + m'$  is even then  $l + l'$  is odd, and when  $m + m'$  is odd then  $l + l'$  is even. For other cases

$$\phi(k) = 0.$$

Going into detail one finds that the matrix  $A_{mn} = 1 - M_{n'l'm}^{nlm}$  may be rearranged to give exactly one nonzero submatrix per closed shell, where closed shells are counted per spin up or down, per isospin up or down and per main quantum number.

As a result of condition (3) and symmetrical dependence of

(2) on  $m$  and  $m'$ , these submatrices can be transformed to two nonzero submatrices. (Figure 1 illustrates the matrix as having zeros everywhere outside the hatched areas).

The particular case shown corresponds to  $^{16}\text{O}$  with closed shells in the terminology used here.

In the  $z$  direction approximation [3] we can rewrite the multiple scattering formula as follows for elastic scattering:

$$F_{ii} = \frac{ip}{2\pi} \int e^{iqb} d^2b \left\{ 1 - \prod_{i+1}^N \left( 1 - N_i \varphi_n^* \gamma (b-s) \varphi_n \right) \right\}. \quad (5)$$

2. The nonantisymmetrized wave function of the nucleus in the oscillator-cluster model can be written as [4]

$$\psi = \varphi_{N_1}(r_1, r_2, r_3) \cdots \varphi_{N_A}(r_{3A-2}, r_{3A-1}, r_{3A}) \chi(\tilde{R}_1, \tilde{R}_2, \dots, \tilde{R}_A). \quad (6)$$

Here the nucleus is pictured as a bag (at radius  $R_A$ ) located at  $\tilde{R}_A$  enclosing  $A$  nucleons with radius  $R_h$  located at  $R_{3i}$ . Using the representations:

$$e^{-3\left(\frac{1}{R_A^2} - \frac{1}{R_h^2}\right)R_i^2} = \left[ \frac{3(R_h^2 - R_A^2)}{\pi R_h^2} \right]^{3/2} \int e^{-6\beta_i \left(\frac{1}{R_A^2} - \frac{1}{R_h^2}\right)R_i} e^{-3\left(\frac{1}{R_A^2} - \frac{1}{R_h^2}\right)\beta^2} d^3\beta \quad (7)$$

and the relation

$$R_i = \frac{r_{3i-2} + r_{3i-1} + r_{3i}}{3}; \quad (i = 1, 2, \dots, A),$$

we can write (5) in a formally factorized form:

$$\begin{aligned} \psi = N \prod_{j=1}^A \exp \left[ -\frac{r_{3j-2}^2 + r_{3j-1}^2 + r_{3j}^2}{R_h^2} - 6\beta_j i \left( \frac{1}{R_A^2} - \frac{1}{R_h^2} \right) \times \right. \\ \left. \times (r_{3j-2} + r_{3j-1} + r_{3j}) - 3 \left( \frac{1}{R_A^2} - \frac{1}{R_h^2} \right) \beta^2 \right] p_n(r_j) Y_{lm}(\theta, \varphi). \end{aligned} \quad (8)$$

For this factorized function we can use the above-mentioned calculation method for the scattering amplitude.

For nucleus-nucleus scattering the proposed method is generalized trivially.

3. Integration by quark coordinates and parameter coordinates in (1) is very cumbersome for the case of nucleus-nucleus scattering.

Moreover, in Glauber's theory the amplitude of nucleus-nucleus scattering is determined by a sum of  $2^{A \cdot B} - 1$  terms [5,6,7] representing different rescattering processes. Among these terms there are many similar ones. That is why the amplitude is actually determined by a smaller number of essentially different terms.

Reduction of similar terms in the scattering amplitude was shown in earlier papers [8,9]. In particular, the general term of the Glauber series can be represented in the form [5]:

$$\begin{aligned} N \frac{i^p}{2\pi} \int \exp \{-x^T Q x - 2bH^T x - 2b^2 c + iqb\} d^{m+n} x d^2 b = \\ = \frac{\pi^{n+m+1}}{\text{Det } W} \exp \left\{ -\frac{q^2}{4} \left| \frac{\text{Det } Q}{\text{Det } W} \right| \right\}. \end{aligned} \quad (9)$$

Using the block structure of the matrices  $Q$  and  $W$  and the scattering diagram method [5], we can write a simple expression for the scattering amplitude. For the case when  $B = 3$  and  $A$  is arbitrary, we have [5]:

$$\begin{aligned} F^{el}(q) = K_A(q) K_B(q) \sum_{\substack{n_1 n_2 \dots n_7 \\ (0 < n_1 + \dots + n_7 \leq A)}} C_A^{n_1 + \dots + n_7} \left( \frac{n_1 + \dots + n_7!}{n_1! \dots n_7!} \right) \times \\ \times \left( -\frac{\tilde{\sigma} a}{2\pi} \right)^{n_1 + n_2 + n_3 + 2(n_4 + n_5 + n_6) + 3n_7} \frac{(\pi^{n_1 + \dots + n_7 + 1})}{\text{Det } W (n_1, \dots, n_7)} \times \end{aligned}$$

$$\times \left(\frac{d}{\pi}\right)^3 \left(\frac{t}{\pi}\right)^{n_1+\dots+n_7} \exp \left\{ -\frac{q^2}{4} \frac{\text{Det } Q}{\text{Det } W} \right\}. \quad (10)$$

4. The microscopic calculation of  $pd$  elastic scattering at high energies seems to be the best source of information about a short-distance nuclear structure [10,11]. Goggi et al. [6] found a definite discrepancy between the experimental data and theoretical calculations in the nucleon model of deuterons.

On the other hand, calculation in the nucleon model with the more realistic Paris wave function of deuteron and a profile-function of the form [12]:

$$\gamma(b) = \frac{1}{\sqrt{\pi}} \left\{ \frac{A_1}{B_1} e^{-\frac{b^2}{2B_1\alpha}} + \frac{A_2}{B_2} e^{-\frac{b^2}{2B_2\alpha}} - \frac{A_2}{B_3} e^{-\frac{b^2}{B_3\alpha}} \right\}, \quad (11)$$

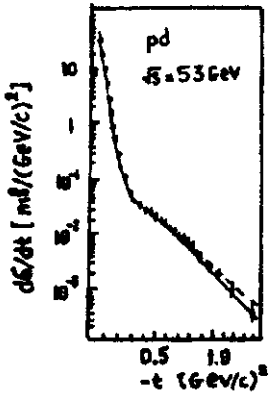


Fig.2

gives a better agreement with experimental results (see fig.2, where the dashed line corresponds to the  $D$  wave contribution). As illustrated, the result of calculation is very sensitive to the form of wave function and profile operator.

Therefore, the microscopic calculation is necessary for precise analysis of data. A deuteron in the quark-cluster model is described as a system of two clusters, antisymmetrized with respect to the quark variables. The following expression is most popular at present for the wave function of deuteron:

$$\psi_d = A \left\{ \psi_p(r_1, r_2, r_3) \psi_n(r_4, r_5, r_6) \chi(R) \right\},$$

where

$$\chi(R) = \frac{1}{\sqrt{4\pi}} \frac{U(R)}{R} \chi_{1M} + \frac{W(R)}{R} \sum (2m1_\mu | 1M) Y_{2m}(\hat{R}) \chi_{1\mu},$$

$$\chi_{1\mu} = \sum \left( \frac{1}{2} \sigma_1 \frac{1}{2} \sigma_2 | 1\mu \right) \left( \frac{1}{2} \tau_1 \frac{1}{2} \tau_2 | 00 \right) \phi_{\sigma_1 \tau_1}^2 \frac{1}{2} \frac{1}{2} \phi_{\sigma_2 \tau_2}^2 \frac{1}{2} \frac{1}{2}.$$

The space parts of the  $S$  and  $D$  state wave functions are taken here from ref. [13]

$$U(R) = \sum_j C_j e^{-m_j R}, \quad (12)$$

$$W(R) = \sum_j D_j e^{-m_j R} \left( 1 + \frac{3}{m_j R} + \frac{3}{m_j^2 R^2} \right). \quad (13)$$

Using the formula

$$\frac{e^{-mr}}{n} = \frac{m^{n-1}}{(n-2)!} \int_0^\infty dR (R-1)^{n-2} \frac{e^{-mRr}}{r}, \quad (14)$$

we can write (12) and (13) in the Gaussian representation, then we can use the above-developed method of microscopic calculation.

The result of the microscopic calculation is shown in fig.2. The parameters of  $qq$ -interactions were taken from ref. [9]

5. Proton-helium scattering [16—20] has been the touchstone of the Glauber theory since its first derivation, showing a remarkable agreement between the predicted and experimental structure of the angular scattering distributions.

A detailed comparison with the most recent and precise data on elastic scattering [21,22], however, seems to display a small but definite discrepancy between the data and some characteristic features of the model, in particular the position of the first diffraction dip, the forward slope of the cross-section and the relative height of the cross-section of the optical point and after the dip. Dakhno and Nicolaev [23] have shown thorough and accurate analysis that Glauber's theory with inelastic shadowing displays again a systematic disagreement with the data.

They found that there is a persistent disagreement between theory and the data of high-accuracy experiments, which cannot be eliminated in the conventional picture of the particle made of four nucleons.

Therefore we employ the nonrelativistic quark-cluster model to describe a nucleon, and treat the particle as a system of four clusters which is totally antisymmetrized with respect to the quark variables. The wave function for the  $\alpha$  particle is thus given by

$$\psi_\alpha = A_\alpha \left\{ \left[ |N\rangle^{(\sigma\tau\chi)} \right]^4 \right\}_{s=0, \tau=0} \phi_{N_1} \phi_{N_2} \phi_{N_3} \phi_{N_4} \chi(\tilde{R}_1, \tilde{R}_2, \tilde{R}_3, \tilde{R}_4). \quad (15)$$

In our case  $A_\alpha$  consists of  $12!/(3!)(4!) = 15400$  terms, which are classified into 12 essentially different terms [24].

The results of the microscopic calculations are shown in fig.3. The solid line corresponds to the cross-section calculated for the direct integral, and the dashed line corresponds to the contribution of exchange integrals [25].

6. In Glauber's theory for description of hadron or light nucleus scatterings by heavy nucleus, any method of approximation can usually be used (optical limit approximation, Gyz—Maximon approximation, rigid projectile approximation, etc.).

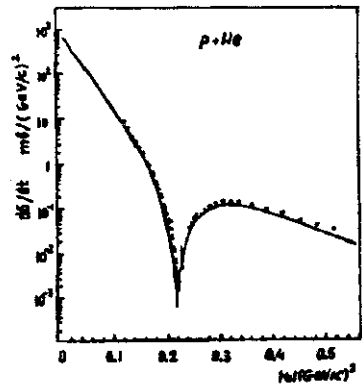


Fig.3

Therefore, the microscopic calculation in case of scattering by heavy nucleus required the introduction of the following intercluster coordinates:

$$\begin{aligned}
 \bar{S}_1 &= S_1 - R_1, \\
 \bar{S}_2 &= S_2 - R_1, \\
 \bar{S}_3 &= S_3 - R_1 = -(\bar{S}_1 + \bar{S}_2), \\
 &\dots\dots \\
 &\dots\dots \\
 \bar{S}_{3A-2} &= S_{3A-2} - R_A, \\
 \bar{S}_{3A-1} &= S_{3A-1} - R_A, \\
 \bar{S}_{3A} &= S_{3A} - R_{3A} = -(\bar{S}_{3A-2} + \bar{S}).
 \end{aligned} \tag{16}$$

In these coordinates the wave function can be written as:

$$\psi = \left( \frac{3\alpha^2}{\pi^2} \right)^{\frac{A}{2}} \prod_{i=1}^A e^{-\alpha^2 (r_{3i-2}^2 + r_{3i-1}^2) - \alpha^2 (r_{3i-2} r_{3i-1})} \chi(R)_i. \tag{17}$$

For the Gaussian parameterization of the profile operator, after integration by intercluster coordinates, we have the following expression for the amplitude of scattering:

$$\begin{aligned}
 E(q) = \frac{ip}{2\pi} \int d^2 b e^{iqb} \langle \chi^*(R) | & \left[ 1 - \prod_{n=1}^A \prod_{k=1}^B \left( 1 - \sum_{i=1}^7 \frac{3\gamma_0 \alpha}{\Delta_i} e^{-\tilde{a}_i (b - R_n + r_{i_k})^2} \right) \right] \times \\
 & \times |\chi(R)\rangle; \quad (j = 1, 2, \dots, 3B), \tag{18}
 \end{aligned}$$

where

$$\tilde{a}_1 = \left[ -2a + \frac{a^2(a + 2\alpha)}{\Delta_1} \right];$$

$$\tilde{a}_2 = \left[ -a + \frac{2a^2\alpha}{\Delta_2} \right];$$

$$\tilde{a}_3 = \left[ -2a + \frac{2a^3}{\Delta_3} \right];$$

$$\tilde{a}_4 = \left[ -a + \frac{a^2(a + 2\alpha)}{\Delta} \right];$$

$$\begin{aligned}\tilde{a}_5 &= [-3a]; \\ \tilde{a}_6 &= \left[ -2a + \frac{a^2(a+2\alpha)}{\Delta_6} \right]; \\ \tilde{a}_7 &= \left[ -a + \frac{a^2(a+2\alpha)}{\Delta_7} \right];\end{aligned}$$

$$\begin{aligned}\Delta_1 &+ \frac{1}{a^2 + 4a\alpha + 3\alpha^2}; & \Delta_5 &= \frac{1}{3}(a + \alpha)^2; \\ \Delta_2 &= \frac{1}{2a\alpha + 3\alpha^2}; & \Delta_6 &= \Delta_1; \\ \Delta_3 &= \Delta_1; \quad \Delta_4 + \Delta_2; & \Delta_7 &= \Delta_2.\end{aligned}$$

The results of the calculation using (10) are shown in fig.4.

The experimental data was taken from ref. [26]. The dashed line corresponds to the microscopic calculation, and the dash double-dotted line corresponds to the nucleon model calculation. For calculating the differential cross-section of the quasi-elastic  $\alpha$ -nucleus scattering (dotted line), an exponential approximation [27–32] was used.

The solid line corresponds to  $\frac{d\sigma^{q-el}}{dt} \rightarrow \frac{d\sigma^{el}}{dt} + \frac{d\sigma^{el}}{dt}$  in the quark model.

Our results can be summarized as follows:

1. Taking into account the quark structure of nucleons, we have significantly improved the description of the data, thus restoring the agreement between the Glauber theory and the experiment.

2. The quark-clusterization effect is not negligible in this case.

3. The microscopic calculation is necessary for precise calculations.

4. The multiple scattering terms which are usually neglected are not unimportant.

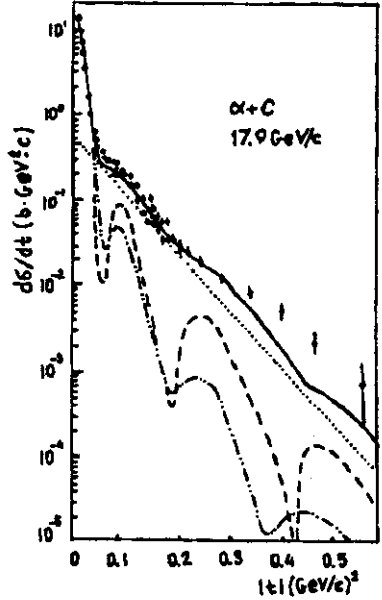


Fig.4



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