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PROTON ELECTROMAGNETIC FORM FACTOR NEAR THE PROTON-ANTIPROTON THRESHOLD AND QUASINUCLEAR BOUND STATE

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1. Introduction

Experiments performed at LEAR at CERN provided new information on the $P\bar{P}$ interaction. This information includes the data both on $P\bar{P}$ elastic scattering [1, 2] and on the proton electromagnetic form factor in the time-like region near $4M^2$ [3]. A common feature of these data is that they are difficult for being interpreted within many models. To overcome these difficulties, some interesting hypotheses are to be put forward. For instance, in ref. [4] for explaining the energy dependence of the ρ -ratio of the real to imaginary part of the forward elastic $P\bar{P}$ scattering amplitude, low-lying resonances are assumed to exist in the P and D waves. In ref. [5], a great role of the P wave in elastic $P\bar{P}$ scattering at low energies is attributed to the bound states in a $P\bar{P}$ system. Analogous conclusions were drawn in refs. [6] and [7] within the phase-shift analysis of elastic $P\bar{P}$ scattering and the ρ- ratio. It is obvious that a bound state in the $P\bar{P}$ system with a small binding energy may influence on the behavior of the proton electromagnetic form factor. The discovery of that influence will indicate its possible quantum numbers.

2. Formulation of the Model

The model is based upon analytic properties of the Dirac and Pauli isoscalar and isovector form factors. The latter are expressed in terms of the experimental electric and magnetic proton form factor by the known formulae

$$G_E^P(S) = [F_1^s(S) + F_1^v(S)] + \frac{t}{4M_p^2}[F_2^s(S) + F_2^v(S)]$$

$$G_M^P(S) = [F_1^s(S) + F_1^v(S)] + [F_2^s(S) + F_2^v(S)]. \tag{1}$$

Analytic properties of the form factors $F_i^{s,v}(S)$ are determined by dispersion relations that permit us to formulate various models. We will now write them without subtraction:

$$F_i^{s,v}(S) = \frac{1}{\pi}\int_{S_i^{s,v}}^{\infty} \frac{Im\, F_i^{s,v}(s)}{S' - S} dS', \tag{2}$$

where $S^s = 4m_\pi^2$ $S^v = 9m_\pi^2$.

For practical use of the relation (2) we should apply to the unitarity condition and connect the expression for $ImF_i^{s,v}(S)$ with amplitudes of other processes. The general form is as follows:

$$Im < 0 \,|\, j_\mu \,|\, N\bar{N} > = \sum_n < 0 \,|\, j_\mu \,|\, n >< n \,|\, T^+ \,|\, N\bar{N} >, \tag{3}$$

where summation runs over the complete set of admissible intermediate states. In our case it is of the form :

$$| n >=| 2\pi >, | 3\pi > \cdots | K, \bar{K} >, | N, \bar{N} > \cdots \quad (4)$$

Frazer and Fulco were the first who computed two-meson contributions [8] and predicted the existence of ρ-meson. Then, along this line being developed, the vector dominance model was formulated [9] that in terms of the dispersion relations (2) looks as follows:

$$Im\, F_i^{s,v}(S) = \sum_{s,v} m_{s,v}^2 \frac{f_{s,v,N\bar{N}}^{(i)}}{f_\rho} \delta(S - m_{s,v}^2). \quad (5)$$

Equation (5) is approximate as the sum contains only one-meson states and thus does not take account of abundant experimental data and theoretical models of two-particle channels [10]. The additive nature of eq.(5) is a consequence of the unitarity condition (3). Below we will employ an analytic and unitarized version of the vector dominance model thoroughly described in ref. [11,12]. Namely, we will apply the latest version in which every of the functions $F_i^{s,v}(S)$ has its own effective threshold $S_i^{s,v} < 4M_p^2$ so that all the form factors $S_i^{s,v} < 4M_p^2$ are complex when $S \geq 4M_p^2$. In this model, the sum in (5) includes mesons $s = (\omega, \omega', \omega''); v = (\rho, \rho', \rho'', \rho''')$. Comparison of the results for $|G| = |G_E^P| \simeq |G_M^P|$ calculated by models [12] and experimental data [3] shows that the theoretical curve does not describe these data (Fig.1). Experimental points are below the theoretical curve just above the threshold $4M_p^2 = 3.523 GeV^2$, when $S \approx 3.75 GeV^2$, they cross the curve, and in the interval up to $S \approx 4.2 GeV^2$ lie above that curve. In ref. [3] a theoretical curve was constructed on the basis of calculations made within the vector dominance model [11] and it lies higher than our curve; as a result, only the last experimental point is above the computed value. Thus, irrespective of the theoretical model used, the common deviation from all theoretical predictions takes place for experimental points, which certainly points to the necessity of inclusion of the contribution of the state $|N\bar{N}>$- to the unitarity condition (3). Earlier we have thoroughly studied the forward elastic $P\bar{P}$ scattering amplitude [7] for which we have constructed an analytic model based on the uniformizing variable:

$$Z = \frac{5}{4}\sqrt{\frac{S-1.44}{S}} - \frac{3}{4}\sqrt{\frac{S-4}{S}}, \quad (6)$$

where S- is a conventional Mandelstam variable, the total energy squared of a $P\bar{P}$ system in the c.m.s. in M_p units. The variable Z contains the thresholds of PP and $P\bar{P}$ reactions important for $P\bar{P}$- scattering at points $S = 0; 4$ and the effective threshold at $S = 1.44$ in the nonobservable region. The threshold of the $P\bar{P} \longrightarrow P\bar{P}$ reaction is given by the point $Z = 1$ (Fig.2) and disposition of four sheets of the

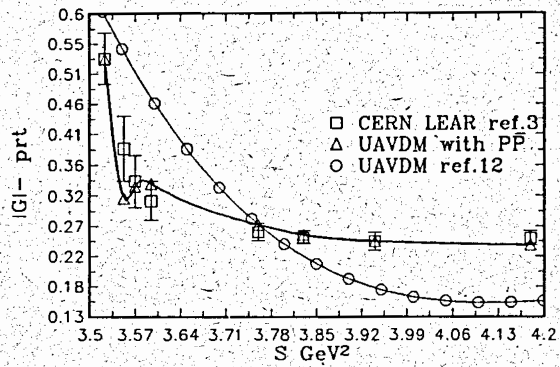

Figure 1: Proton form factor for $S \geq 4M_p^2$ when a quasinuclear bound state of $P\bar{P}$ is present

Riemannian surface of the function $Z(S)$ can be seen from the Fig.2. As has been shown [7], the experimental data on $\rho = ReT_{P\bar{P}}/ImT_{P\bar{P}}$ and σ_{tot} can be explained only if we assume that a $P\bar{P}$ system possesses a quasinuclear state with the binding energy $E = (1.88 \pm 0.05) MeV$ and width $\frac{\Gamma}{2} = (0.80 \pm 0.05) MeV$. The amplitude was assumed in the following form:

$$T_{P\bar{P}}(S) = \sum_{n=0}^{N} A_n (1-Z)^n + \frac{C_\rho}{Z-(Z_\rho)_1} - \frac{C_\rho}{Z-(Z_\rho)_2}, \quad (7)$$

where $(Z_\rho)_{1,2} = 1 \mp \gamma \pm i\delta$ and $N = 2$. The pole terms describe the quasinuclear state; whereas the sum, a nonresonance background of the S-,P- and D-partial waves. The amplitude (7) well describe the experimental data in the region up to $4.4 GeV^2$ in the variable S. Around the threshold $S = 4$ poles of the quasinuclear state are dominating in (7). In the pole approximation, the unitarity condition (3) reduces to Riemann boundary-value problem [14] that can be solved. In the ring, containing the unit circle (Fig.2) the solution looks as follows:

$$G_{pol} = \frac{C(z)}{\prod_{i=1}^{2}(z-(z_\rho)_i)(z+(z_\rho)_i^*)}\Big[\frac{(4z^2-1)(4-z^2)}{9z^2}\Big]^{1/2}; \quad z \approx 1 \quad (8)$$

where $C(z) = C(-z)$. So, by choosing $C(Z)$ of the form

$$C(z) = A_1\Big\{\Big(\frac{1}{z-z_1} - \frac{1}{z-z_2}\Big) - \Big(\frac{1}{z+z_1^*} - \frac{1}{z+z_2^*}\Big)\Big\} +$$

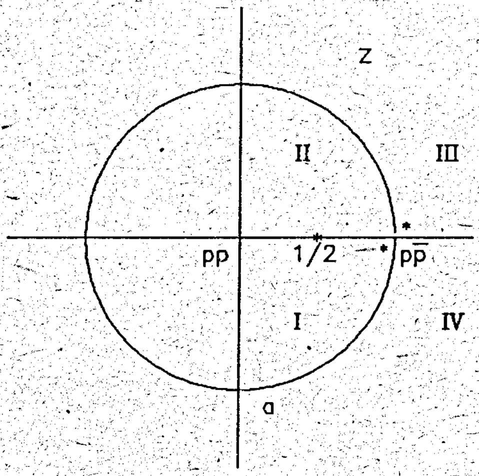

Figure 2: Disposition of four sheets of the riemannian surface of function Z(S)

$$+A_2\{(\frac{1}{z-z_1}+\frac{1}{z-z_2})-(\frac{1}{z+z_1^*}+\frac{1}{z+z_2^*})\}, \qquad (9)$$

we get the following expression for the form factor G :

$$G = G_{UAVDM} + G_{pol}, \qquad (10)$$

where G_{UAVDM} is taken from [12] and G_{pol}- is defined by (9). Formula (10) with two parameters A_1 A_2 is a consequence of the unitarity condition (3) being additive. A rapid change of the derivative of $|G(S)|$ around the threshold $S = 4$ is determined by the pole term in (10), i.e. by strong $N\bar{N}$- interaction on the background of the complex contribution G_{UAVDM} . Therefore, the observed behavior of $|G(S)|$ shows the influence of the pole term on the real and imaginary parts of G_{UAVDM}.

3. Analysis of Experimental Data

We will analyse the experimental data in the range $3.523GeV^2 < S < 4.19GeV^2$ only on the basis of the results derived in ref. [3] as they provide basic information on $|G|$ in that range. The position of poles is fixed by the results from ref. [7] and is determined by the quantities δ and γ: and $10^2\delta = 3.46 \pm 0.1$, $10^2\gamma = -0.72 \pm 0.03$. The quantities A_1 and A_2 are free parameters. When $A_1 = A_2 = 0$, then $\chi^2 \sim 450$. The parameter A_1 can lower the value of χ^2 down to ≈ 150. The results of the analysis with both parameters are listed in Table . Though the quantity $\frac{\chi^2}{NDF} = \frac{4.9}{6}$ is small, it can be reduced because the arguments of three points belong to the intervals.

$S\ GeV^2$	$\|G\|_{UAVDM}$	$\|G\|$	$\|G\|_{exp}$	χ_i^2
3.523	0.604	0.529	0.53 ± 0.02	$\sim 10^{-3}$
3.546-3.56	0.536	0.313	0.39 ± 0.05	2.37
3.56-3.58	0.509	0.333	0.34 ± 0.04	$\sim 10^{-2}$
3.58-3.61	0.468	0.336	0.31 ± 0.03	0.75
3.76	0.273	0.273	0.262 ± 0.014	0.62
3.83	0.224	0.257	0.253 ± 0.01	0.16
3.94	0.175	0.247	0.247 ± 0.014	$\sim 10^{-3}$
4.18	0.154	0.241	0.252 ± 0.011	1.0
$10^3A_1 = 5.18 \pm 2.9$	$10^3A_2 = -8.01 \pm 0.94$		$\chi^2 = 4.93$	

4. Discussion of the Results

Assuming that the elastic $P\bar{P}$ scattering amplitude has near-threshold poles responsible for the quasinuclear bound state of a $P\bar{P}$ system we could derive formula (10) for the proton electromagnetic form factor that well describes the experimental results [3]. This formula looks like the formula for the $(\rho - \omega)$- interference in the time-like region of the pion electromagnetic form factor [13]. The derivative $\frac{d|G|}{dS}$ at the $S = 4M^2$ threshold gets infinite, in agreement with the conclusions drawn in ref. [15]. Permissible quantum numbers of the quasinuclear state are determined by the one-photon mechanism of its production and may be $^3S_1$, $^3D_1$. Such states appear in a number of potential and one-channel optical models [16,17], and the range of variation of binding energies and widths is large. In this connection we mention once again the discussion of the influence of wide and narrow D- resonances on elastic $P\bar{P}$- scattering, σ_{tot} and ρ . The disposition of poles of the quasinuclear bound state on the four-sheeted Riemannian surface is such (Fig.2) that one of them may appear as a low-energy resonance. This possibility can be investigated if parameters δ end γ are free, but the appropriate set of experimental data is scarce. Therefore it is important to study the behavior of σ, ρ and polarization of elastic $P\bar{P}$- scattering for $P_L \leq 180\,MeV/c$ and proton electromagnetic form factor G near the threshold. This recommendation is in complete agreement with the conclusions of ref. [18] where experimental aguments are given in favor of narrow $(P\bar{P})$ bound states near the thesold.

Thus, just one system of poles generated by the quasinuclear bound state allowed us to interpret both the peculiarities of forward elastic $P\bar{P}$ scattering and the new experimental data on the proton electromagnetic form factor measured in experiment PS-170 at LEAR, CERN.

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