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THE HEISENBERG — PAULI QUANTIZATION OF GRAVITY

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1. Introduction

The main difficulty in the quantization of gauge theories is the nonphysical character of gauge field components which have the zero canonical momentum, or are not included in the action owing to the gauge invariance.

And the following dilemma arises: i) to quantize only the physical part of the components, contradicting the manifest Lorentz invariance, but conserving all quantum and gauge principles; or ii) to quantize all components by a relativistic invariant approach, but not being anxious about the nonconservation of quantum and gauge principles in this approach. We shall call these approaches the "quantum" and "relativistic" ones.

The first "quantum" approach to the quantization of electrodynamics was considered in the pioneer papers by Heisenberg and Pauli [1, 2], where the gauge theory was treated as one of the types of relativistic quantum mechanics in accordance to the Weyl formulation [3].

The highest achievement of the "relativistic" approach is the Faddeev-Popov (FP) method [4] based on the Dirac quantization [5]. The simplicity and efficiency of this method stimulated in many aspects the contemporary development of gauge field theory. In the context of this development the old "quantum" approach is conventionally considered as a particular case of the choice of a nonrelativistic gauge, which is needed only to demonstrate that all quantum principles are fulfilled for all gauges due to their equivalence.

The theorem of equivalence of different gauges is the one of the basic elements of the FP method and defines the region of its validity.

It is worth to recall that the equivalence theorem is strictly proved [5, 6] only for asymptotically free states of the elementary particles (on their mass-shell), or for the asymptotically flat space-time in the case of gravity, which restricts the region of the application of the FP method by the scattering problem and the "island" Universe.

The fact of the nonequivalence of different gauges off mass-shell is well known and moreover it is used for the derivation of the Ward-Slavnov-Taylor identities [5].

The same off mass-shell nonequivalence of different time-axes of the Heisenberg-Pauli (HP) quantization [1, 2] of electrodynamics is also known in the atomic physics [6].

There are two different points of view on this time-axis dependence of the HP quantization: i) the "theoretical point" [5] is to consider the time-axis dependence as the defect of the quantization scheme and to try to get rid of this time-axis by the transition to a relativistic invariant gauge, and ii) the "practical point" [6] is the description of an atom in the rest frame and a moving atom by different time-axes (gauges) with the rest, or moving, Coulomb field, correspondingly (the choice of the rest time-axis for a moving atom breaks down the relativistic dispersion law [7]). In other words, in the atomic physics this "theoretical defect" is used just for the relativistic covariant description of bound states formed by elementary particles off mass-shell. In the last case the time-axis dependence of the HP quantization is not a "defect", but a useful physical fact.

Thus, the experience of the atomic physics not only testifies to the dependence of the off mass shell physical results on gauges but also uses this dependence for relativistic transformations in accordance with the initial interpretation of the HP quantization. In the light of this fact, a relativistic invariant gauge of the FP method looks as only the technical reception for the calculation of scattering amplitudes on mass–shell, and is not suitable for the description of the bound state physics and of nonflat space-time in gravity. The latter was emphasized by the authors of the FP method [8]. Schwinger [9] was the first who pointed out (before the FP method formulation [4]) in a sharp form the possible difference between the HP quantization and the relativistic gauge one as the consequence of the noncommutativity of constraining and quantizing.

In this paper we shall apply the non-Abelian generalization [6, 10, 11] of the HP method [1, 2] to quantization of the time-reparametrization invariant theories, including the relativistic particle, string and the Einstein gravity, and shall consider the concrete examples of the physical nonequivalence of the two approaches: the "quantum" and the "relativistic" one.

In the second section we discuss the minimal HP quantization for the simplest example of the zero-dimension gravity.

In section 3 the models of relativistic particle and strings are considered.

Section 4 is devoted to the Einstein gravity in n+1-dimensional space-time.

In section 5 we consider the Friedmann approximation.

In section 6 the influence of inhomogeneity and the evolution of the Newton law are estimated.

2. The Zero-Dimension Gravity

To emphasize the difference between the HP and FP schemes of quantization we consider at first the case of the zero-dimension gravity:

$$W = \int_0^T dt\mathcal{L}; \quad \mathcal{L} = -\frac{1}{2}\Big[\frac{1}{\alpha}\dot{q}^2 + \alpha F(q)\Big]; \quad \dot{q} = \partial_t q, \tag{1}$$

where F(q) is an arbitrary function over the variable q.

There are two variables α and q with the canonical momenta

$$P_\alpha = \frac{\partial \mathcal{L}}{\partial \dot{\alpha}} \equiv 0; \quad P = \frac{\partial \mathcal{L}}{\partial \dot{q}} = -\frac{1}{\alpha}\dot{q} \tag{2}$$

and the Hamiltonian

$$H = \dot{\alpha}P_\alpha + \dot{q}P - \mathcal{L} = \alpha\mathcal{H}; \quad \mathcal{H} = \frac{1}{2}\Big[F^2 - P^2\Big]. \tag{3}$$

The classical motion equation for α

$$\{H, P_\alpha\} = \mathcal{H} = 0; \quad \left(\left(\frac{1}{\alpha}\dot{q}\right)^2 = F^2(q)\right) \tag{4}$$

allows one to define the proper invariant time

$$dT_F = \alpha dt \quad \mapsto \quad T_{F(\pm)} = \pm \int_0^q \frac{dq}{F} \tag{5}$$

which coincides with the Friedmann time in the homogeneous cosmology.

The classical equation for the variable q faithfully copies eqs.(2), (4) and does not contain any new information.

In the quantum theory according to the correspondence principle we should reproduce the Friedmann evolution.

Let us apply at first the conventional quantization [4, 12], where both the variables are included in the canonical scheme, and constraints $P_\alpha = 0, \ \mathcal{H} = 0$ are imposed on the wave function. The last equation

$$\mathcal{H}\Psi = 0 \Rightarrow \left[\left(\frac{d}{dq}\right)^2 + F^2(q)\right]\Psi(q) = 0 \tag{6}$$

is well known in quantum gravity as the Wheeler-DeWitt equation [12, 13]. This constraint should be completed by "gauges" $f(q) = 0, \alpha = 1$, restricting the group of gauge invariance. This result is interpreted as the stationary picture without any evolution [13].

The minimal quantization [6, 10, 11] (which reproduces the HP quantization in electrodynamics [1, 2]) is based on the construction of the minimal set of dynamical variables by explicitly solving the classical equations for the components without time derivative in the action. This quantization at all the steps conserves all quantum principles (uncertainty, observability, correspondence) and gauge invariance. For example, the initial theory (1) cannot be considered as quantum with respect to the component α as the constraints (2), (4) fix simultaneously the canonical momentum P_α and "coordinate" α, and contradict the uncertainty principle. To conserve this principle we should use for the construction of the canonical scheme only the initial action (1), or its first-order formalism version

$$W_I = \int_o^T dt(\dot{q}P - \alpha\mathcal{H}), \tag{7}$$

taken onto explicit solutions of the classical equation for α (4). It is very useful to introduce here the notions of the surface of admissible dynamics (SAD) defined by eq.(4), and of the minimal action on SAD:

$$W_\pm^{Min} = \pm \int_0^T dt\dot{q}F(q) = \pm \int_0^{q(T)} dqF(q). \tag{8}$$

The minimal action is invariant under the reparametrizations of time: $t \to t'(t)$ and depends on the dynamical variable at the boundary of the Friedmann time T_F. The action (8) has a nontrivial canonical momentum $P = \mp F$ and leads to quantum theory with the wave function:

$$\Psi^{Min}(q) = A^{(+)}e^{iW_{(+)}} + A^{(-)}e^{iW_{(-)}} = \Psi_{(+)} + \Psi_{(-)}, \tag{9}$$

where $A^{(\pm)}$ are the coefficients of the decomposition of the wave function in accordance with the two types of SADs. This wave function does not coincide for $F' \neq 0$ with the Wheeler-DeWitt one (6) as it satisfies equations

$$(\hat{P} \pm F)\Psi_{(\pm)} = 0; \quad \left(\hat{P} = \frac{1}{i}\frac{d}{dq}\right). \tag{10}$$

According to the principle of observability, the quantum theory (9) is expressed only in terms of gauge invariant magnitudes.

According to the principle of correspondence, in quantum theory there is the generator of evolution which should reproduce the Friedmann equation of the "boundary" evolution (5).

It is easy to prove that the role of this evolution generator is played by the Hamiltonian of the initial theory (3), and that the Heisenberg equation

$$\frac{1}{\alpha}\dot{q} = \frac{1}{\alpha}i[H, q] = -p = \pm F \tag{11}$$

reproduces the Friedmann one in the classical theory (5). We can see that the Hamiltonian of the initial theory plays simultaneously two roles: *i*) the role of the constraint (4) for the invariant time interval $0 \leq t < T_F$, and *ii*) the role of the generator of evolution (11) at the boundary of the time interval $t = T_F$.

The second role of the initial Hamiltonian transforms the "stationary" wave function (9) into the Green function of the boundary Friedmann evolution

$$G(0|T_F) = \Psi^{Min}[q(T_F)]. \tag{12}$$

It is useful to represent this Green function in the form of the FP functional integral in the "boundary" gauge

$$q(t) = q(T_F) \tag{13}$$

in terms of the initial action (7) and the variables $P,\ q,\ \alpha$:

$$G(0|T_F) = \int Dq\ D\alpha\ DP\Delta_{FP}\delta(q - q(T_F)e^{iW_I(\alpha,q,P)}, \tag{14}$$

where $\Delta_{FP} = \{\mathcal{H}, q\}$ is the FP determinant.

By the example of this integral it is easy to see the main difference of the minimal HP quantization from the conventional approach with the fixation of an arbitrary gauge: the "boundary gauge" (13) is the final result of the SAD construction of the invariant variables, but not the initial supposition. This SAD construction leads to the boundary dynamics $q(T_F)$, which has all attributes of quantum dynamics: Hamiltonian, the Heisenberg equation and the Green function of the evolution.

The "boundary gauge" gives the definite rule of the ordering of the operator $P,\ q$ in the constraint $\mathcal{H} = 0$, so that this constraint becomes equivalent to eq. (10).

The variable of the type of $q(T_F)$ in the role of the physical time, undressed from all attributes of quantum dynamics, is applied in the models of relativistic particle and string [14, 15].

3. Relativistic Particle and String

Let us consider the action of a relativistic particle

$$W = -m \int_0^T d\tau \sqrt{\dot{X}_\mu \dot{X}^\mu}; \quad X_\mu X^\mu = X_0^2 - X_i^2 \tag{15}$$

in the first-order formalism

$$W = \int_0^T d\tau [\dot{X}_i P_i + \dot{X}_0 P_0 - \alpha \mathcal{H}], \tag{16}$$

$$\mathcal{H} = \frac{1}{2}(\omega^2 - P_0^2); \quad \omega^2 = P_i^2 + m^2. \tag{17}$$

The surface of admissible dynamics (SAD) is defined by equation $\mathcal{H} = 0$, or

$$P_{0(\pm)} = \pm\omega(P_i), \tag{18}$$

and the primary quantization of the "boundary evolution" (9), (12)

$$G(0|X_0) = \int d^3PN \left[a_{P_i}^{(+)} e^{iX_jP_j - iX_0\omega} + a_{-P_i}^{(-)} e^{-iX_jP_j + iX_0\omega}\right] \tag{19}$$

coincides with the representation of the secondary quantization for a scalar relativistic field up to the normalization N.

The equation of the boundary dynamics

$$\frac{1}{\alpha}\frac{dX_0}{d\tau}|_{m\tau = t} = \frac{m}{\alpha}\frac{dX_0}{dt} = \pm\omega; \quad dT_F = \alpha dt \tag{20}$$

is the analogy of the Friedmann eqs. (4),(5). The solution of eq. (20)

$$X_0(T_F) = T_F\frac{\omega}{m} = T_F\frac{1}{\sqrt{1 - V_i^2}}; \quad V_i = \frac{P_i}{\omega} \tag{21}$$

is the Lorentz transformation which gives the connection between the Friedmann proper time T_F and the time T_Q of the spectral decomposition of the Green function over the eigenvalues of the physical Hamiltonian (18) on SAD. In the considered case this "quantum" time T_Q coincides with the variable X_0 ($X_0 = T_Q$), which is distinguished from the variables X_i by the opposite sign of its canonical momentum term in the initial Hamiltonian (17).

By the same way one can construct the Green function of the "boundary evolution" of the relativistic strings [16, 17, 18]. From the very beginning we choose the first-order formalism

$$W = \int_0^T d\tau \int_0^\pi d\sigma \{\dot{X}_i P_i + \dot{X}_0 P_0 - \alpha\mathcal{H} + \beta\mathcal{P}\}, \tag{22}$$

where

$$
\begin{aligned}
\mathcal{H} &= \frac{1}{2}\Big[P_i^2 + \gamma^2 X_i'^2 - P_0^2 - \gamma^2 X_0'^2\Big], \\
\mathcal{P} &= \gamma X_0' P_0 - \gamma X_i' P_i
\end{aligned}
\tag{23}
$$

are the total energy and total momentum, which act as constraints

$$
\mathcal{H} = 0; \quad \mathcal{P} = 0. \tag{24}
$$

Let us decompose the space coordinates and momenta over harmonics

$$
\begin{aligned}
X_j(\tau,\sigma) &= \frac{1}{\pi}\bar{X}_j + \frac{i}{\sqrt{\pi\gamma}}\sum_{n\neq 0}\frac{\cos n\sigma}{n}\ a_j(\tau/n), \\
P_j(\tau,\sigma) &= \bar{P}_j + \sqrt{\frac{\gamma}{\pi}}\sum_{n\neq 0}\cos n\sigma\ a_j(\tau/n),
\end{aligned}
\tag{25}
$$

where $\bar{X}_j$, $\bar{P}_j$ are the total space coordinates and momenta, $a_j(\tau/n)$ are harmonics.

Then the explicit solution of eq. (24) can be represented in the form

$$
P_{0(\pm)} = \mp\frac{1}{2}[P_{(+)} + P_{(-)}]; \quad \gamma X'_{0(\pm)} = \mp\frac{1}{2}[P_{(+)} - P_{(-)}], \tag{26}
$$

$$
P_{(\pm)} = \Big[\bar{P}_j^2 + \hat{m}^2 + \sum_{n=1}^{\infty} e^{\mp in\sigma} L_{(\pm n)}(\tau)\Big]^{1/2}, \tag{27}
$$

$$
\hat{m}^2 = \sum_{n\neq 0} a_j(\tau/-n)\ a_j(\tau/n), \tag{28}
$$

$$
L_{(\pm)}(\tau) = \sum_{k=-\infty}^{\infty} a_j(\tau/\pm n - k)\ a_j(\tau/k). \tag{29}
$$

The initial action (22) on SAD (26) takes the form

$$
\begin{aligned}
W^{Min}_{(\pm)} &= \int_0^T d\tau \int_0^\pi d\sigma[\dot{X}_i P_i \mp \dot{X}_0(\tau,\sigma)\frac{1}{2}(P_{(+)} + P_{(-)})] \\
&= \int_0^T d\tau \int_0^\pi d\sigma(\dot{X}_i P_i) \mp \bar{X}_0(T)\sqrt{\bar{P}_i^2 + \hat{m}^2}.
\end{aligned}
\tag{30}
$$

We have taken into account here the solution of eq. (26) for X_0

$$
\dot{X}_0(\tau,\sigma) = \frac{1}{\pi}\dot{X}_0(\tau) \pm \frac{1}{2\gamma}(P_{(+)} - P_{(-)}) \tag{31}
$$

and the disappearance of all inhomogeneous terms owing to the integration over σ.

The Green function for the minimal action (30) is constructed by the spectral decomposition over the eigenfunctions of the effective Hamiltonian:

$$
\sqrt{\bar{P}_i^2 + \hat{m}^2}\,\Phi_\nu = \omega_\nu \Phi_\nu; \quad \nu = \{\nu_{n_j}, Pj\} \tag{32}
$$

which represent the states with different spins and masses formed by the ν_{n_j}–fold action of the different mode operators $a_j(0|n>0) = a_j^+$ on the vacuum: $\Phi_\nu = \Pi(a_{n_j}^{(+)})^{\nu_{n_j}}/\sqrt{\nu_{n_j}!}|0>$. This Green function has the form:

$$G(0|\bar{X}_0) = \sum_\nu \int d^3\bar{P}N_p\Big[A_\nu^{(+)}e^{i\bar{X}_jP_j - iX_0\omega_\nu}\Phi_\nu + A_\nu^{(-)}e^{-i\bar{X}_jP_j+iX_0\omega_\nu}\tilde{\Phi}_\nu\Big], \tag{33}$$

where $A_\nu^{(\pm)}$ are the creation and annihilation operators of the string with the sets of their quantum number ν. The Green function (33) reproduces the results by refs.[18].

We would like to emphasize here that the minimal (first) quantization of the particle and string corresponds to the second quantization. This fact points out that in gravity field theory we can get the third quantization.

4. Minimal Quantization of the n+1 Dimensional Gravity

We shall consider the Einstein theory in (n+1) dimension.

$$W = \int d^nxdx^0[\mathcal{L}_G + \mathcal{L}_M]; \quad \mathcal{L}_G = -\frac{1}{2\kappa^2}{}^{(n+1)}R(g_{\mu\nu})\sqrt{-g}, \tag{34}$$

where $\mathcal{L}_M$ is the matter Lagrangian, and choose the ADM metric [19] (which is used for the canonical quantization) with the factorization of the "scale-space variable" [20] $a(x) = \exp\mu(x)$, and the conformal-invariant "graviton" h_{ij}

$$(ds)^2 = g_{\mu\nu}dx^\mu dx^\nu = \alpha^2(dx^0)^2 - a^2h_{ij}(dx^i - \beta^i dx^0)(dx^j - \beta^j dx^0), \tag{35}$$

$$\sqrt{-g} = \alpha a^n; \quad deth = 1. \tag{36}$$

The curvature of the (n+1) dimensional space (34) is decomposed in a "kinetic" term ($\mathcal{K}$), and the "potential" (${}^{(n)}R$), and "surface" (Σ) ones

$${}^{(n+1)}R = \mathcal{K} + {}^{(n)}R + 2\Sigma, \tag{37}$$

where

$$\mathcal{K} = n(n-1)\frac{1}{\alpha^2}\Pi^2 - \frac{1}{4\alpha^2}\Pi_l^k\Pi_k^l; \quad (\Pi_k^k = 0), \tag{38}$$

$$\Pi = \partial_0\mu - \frac{1}{na^n}\partial_k(a^n\beta^k),$$

$$\Pi_l^k = \partial_0 h_l^k - \beta^k{}_{,l} - \beta_{l,}{}^k + \frac{2}{n}\delta_l^k\partial_i\beta^i, \tag{39}$$

$${}^{(n)}R(a^2h) = R(a^2) + \frac{1}{a^2}R(h); \quad (\partial_\mu h_l^k = h^{ki}\partial_\mu h_{il}), \tag{40}$$

$$R(a^2) = \frac{4(n-1)}{(n-2)a^2}\left(a^{-\frac{n-2}{2}}\Delta a^{\frac{n-2}{2}}\right) = \frac{2(n-1)}{a^2}\left[\Delta\mu + \frac{n-2}{2}\partial_k\mu\partial^k\mu\right], \quad (41)$$

$$R(h) = \frac{1}{4}\partial_j h_l^k(\partial^j h_k^l - 2\partial^l h_k^j) + \partial_k\partial_l h^{kl}, \quad (42)$$

$$\Sigma = \frac{1}{\alpha a^n}\left\{\partial_k\left[a^{n-2}\partial^k\alpha + \frac{na^n}{\alpha}\beta^k\Pi\right] - n\partial_0\left[\frac{a^n}{\alpha}\Pi\right]\right\}, \quad (43)$$

where $\beta^k_{;l}$ is a covariant derivative in metric h_{ij}, $\Delta\phi = \phi_{,kl}h^{kl}$.

One can see that the kinetic term (38) is split into positive and negative parts, and the role of the variable of the "boundary dynamics" (with the negative momentum squared) plays the scale of space a(x). The components α, β_k do not have canonical momentum and are nonphysical fields. The classical equations for the fields α, β_k, a in terms of the definition (37)-(43) have the form

$$\frac{1}{2\kappa^2}[\mathcal{K} -^{(n)} R] = T_0^0(M), \quad (44)$$

$$\frac{1}{2\kappa^2}\left[2(n-1)\partial_k\left(\frac{\Pi}{\alpha}\right) - \frac{1}{a^n}\left(\frac{1}{\alpha}\Pi^l_k a^n\right)_{;l}\right] = -\bar{T}_k^0(M), \quad (45)$$

$$\frac{1}{2\kappa^2}[2(n+1)(\mathcal{K}+\Sigma) - (n-2)(\mathcal{K} -^{(n)} R)] = -T_k^k(M), \quad (46)$$

where $T_\nu^\mu(M)$ is the matter energy-momentum tensor. (We have used the expression $\bar{T}_k^0 = \alpha T_k^0$, which in terms of canonical momenta does not depend explicitly on α).

In the first-order formalism with respect to the time derivative the action (34) has the form

$$W_I = \int d^n x dx^0 [P\partial_0\mu + K_n^l\partial_0 h_l^n + P_M\partial_0 M - \alpha\mathcal{H} + \beta^k\mathcal{P}_k] + W_{\Sigma'}. \quad (47)$$

Here we introduce the brief denotion for matter fields M and their canonical momenta P_M; $\mathcal{H}$ and $\mathcal{P}_k$ are the densities of the total Hamiltonian and total momentum:

$$\mathcal{H} = a^n\left[-\frac{1}{2}\kappa_n^2\frac{P^2}{a^{2n}} + \frac{R(a^2)}{2\kappa^2} + T_0^0(h,M)\right]; \quad \left(\kappa_n^2 = \frac{\kappa^2}{n(n-1)}\right), \quad (48)$$

$$\mathcal{P}_k = \frac{a^n}{n}\partial_k\left(\frac{P}{a^n}\right) + 2K^i_{k,i} - a^n\bar{T}_k^0(M), \quad (49)$$

$$T_0^0(h,M) = T_0^0(M) + T_0^0(h),$$

$$T_0^0(h) = \frac{2\kappa^2 K_l^i K_i^l}{a^{2n}} + \frac{R(h)}{2\kappa^2 a^2}, \quad (50)$$

$W_{\Sigma'}$ is the surface term

$$W_{\Sigma'} = \int d^n x dx^0\left\{-\frac{\partial_0 P}{n-1} - \partial_k\left[\frac{a^{n-2}\partial^k\alpha}{\kappa^2} - \frac{P\beta^k}{n(n-1)} + 2K_l^k\beta^l\right]\right\}. \quad (51)$$

The explicit solving of the constraints

$$\mathcal{H} = 0; \quad \mathcal{P}_k = 0 \quad (52)$$

leads to two surfaces of admissible dynamics which correspond to a creation (+) and an annihilation (-) of the Universe:

$$P_{(\pm)} = \mp F_{(pm)};\quad F_{(\pm)} = \frac{a^n}{\kappa_n}\left[2T^0_{(\pm)0}(h, M) + \frac{R(a^2)}{\kappa^2}\right]^{1/2}, \tag{53}$$

$$K^i_{(\pm)k,i} = \pm\frac{1}{2}\frac{a^n}{n}\partial_k\left(\frac{F_{(\pm)}}{a^n}\right) + a^n\bar{T}^0_k(M). \tag{54}$$

The last equation defines the covariant-longitudinal part of the graviton canonical momentum

$$K^i_{(\pm)l} = K^{Ti}_l + \eta^i_{(\pm),l} + \eta_{(\pm)l,}^{\ i} - \frac{2}{n}\delta^i_l(\partial_k\eta^k_{(\pm)});\quad K^{Ti}_{l,i} = 0 \tag{55}$$

as the function over independent variables. The minimal action has the form

$$W^{Min}_{\pm} = \int d^n x dx^0[\partial_0 h^{Ti}_l K^{Tl}_i + \partial_0 M P_M \mp \partial_0\mu F_{(\pm)}] + W_{\Sigma'}(P = \mp F_{(\pm)}). \tag{56}$$

The transversality constraint (55) dictates the choice of gauge for gravitons

$$(\partial_0 h^{Ti}_l)_{,i} = 0 \tag{57}$$

which is used also in non-Abelian theories [10, 21], and is analogous to the radiation gauge [1, 2].

The main differences of the minimal quantization from the conventional approach [4, 5] are i) the single-value definition of the "boundary" gauge for the scale-space component

$$a(x) = a(T, x_i) = \exp\mu(T, x_i), \tag{58}$$

and ii) the definition of the physical time T.

Formally we can write the FP integral for the gauges (57), (58), omitting the matter field for simplicity:

$$Z(0|T)^{FP} = \int(Dh_{ij}DK^{kl})(D\mu DP)(D\alpha D\beta^k)\delta(\partial_0 h^{Ti}_{l,i})\delta(\mu - \mu(T, x))\Delta_{FP}e^{iW_I}, \tag{59}$$

where W_I is defined by eq.(47), Δ_{FP} is the FP-determinant [8]

$$\Delta_{FP} = \int D\theta_\nu D\bar{\theta}_\mu \exp\Big\{-i\int\int_0^T d^4x dt\Big[\bar{\theta}^0\left(\frac{\kappa_n^2 P}{a^n}\theta^0 + \frac{\partial_k(a^n\theta^k)}{na^n}\right) + \bar{\theta}^{(l,k)}\left(\frac{\kappa^2 4K_{lk}}{a^n}\theta^0 + 2\theta_{(l,k)}\right)\Big]\Big\}. \tag{60}$$

The physical time T is connected with the proper inhomogeneous time

$$dT = \alpha(x)dx^0;\quad \alpha_{(\pm)} = \pm\frac{\partial_0\mu a^n}{F_{(\pm)}\kappa_n^2}. \tag{61}$$

In the case of the frame $\beta_k = 0$ one can convince oneself that the Hamiltonian of the theory (48)

$$H = \int d^n x \alpha \mathcal{H} \tag{62}$$

is the generator of the evolution for the quantum scale $\mu(x)$ and its momentum $P = -i\delta/\delta\mu(x)$ with respect to the time (61).

The Heisenberg equations

$$\frac{1}{\alpha}\partial_0\mu = \frac{i}{\alpha}[\hat{H}, \mu(x)] = -\frac{\kappa_n^2 P}{a^n}, \tag{63}$$

$$\frac{1}{\alpha}\partial_0 P = -a^n\left[n\frac{\kappa_n^2 P^2}{2a^{2n}} + \frac{n-2}{2\kappa^2}R(a) + \frac{n-1}{\kappa^2 a^n \alpha}\partial_k(a^{n-2}\partial^k\alpha) + T_k^k(h,M)\right], \tag{64}$$

$$T_k^k(h,M) = T_k^k(M) - n\frac{2\kappa^2 K_j^i K_i^j}{a^{2n}} + \frac{n-2}{2\kappa^2 a^2}R(h), \tag{65}$$

completely coincide with the classical Einstein equation (44), (46), and are quantum equations of the boundary evolution on SAD (52).

In the general case $\beta_k \neq 0$ the "boundary dynamics" is defined from the set of eqs.(44)-(46). Thus, in the minimal HP quantization the "gauge" is calculated together with the Friedmann time T_F and quantum time T_Q of the Green function spectral decomposition. Let us consider this problem of the calculation of "quantum" time at first for the Friedmann homogeneous Universe.

5. The Friedmann Universe

To get the Friedmann Universe it is enough to neglect the nondiagonal elements of the energy momentum tensor $T_k^0 = 0$; $\eta^i = 0$ in eqs.(52)-(55). In this case the density momentum conservation law $\mathcal{P}_k = 0$ turns into the equation for the isotropic homogeneous Universe

$$\partial_k\left(\frac{F}{a^n}\right) = 0; \quad F = \frac{a^n}{\kappa_n}\left[2T_0^0 + \frac{R(a)}{\kappa^2}\right]^{1/2} \tag{66}$$

for which the curvature can have three types

$$\frac{1}{\kappa^2}R(a) = -\frac{1}{\kappa_n^2}\frac{1}{r_o^2 a^2}k; \quad k = 0, \pm 1, \tag{67}$$

where r_0 is the constant of the dimension of length. (From the point of view of quantum field theory this case can be also a vacuum, as the state without any local excitations with positive energy of the type of the vacuum with respect to radiation in the Heisenberg-Pauli electrodynamics [1, 2].)

The Friedmann evolution is described by the homogeneous form of eq. (61)

$$T_F = \pm \int_0^a \frac{d\bar{a}\bar{a}^{n-1}}{F(\bar{a})\kappa_n^2}. \tag{68}$$

The Friedmann evolution (68) is reflected in the "boundary evolution" of the functional integral (59) where the Friedmann sector looks as the collective global excitation of the physical space [22], which should be considered as the zero harmonics $\bar{X}_\mu(\tau)$, $\bar{P}_\mu$ in the string model (25), (31), (32).

The Green function of the evolution of the homogeneous Universe can be also constructed by direct minimal quantization of the Einstein action in the Friedmann approximation (66), which represents one of the models of the zero-dimension gravity (see Section 2). The final result can be got from eq. (56) in the form of the "third" quantization (9) [23]:

$$Z_F(0|T_F) = A_F^{(+)} e^{-iW^{Min}(a)} + A_F^{(-)} e^{iW^{Min}(a)}, \tag{69}$$

$$W^{Min}(a) = V_n(r_0)\left[\int_0^a \frac{d\bar{a}}{\bar{a}} F(\bar{a}) - \frac{F(a)}{n-1}\right], \tag{70}$$

where $A^{\pm}$ are the operators of creation and annihilation of the Universe. The origin of the last term in the minimal action (70) is $W_{\Sigma'}$ in eq. (56). $V_n(r_0)$ is the volume of the n-dimensional space, in the case of the positive constant curvature it is equal

$$V_n(r_0) = r_0^n 2\pi^{\frac{n+1}{2}} \frac{1}{\Gamma\left(\frac{n+1}{2}\right)}. \tag{71}$$

Let us choose the example of radiation

$$T_0^0(R) = \frac{\varepsilon_R}{2a^{n+1}V_n(r_0)r_0} \tag{72}$$

for which the minimal action (70) for all the three types of space coincides with the conformal time [22]

$$W^{Min}(a) = \frac{\varepsilon_R}{2}\eta(a|k); \quad \eta = \int^a \frac{dT_F(\bar{a})}{\bar{a}r_0}, \tag{73}$$

where

$$\eta(a|0) = \frac{2A}{n-1}; \quad \eta(a|+1) = \frac{2\arcsin A}{n-1},$$

$$\eta(a|-1) = \frac{2\ln(A+\sqrt{1+A^2})}{n-1}, \tag{74}$$

$$A = [2T_0^0(a)]^{-1/2} a r_0 \kappa_n \sim a^{\frac{n-1}{2}}. \tag{75}$$

We can see that in the case of radiation the conformal time plays the role of the "quantum" time T_Q of the spectral representation like X_0 in the models of relativistic particle and string:

$$W^{Min}(a) = E_Q T_Q; \quad \left(T_Q = r_0\eta; \ E_Q = \frac{\varepsilon_R}{2r_0}\right). \tag{76}$$

The expression (76) follows also from the quantum energy conservation law [23]

$$\frac{dE_Q}{dT_Q} = 0, \tag{77}$$

which can be used to define the physical time of a quantum observer. T_Q is connected with the Friedmann time by the nonlinear generalization of the Lorentz transformation (21) of the proper time in the rest frame. Recall that just for radiation the notion of the "rest frame" is absent.

In a three dimensional space with positive constant curvature this nonlinear "Lorentz transformation" has the form

$$T_F(T_Q) = r_0\sqrt{\bar{\varepsilon}}\sin(T_Q/r_0); \quad \left(\bar{\varepsilon} = \frac{\kappa_3^2\varepsilon_R}{V_3(1)r_0^2}; \;\; V_3(1) = 2\pi^2\right) \tag{78}$$

and the quantum time strongly differs from the Friedmann one, which violates the causality.

For the case of a dust

$$T_0^0(d) = \frac{M_d}{V_3(r_0)a^3}; \quad \left(\bar{M} = \frac{\kappa_3^2 M_d}{V_3(1)r_0}\right), \tag{79}$$

where the "rest frame" is well defined, we can demand the identity of these two times:

$$T_Q = r_0(\eta + \sin\eta); \quad T_F = r_0\bar{M}(\eta + \sin\eta). \tag{80}$$

As a result we get the simplest version of the Schwinger-Dyson equation for the Newton selfinteraction

$$\frac{\kappa_3^2 M_d^2}{2\pi^2 r_0} = M_d \mapsto \bar{M} = 1 \tag{81}$$

which suits for the observable Universe with the mass and radius:
$Md \sim 10^{80}GeV; \quad \kappa_3^2/2\pi^2 \sim 10^{-38}GeV^{-2}; \quad r_0 \sim 10^{42}GeV^{-1}$.
The case of the sum of radiation and dust $T_0^0 = T_0^0(R) + T_0^0(d), \;\; k = +1$ is described by the formulae:

$$\hat{E}_Q = \bar{V}_{(+1)}[\bar{\varepsilon} + \bar{M}^2]; \quad ; \bar{V}_{(+1)} = \frac{V_3(1)r_0}{2\kappa_3^2} \tag{82}$$

$$\frac{T_Q(\eta)}{r_0} = \eta + \cos\eta_0[\sin\eta_0 + \sin(\eta - \eta_0)]; \quad a = \bar{M}\left(1 - \frac{\cos(\eta - \eta_0)}{\cos\eta_0}\right);$$

$$\frac{T_F(\eta)}{r_0\bar{M}} = \eta + \cos^{-1}\eta_0[\sin\eta_0 + \sin(\eta - \eta_0)]; \quad \cos\eta_0 = \frac{\bar{M}}{\sqrt{\bar{M}^2 + \bar{\varepsilon}}} \tag{83}$$

which can be got as the generalization of the well known results [24]. For any radiation the Friedmann time violates the causality. In accordance with the conservation law of the quantum energy (77) an observer sees indeed the "quantum" time and the "quantum" Hubble "constant"

$$H_Q = \frac{\sin(\eta - \eta_0)}{[(\cos\eta_0 - \cos(\eta - \eta_0)]T_Q'(\eta)}; \quad T' = \frac{d}{d\eta}T, \tag{84}$$

but not the classical Friedmann time with the "constant"

$$H_F = H_Q \frac{T'_Q}{T'_F} \tag{85}$$

which defines the critical density

$$\rho_{cr} \equiv \frac{3}{\kappa^2} H_F^2 = T_0^0 - \frac{3k}{\kappa^2 r_0^2 a^2}. \tag{86}$$

In particular, even in the case of the infinitely small radiation $\bar{\varepsilon} = 2\gamma \bar{M}^2$, $2\gamma \ll 1$ the relation of the classical (H_F) and quantum (H_Q) Hubble constants oscillates, or twinkles, with the period, which does not depend on γ.

Let us consider the Universe with a dust and radiation for other types of space: $k = -1, 0$. In this case the volume $V_3(r_0) = r_0^3 V_3(1)$ is arbitrary.

For $k = -1$, $\bar{\varepsilon} < \bar{M}^2$ we got the times T_Q, T_F as functions of the conformal time η.

$$\begin{aligned} \frac{T_Q(\eta)}{r_0} &= -\eta + \cosh\eta_0[\sinh\eta_0 + \sinh(\eta-\eta_0)]; \quad a(\eta) = \bar{M}\left(\frac{\cosh(\eta-\eta_0)}{\cosh\eta_0} - 1\right), \\ \frac{T_F(\eta)}{r_0\bar{M}} &= -\eta + \cosh^{-1}\eta_0[\sinh\eta_0 + \sinh(\eta-\eta_0)]); \quad \cosh\eta_0 = \frac{\bar{M}}{\sqrt{\bar{M}^2 - \bar{\varepsilon}}}. \end{aligned} \tag{87}$$

In the limit $\bar{M}^2 \gg \bar{\varepsilon}$; $\cosh\eta_0 = 1$ these times should coincide, therefore $\bar{M} = 1$. The action (70) has the form:

$$W = E_Q T_Q; \quad E_Q = \bar{V}_{(-1)}(\bar{M}^2 - \bar{\varepsilon}).$$

For a flat space we got the action with the quantum energy and time

$$\begin{aligned} E_Q &= \bar{V}_{(0)}(\bar{M} + \bar{\varepsilon}), \\ T_Q(a) &= \frac{2r_0}{3\bar{M}(\bar{M}+\bar{\varepsilon})}\left[(2\bar{M}a + \bar{\varepsilon})^{3/2} - \bar{\varepsilon}^{3/2} - \frac{3\bar{M}a}{2}(2\bar{M}a+\bar{\varepsilon})^{1/2}\right]. \end{aligned} \tag{88}$$

The latter coincides with the Friedmann time

$$T_F(a) = \frac{r_0}{6\bar{M}^2}[(2\bar{M}a+\bar{\varepsilon})^{3/2} - 3\bar{\varepsilon}(2\bar{M}a+\bar{\varepsilon})^{1/2} + 2\bar{\varepsilon}^{3/2}]$$

in the limit $\bar{\varepsilon} = 0$: $T_Q = T_F = r_0(2\bar{M}a)^{3/2}/6\bar{M}^2$, and with the conformal time $r_0\eta = r_0 a/\bar{\varepsilon}^{1/2}$ in the limit $\bar{M} \to 0$.

The example of string with nontrivial vacuum energy of local excitations (28) points out to one more difficulty of the definition of the type of a space by eq. (86), connected with the above–mentioned vacuum part of the homogeneous energy density operator: $\varepsilon_R = \varepsilon_V + 2\sum_i a_i^{(+)} a_i^{(-)}\omega_i$, $M_d = M_V + \sum_i \psi_i^{(+)}\psi_i^{(-)} M_i$, where $a^{(\pm)}$, $\psi^{(\pm)}$ are the operators of radiation and dust, correspondingly. This vacuum part (ε_V, M_V) is not directly observed and plays the role of the hidden mass. On the other hand, the large-scale periodic structure of the Universe [25] can testify, in the light of the definition of the quantum time, that we are living in the closed oscillating Universe.

From this point of view it is very interesting to investigate the boundaries of large and small Hubble constant for the estimation of the behavior of an inhomogeneity at the moment of a compression of the Universe.

In the next section we consider this question for the radiation Universe when the conformal time coincides with the physical (quantum) one and we can use the cosmological perturbation theory [26].

6. Inhomogeneity

Let us consider the "boundary" dynamics of the scale component $a(x)$ which is given by inhomogeneities of the energy-momentum tensor

$$T^{\mu}_{\nu} =< T^{\mu}_{\nu} > +\delta T^{\mu}_{\nu}; \quad R(a^2) = R(a_0^2) + \frac{1}{a^2} R(e^{-2\Psi}). \tag{89}$$

We shall use the conventional notation [26]

$$\alpha = a_0 e^{\Phi}; \quad a = a_0 e^{-\Psi}; \quad \frac{d}{d\eta}\mu = H_Q - \Psi'. \tag{90}$$

The homogeneous and inhomogeneous parts of eq.(61) take the forms

$$H_Q = [2\kappa_n^2 < T_0^0 > a_0^2 - 1]^{1/2}, \tag{91}$$

$$\Psi' + \frac{\tilde{T}_0^0 - \Delta\Psi}{nH_Q} + H_Q \Phi = 0, \tag{92}$$

$$\left(\tilde{T}_0^0 = \delta T_0^0 \frac{a_0^2 \kappa^2}{n-1} \right).$$

The function Φ is defined from eq.(64)

$$[\Delta - 2n(n-1)H_Q^2]\Phi - nH_Q\Phi' + n(H_Q\Psi' + \Psi'') = -\tilde{T}_k^k + (n-2)\tilde{T}_0^0. \tag{93}$$

Here Φ and Ψ are the eigenfunctions of the operator Δ with the eigenvalues $\Delta = -k^2 \neq 0$. (For the case of $\beta_i \neq 0$ one needs to take into account eq.(45).)

The solution of eq.(92) can be written in the form:

$$\Psi = -e^{-k^2 f(\eta)} \int\limits_0^{k^2 f(\eta)} (df(\bar{\eta})) e^{k^2 f(\bar{\eta})} [\tilde{T}_0^0 + H_Q^2 n \Phi]; \quad \left(df = \frac{d\eta}{nH_Q} \right). \tag{94}$$

In particular, for the stationary inhomogeneity $\tilde{T}_0^{0\prime} = 0$ and $\Phi = 0$ we get the evolution of the Newton law

$$\Psi = -\frac{1}{k^2}[1 - e^{-k^2 f(\eta)}]\tilde{T}_0^0; \quad f(\eta) \sim \eta^2. \tag{95}$$

In the ultraviolet limit $k^2 \to \infty$ or $H_Q \to 0$ eq.s (92), (93) turn into the Newton law

$$\Psi = -\frac{1}{k^2}\tilde{T}_0^0; \quad \Phi = \frac{1}{k^2}[\tilde{T}_k^k - (n-2)\tilde{T}_0^0]. \tag{96}$$

One can see that when H_Q goes to zero, $f(\eta) \to \infty$, the whole boundary dynamics of inhomogeneities disappears and in this limit we have only the stationary Newton interaction of stationary inhomogeneities.

Just in this case the "boundary" gauge of the minimal quantization (58) coincides with the Faddeev-Popov one [8].

The evolution of approaching to a singularity ($f(\eta) \to 0,\ H_Q \to \infty$) reflects the approximate solution (95), which disappears quicker than the first term of the spectral decomposition (73) $W^{Min} \sim \frac{\epsilon_R}{2}\eta$. Thus, the evolution of the Universe in the vicinity of the point of a singularity is described by the homogeneous Green function (69) which at the time of the complete compression of the Universe has no peculiarities. Recall that the same situation takes place for the relativistic oscillating string [14]. The external observer sees only the spectrum of string and the regular wave function of the probability amplitude.

7. Effective Hamiltonian

The minimal action (56) on SAD can be used for the construction of the effective Hamiltonian density which is formed by the two last terms in (56)

$$\pm\left[F_{(\pm)}\partial_0\mu - \frac{\partial_0 F_{(\pm)}}{(n-1)} - \frac{\partial_k(a^{n-2}\partial^k\alpha_{(\pm)})}{\kappa^2}\right] = \mp\alpha_{(\pm)}a^n T_{(\pm)eff} \tag{97}$$

where $a,\ \alpha_{(\pm)}$ form the invariant volume: $(d^n x dx^0 \alpha_{(\pm)} a^n)$ and are defined together with $F_{(\pm)}$ by eqs.(52)-(54), (63)-(65) or (89)-(93). These equations can be represented in the form

$$F_{(\pm)}\partial_0\mu = a^n\alpha_{(\pm)}\left[-2T^0_{0(\pm)}(h,M) - \frac{R(a)}{\kappa^2}\right];$$
$$-\left(\frac{\partial_0 F}{(n-1)} + \frac{\partial_n(a^{n-2}\partial^n\alpha_{(\pm)})}{\kappa^2}\right) = a^n\alpha_{(\pm)}\left[\frac{R(a)}{\kappa^2} + T^0_{0(\pm)}(h,M) + \frac{T_{(\pm)}(h,M)}{n-1}\right], \tag{98}$$

where $T_{(\pm)}(h,M)$ is the trace of the total energy-momentum tensor (50), (65): $T_{(\pm)} = T^0_{0(\pm)} + T^k_{k(\pm)}$.

The substitution of eqs.(98) into the definition (97) leads to the following effective Hamiltonian density expressed in terms of the minimal set of physical variables

$$T_{\pm eff} = T^0_{0(\pm)}(h,M) - \frac{T_{(\pm)}(h,M)}{(n-1)}. \tag{99}$$

For the case n=3 this Hamiltonian density coincides up to the factor (1/2) with the Tolman one [27]. To get the classical static limit it is enough to add to the expression (99) the total space derivative [8, 27], describing the stationary distribution of the matter in the Universe.

Let us consider the pure dynamical example of a single nonlinear graviton, which propagates in the direction of the axis (n)

$$h_{ij} = \begin{pmatrix} h_{AB}(\tau + \xi^{(n)}) & 0 \\ 0 & 1 \end{pmatrix}, \tag{100}$$

where $d\tau = \alpha dx^0$; $\underline{d}\xi^n = a dx^n$ are the proper time and coordinate correspondingly.

Then the constraints (52), (64) turn into the equations

$$\alpha = a; \; \ddot{\mu} = -\frac{\kappa^2}{n-1}T_0^0(h); \;\; T_0^0(h) = -T_n^n(h) = T_n^0(h), \;\; (\dot{\mu} = \partial_\tau \mu) \tag{101}$$

which point out that the proper coordinates τ, $\xi^{(n)}$ on the class of functions $f(\tau + \xi^{(n)})$ become integrable

$$(\partial_\tau \underline{\partial}_n - \underline{\partial}_n \partial_\tau) f(\tau + \xi^{(n)}) = 0; \;\; \left(\underline{\partial}_n = \frac{1}{a}\frac{\partial}{\partial x^n} \right). \tag{102}$$

As a result the effective Hamiltonian density (99) coincides with the conventional one for graviton (101).

Conclusion

We considered the minimal quantization of the time-reparametrization invariant theories. "Minimal" means that one uses only a minimal set of gauge invariant variables selected by the explicit solving of equations for the time components before quantizing.

This quantization differs from the conventional approach, where the complete set of components are considered as the variables of the canonical scheme. In the last case the set of constraints, including the relativistic gauge, on the quantum level contradicts the uncertainty principle as it fixes simultaneously the fields and their momenta. The relativistic gauges also restrict the group of gauge transformations as Schwinger has pointed out [9], and thereby change the off mass-shell physics in the comparison with the theory with the initial gauge group. Finally, the conventional quantization of gravity loses the dynamics of the homogeneous Universe, that contradicts the correspondence principle.

The minimal scheme [1, 2, 6, 10, 11] does not have these defects. Considering the initial action onto the surface of admissible dynamics (SAD), defined by the time component equations,we conserve all quantum principles and the "gauge" invariance on the level of the physical variables and coordinates, and can reproduce the dynamics of the homogeneous Universe in the quantum theory.

The main difficulty of the minimal scheme is to realize the double roles of the Einstein Hamiltonian (as a constraint and as the evolution generator), of the Green function of the evolution (which looks as a stationary wave function with respect to the scale-space field), and of the dynamical field $a(x)$, which plays also the role of physical time.

The first role of the scale-space component as a dynamical variable points out that in the spectrum of elementary excitations of the Einstein theory there is the collective excitation of the physical space (of the type of the superfluid motion of quantum liquid) bidden at the boundary of the time interval. This collective excitation can be considered as the reason of the Universe expansion.

The calculation of the Green function of the expansion recovered the possibility to introduce the physical time of the spectral representation with the conservation of the quantum energy.

In the light of this definition of the "quantum" time the behavior of a closed oscillating Universe filled in a dust and radiation looks more attractive than in the case of the Friedmann time, and can by used for the description of the large-scale periodic structure of the Universe discovered recently.

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