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EXCITED LANDAU LEVELS, ORBITAL ANGULAR MOMENTUM AND AXIAL ANOMALY

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1. Introduction

The large Axial Anomaly contribution to the proton spin structure (see, e.g., [1] and Ref. therein)**naturally puts** a question of its partonic interpretation. This problem was solved by Mueller [2] in the framework of Landau levels flow approach to the Axial Anomaly [3]. Proceeding in this direction it appeared possible to describe Axial Anomaly even in a semiclassical way [4, 5].

The key element of the Mueller approach allowing to generalize the Gribov treatment of the massless fermions to the massive case is the cancellation between the anomalous and explicit chiral symmetry breaking. This phenomenon is very transparent when the standard UV approach to the anomaly is adopted. Really, the anomaly arises when the contribution of the regulator fermions (with the nonzero infinite mass limit) is subtracted from the axial current matrix element. Therefore, the anomalous term differs only in sign (coming from the subtraction) from the "normal" one, arising from the physical fermion mass, if the latter is much larger than all the kinematical variables.

The Landau levels picture manifests that the anomalous chirality flow cancels the normal one only for vacuum states with the negative energy.**If one considers** the real positive energy states, the straightforward generalization of Mueller arguments shows the explicit chiral symmetry breaking to be of the same sign with the anomalous one. Therefore, one may expect that in massless QCD Axial Anomaly would generate a variety of the helicity-flip spin effects, reproducing the effect of the large quark mass.

Making use of the normal-anomalous cancellation in the "physical" Landau levels framework it is desirable to have its explanation in "physical" language. It is just the subject of the present Letter. It appears that the natural IR cutoff leads to the chirality flow associated with the excited Landau levels and compensating the ground state contribution (Section 2). The relation of this procedure to the supersymmetric quantum mechanics is the subject of Section 3. The possible implications for the Axial Anomaly at $2+1$ dimensions and Anyon Superconductivity problem are discussed in Section 4, while the concluding remarks are presented in Section 5.

2. Orbital momentum cutoff and the chirality of excited Landau levels

The eigenstates in the constant magnetic fields [6] are highly degenerated. The rate of this degeneracy per unit area is nothing else than the number of the magnetic flux quanta

$$N_{\perp} = \frac{eH\Delta x\Delta y}{2\pi\hbar c}. \tag{1}$$

To obtain the familiar expression [2, 3] for the chiral charge anomalous nonconservation

$$\Delta Q_5 = \frac{e^2}{2\pi^2\hbar c} EH\Delta V_{(4)}, \tag{2}$$

one should multiply it by the longitudinal degeneracy rate in the constant electric field $E \parallel H$

$$N_{\parallel} = \frac{eE\Delta t\Delta z}{2\pi\hbar}, \tag{3}$$

and by $2\hbar$ — the chirality change induced by each pair of the states crossing the zero energy level [3]. This derivation makes clear the topological nature of the anomaly even in the one-flavour QED case because of the topological origin of the magnetic flux quantum. The duality between its perturbative and nonperturbative treatment is also manifested: each power of small parameter e comes from the large number N.

Choosing the symmetric gauge one obtains the two-dimensional Hamiltonian (in the four-dimensional *relativistic* case one may substitute the energy ϵ in the r.h.s. of the Schrödinger equation for $\sqrt{\epsilon^2 - p_z^2 - m^2}/2m$):

$$\hat{H} = \frac{1}{2}(\hat{H}_0 - \hat{L}) - \hat{S}. \tag{4}$$

Hereafter magnetic units are used. L and S are the orbital and spin angular momentum respectively (note that the single component differs from zero). H_0 is the Hamiltonian of the *two-dimensional* harmonic oscillator with the eigenvalues

$$\epsilon_0 = l + 1. \tag{5}$$

l is the maximum value of the orbital angular momentum among its $l+1$ degenerated eigenstates. The basis is of course chosen to diagonalize $\hat{L}$. The eigenvalues of the full Hamiltonian are:

$$\epsilon = \frac{l-m}{2} - s + \frac{1}{2} = n + \frac{1}{2} - s = n_s, |s| = \frac{1}{2}. \tag{6}$$

Note that the orbital momentum is quantized modulo 2, because the parity is just $(-)^m$ in the two-dimensional case. $l - m = 2n$ is then even and (6) correctly reproduces the familiar expression for the Landau levels. The quantum numbers are, however, nonconventional. In particular, l has no direct physical interpretation. It is the price I had to pay to express ϵ in terms of the angular momentum variables.

Consider first the ground Landau level. It is degenerated according to the orbital momentum variation: $s = 1/2, l = m = 0, 1, 2...$ To have the finite degeneracy rate, one should restrict the maximum orbital momentum: $l_{max} = m_{max} = N_\perp - 1$. This restriction is well-known as the Aharonov—Casher theorem [7] in the case of the magnetic flux confined to the finite space region while the charge is allowed to move in the infinite flat. It is just the normalizability condition: the high orbital momentum states are nondecreasing at the infinity.

Passing on to the excited levels let us first omit the spin degree of freedom. The first excited level of the spinless charge is realized via $(l = 1, m = -1), (l = 2, m = 0)$, *etc.* How should one restrict these quantum numbers to obtain the finite degeneracy rate for all the excited levels?

My main postulate is: $l \leq l_{max} = N_\perp - 1$ for *all* levels.

It immediately restricts the m values also, because $m \leq l$ by definition. Although the direct physical meaning of l is absent, it is just l that determines the rate of the wave functions decrease at infinity, governed by the $k = 0$ term in the following representation:

$$\Psi_{l,m}(r, \phi) \sim e^{im\phi - \frac{r^2}{4}} \sum_{k=0}^{\frac{l-|m|}{2}} \frac{(-)^k r^{l-2k}}{k!(\frac{l+m}{2} - k)!(\frac{l-m}{2} - k)!}. \tag{7}$$

Unfortunately, it is impossible to use the Aharonov—Casher normalizability argument here: the excited levels wave functions are nonnormalizable at all under their theorem conditions mentioned above. The proposed orbital momentum cutoff may be, however, considered as an IR regularization. Its main objective is to provide the finite degeneracy rate in the translation-invariant manner.

This regularization immediately leads to the following important consequences:

i) The degeneracy rate is energy dependent. It is obvious that the energy increase by unity results in the degeneracy rate decrease to the same amount: $N_\epsilon = N_\perp - n$.

ii) As the degeneracy rate is by definition positive the spectrum should be limited from above: $n_{max} = N_\perp - 1 = l_{max}$. The IR regularization induces the UV one! This phenomenon is qualitatively transparent: the wave function becomes more "wide" while the energy increases. Although this width is usually neglected when the degeneracy is calculated, one cannot place even a single state of the high enough energy in the fixed area.

Let us return to the actual spin-1/2 case. Each excited levels is degenerated according to the spin flip. However, the adopted regularization partially removes this degeneracy. Each $n_s \geq 1$ is realized as $(n = n_s, s = 1/2)$ and $(n = n_s - 1, s = -1/2$. If all one-particle states are occupied, the degeneracy difference results in the $s = -1/2$ spin for each excited level. Note that there are just $N_\perp$ excited levels in the spin-1/2 case exactly compensating the ground state spin. It is just the required mechanism of normal–anomalous cancellation.

It is not obvious which contribution should be identified with the normal and anomalous chiral symmetry breaking. The "classical" orbital momentum direction is negative [4] (we consider the positive charge: orbital momentum direction coincides with the magnetic moment one) reflecting the diamagnetic properties of the classical bounded charge. That is why m is positive for the ground level and becomes negative for the excited ones representing the semiclassical behaviour. However, passing from the massless to the massive case the chirality breaking associated with the ground level, changes sign [2]. The excited levels contribution should be identified as an anomalous one.

It is interesting that one can observe some cancellation between the contributions of different Landau levels to the orbital angular momentum, too. Note that the occupied ground state orbital momentum is:

$$L_0 = \sum_{m=0}^{l_{max}} m = \frac{l_{max}(l_{max}+1)}{2} = \frac{N_\perp(N_\perp - 1)}{2}. \quad (8)$$

This quadratically grown expression is looking strange, at first sight. It is however familiar in the framework of the Laughlin approach to the Quantized Hall Effect [8]. The famous Laughlin wave function describes the ground states as consisting of the electron *pairs* with unit relative orbital momentum, justifying Eq.(8). Its straightforward generalization for the excited level is:

$$L_n = \sum_{m=-n}^{l_{max}-2n} m = \frac{(l_{max}-3n)(l_{max}-n+1)}{2}. \quad (9)$$

One can easily check that

$$\sum_{n=0}^{l_{max}} L_n = 0. \quad (10)$$

Total orbital angular momentum of all Landau levels is zero! This cancellation is even more subtle than the cancellation of spin angular momentum discussed before. The excited levels orbital momentum does not manifest sharp sign change, contrary to the spin one. It varies smoothly from the maximal $l_{max}(l_{max}+1)/2$ value to the minimal $-l_{max}$, turning to zero at $n = l_{max}/3$. The identification of normal and anomalous contributions, if possible, requires further investigations.

3. Axial Anomaly and Supersymmetric Quantum Mechanics

The charged spin-1/2 particle motion in the magnetic field with uniform direction is the realization of supersymmetric quantum mechanics [9]. In fact, the main consequences of supersymmetry were discovered already in the mentioned paper by Aharonov and Casher [7].

First, the cancellation of bosonic and fermionic vacuum oscillations leads to the zero mode appearance. It is this level, connecting the positive and negative energy states for massless fermions, which is crucial for the Axial Anomaly manifestation [3, 2].

Second, all the excited levels are degenerated with respect to spin-flip. This degeneracy excludes the excited levels from the chirality balance in the standard treatment of the Anomaly [3, 2].

Therefore, Axial Anomaly, usually described as a *violation* of the (chiral) symmetry, may be thought of as a *manifestation* of the (super)symmetry.

It is instructive to study the action of the supersymmetry generators on the eigenstates of the adopted angular momentum basis:

$$Q_\pm |l, m, s = \mp 1/2\rangle \sim |l \pm 1, m \mp 1, s = \pm 1/2\rangle. \quad (11)$$

Note that spin, orbital momentum and coordinate dependence are all changed, while the total angular momentum is conserved. This property is easy to verify in the case of the arbitrary axially symmetric magnetic field. The relations between supersymmetry and angular momentum conservation probably may be extracted also from the fact that spin-1/2 angular momentum density is proportional to the axial current, while the orbital momentum density—to the derivative of the vector current. The nonrenormalization of the conserved operator allows one to relate the one-loop QED anomalous magnetic moment to axial anomaly [10]. One may expect that the exploring of the SuSy QCD in the EMC Spin Crisis resolution [11] is possible to justify with the help of the total angular momentum conservation.

Finally, one should notice that the proposed IR cutoff obviously violates one of the two basic SuSy predictions, namely, the degeneracy of the excited levels reflecting the chiral symmetry. One may say that although the axial anomaly in massless case is a consequence of the supersymmetry, its description in massive case requires very specific explicit supersymmetry violation.

4. IR cutoff, Axial Anomaly at 2+1 dimensions and Anyon Superconductivity

As the charge motion in the magnetic field is in fact two-dimensional, the whole analysis of Section 2 is applicable at 2+1 dimensions. The basic difference is the absence of spin degree of freedom: as a result, spin projection onto the magnetic field direction should be substituted for the charge. The standard Gribov treatment in the massless case results then in the induced vacuum charge and Chern–Simons (CS) term [12].

The IR cutoff allows a similar result in the massive case, too, in complete similarity to the 3+1-dimensional situation. The cancellation of two contributions is of additional physical interest here because of remarkable phenomena of Anyon Superconductivity [8], requiring just the cancellation of the bare and induced CS terms [13]. The physical reason for the IR cutoff may be the finite, but very large (macroscopic) area of the sample [14]. The density of the wider excited states should then be made decreasing in the translationally invariant manner, the latter being just the proposed cutoff. The translation invariance leads to the conservation of the momentum, which seems to be of interest if we should take into account the extra third dimension.

Dealing with 2+1-dimensional models of anyon superconductivity one should relate them to the real 3+1-dimensional world. Due to the uncertainty principle it is impossible to neglect the transverse degree of freedom in the coordinate and momentum space simultaneously. Usually one makes this reduction in the coordinate space, assuming the energy to be low enough to excite the transverse degree of freedom. The mentioned translation invariance and momentum conservation inside the plane make it natural to start with the 2+1-dimensional momentum space. The same result was achieved in recent study of the anomalous electron attraction [15] owing to dealing with the scattering amplitudes in the momentum representation.

As high-T_c superconductors represent the layer structure, the uncertainty principle immediately leads to the important conclusion: the electron cannot be confined to the single CuO layer and the coherent multilayer behaviour is of major importance. Therefore, one should choose the Neumann boundary conditions for the transverse Schrödinger equation and obtain the nonzero transverse electromagnetic current. As a result, it is not conserved.

in 2+1 dimensions: the effective theory should be non-gauge-invariant! Note that the nonconservation of the vector current in 2+1 can lead to the appearance of a zero-mass pole completely analogous to the ghost pole in QCD [16] (see also [1] and Ref. therein). As this pole is a signal of superconductivity [17], we have the new mechanism of it. The corresponding physical picture is the charge escape in the transverse direction and the return to another place, i.e., some 'wormhole' in 2+1-dimensional space. Note that the commonly accepted mechanism of high-T_c superconductivity in the Luttinger liquid model also requires the interlayer coherent transport, when passing below T_c (see, e.g., [18]). As this theory (and, in particular, the transverse coherent transport below T_c[19]) is strongly supported by the experimental data, the incorporation of the transverse dynamics into the anyon superconductivity theory via the electromagnetic current nonconservation seems to be reasonable. Recent papers [20],[21] are dealing with two-layer systems, but I would like to stress that a macroscopically large number of layers is required to obtain the 2+1-dimensional momentum space.

These qualitative arguments seem to indicate that the relation of the Axial Anomaly in 3+1-dimensional QED to the nontrivial physics and topology at 2+1 dimensions is not exhausted by the fact that the CS term is the surface one for the anomalous divergence. It is possible to add that the "vortex" in the mentioned semiclassical derivation of the anomaly equation [4] is nothing else than Anyon. The only difference is a factor of 2 resulting from the fact that the constant magnetic field was considered [4] rather than a monopole-like field of the Anyon problem.

5. Conclusions

The natural orbital angular momentum cutoff incorporated to the classical Landau levels problem allows a physical interpretation of the subtle property of cancellation of explicit and anomalous chiral symmetry breaking. The Axial Anomaly, being a manifestation of the supersymmetric quantum mechanics in the massless fermions case, requires the explicit violation of supersymmetry for massive fermions by this cutoff which is, in fact, the IR one. The necessity of such a cutoff is not deduced rigorously, but has some physical motivations. It provides the decreasing of broadening excited levels density in a translationally invariant manner. At 2+1 dimensions the analogous procedure leads to the zero total CS term, indicating the possibility of the Anyon Superconductivity. The latter may be related to the coherent multilayer effects.

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