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THE EXISTENCE OF ANTIPARTICLES SEEMS TO FORBID VIOLATIONS OF STATISTICS

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1 Introduction

The impossibility of small violations of Fermi or Bose statistics within the local quantum field theory using the deformation of the trilinear commutation relations of H. S. Green [1] has been studied in a set of papers [2–7]. It was found that the negative squared norms appear in many–particle states in that theory [4–6] in accordance with the general theorem proved earlier [8]. This theorem reflects the fact that there do not exist generalizations of usual Fermi and Bose statistics other than the para-Fermi and para-Bose statistics of finite orders in the framework of the local algebra of observables in the usual 3+1 dimensional Minkowski space [9, 10][1]. Really, the orders of these statistics are discrete: 1, 2, 3,... and there are no continuous transitions between statistics of different orders. In particular, it is impossible to have a small violation of the usual Fermi or Bose statistics that correspond to the parafermions or parabosons of the first order.

The classification of particle statistics in greater than two space dimensions [9, 10] contains only one possibility in addition to para-Bose and para-Fermi statistics of finite orders: the *infinite* statistics when all representations of the permutation group can occur. But according to the Fredenhagen theorem [11] this statistics cannot be embedded in the local algebra of observables[2].

[1]The *para-Fermi* and *para-Bose* statistics of order M are defined as the identical particle statistics in the three dimensional space under the restriction of a possible number of particles in the *symmetric* or *antisymmetric* state, respectively, by some positive integer number M. So, the number of particles in such states equal to or more than $M+1$ are forbidden. It is clear that cases with $M=1$ correspond to the ordinary Fermi and Bose statistics.

[2]A simplifying understanding of this theorem can be achieved for parafermions on the basis of the following reasoning. It is well known that soon after his discovery of the states with the negative energy for the relativistic particle with the half spin Dirac proposed to fill in these states beforehand by particles in accordance with the Pauli principle. A hole in this Dirac vacuum can be considered as an antiparticle with the particle mass and opposite charge. Obviously, one can fill in the negative-energy states not only by one, but also by two, by three and thus by any finite number of particles according to the para-Fermi statistics of a given finite order. A hole in this para-Dirac vacuum could be considered as an antiparafermion with the paraparticle mass, opposite charge and opposite hidden internal quantum numbers. However, we cannot fulfil this vacuum if the order of para-Fermi statistics goes to infinity. In this case we cannot define the antiparticle as a hole in the Dirac vacuum. Fredenhagen [11] proved that the conjugate (antiparticle) sector always exists in the local algebra of observables. Thus, the infinite statistics cannot be embedded in this algebra for lack of that sector.

So, if we insist on the violation of the statistics at any price, we are forced to try for this purpose to attract the infinite statistics corresponding to the nonlocal quantum field theory.

Recently Greenberg [12] has proposed to explore the so called "quon" statistics which are described by the q-deformed *bilinear* commutation relations. Really, this statistics allows a small deflection of the parameter of deformation q from 1 or -1, which are values corresponding to the Bose and Fermi statistics, respectively. It is much important that the many-particle states for quons have positive-definite squared norms for state vectors. However, Greenberg succeeded only in the non-relativistic quantum theory. In fact, each of quon statistics is an infinite statistics, and, due to the locality problem for the infinite statistics, the status of a relativistic field theory of quons is doubtful [12].

Here I shall try to consider the infinite statistics as the limiting case for finite parastatistics when their orders go to infinity. In this case we can control the conversion of the local quantum field theory into the nonlocal one. However, the result is negative: there is the only infinite statistics corresponding to $q = 0$. Any other values of the parameter q are forbidden just because of *the existence of antiparticles.* Thus, we conclude that the limiting approach forbids a small violation of Fermi or Bose statistics due to the impossibility of a continuous transition between admissible cases $q = 0$ and $q = \pm 1$.

The infinite statistics corresponding to $q = 0$ coincides with the classical Maxwell–Boltzmann statistics. We can comprehend this connection between quantum and classical statistics by means of the following reasoning. It can be suggested that our quantization, as any parastatistics scheme, should correspond to the usual Fermi or Bose statistics of identical particles with an infinite number of internal degenerated degrees of freedom, which is equivalent to the statistics of nonidentical particles since they are distinguishable (in principle) in their internal states [13–15].

Finally, we conclude that in our field theoretical approach admissible statistics (in three dimensional space) are: para-Fermi and para-Bose statistics of finite orders and the only infinite statistics which coincides with the classical Maxwell–Boltzmann statistics.

In conclusion I shortly compare my approach to the infinite statistics with others.

2 The deformed Green's paraquantization

For definiteness we consider the simplest examples of spin-integer and spin-half-integer fields: the scalar field

$$\varphi(x) = (2\pi)^{-3/2}\int d^3k(2E_{\mathbf{k}})^{-1/2}(a_{\mathbf{k}}e^{-ikx} + b^+_{\mathbf{k}}e^{ikx}), \tag{1}$$

and the Dirac field

$$\psi(x) = (2\pi)^{-3/2}\int d^3k(m/E_{\mathbf{k}})^{1/2}\sum_{\sigma=\pm 1/2}[a_{\sigma,\mathbf{k}}u(\sigma,\mathbf{k})e^{-ikx} + b^+_{\sigma,\mathbf{k}}v(\sigma,\mathbf{k})e^{ikx}], \tag{2}$$

where $\mathbf{k}$ is the particle momentum and $k_0 \equiv E_{\mathbf{k}} = (\mathbf{k}^2 + m^2)^{1/2}$. The magnitude of σ characterizes a spin state and $u(\sigma,\mathbf{k})$ and $v(\sigma,\mathbf{k})$ are well known Dirac's (bi)spinors corresponding to the positive- and negative-frequency solutions of the Dirac equation (see, for example, [16]).

For these fields we postulate the q-deformed Green trilinear commutation relations: for the scalar field

$$[[\varphi(x),\varphi^+(y)]_{-q},\varphi(z)]_- = i\rho\Delta(z-y)\varphi(x), \tag{3}$$

where φ^+ is the Hermitian conjugate field and the $\Delta(x)$ is the well-known Pauli–Jordan function

$$\Delta(x) = \frac{-i}{2(2\pi)^3}\int\frac{d^3k}{E_{\mathbf{k}}}(e^{-ikx} - e^{ikx}), \tag{4}$$

which is singular on the light cone and vanishes beyond it, and for Dirac field

$$[[\psi(x),\tilde{\psi}(y)]_{-q},\psi(z)]_- = -i\rho S(z-y)\psi(x), \tag{5}$$

where $\tilde{\psi} = \psi^+\gamma_o$ and

$$S(x) = -(i\gamma^\mu\partial_\mu + m)\Delta(x). \tag{6}$$

The q-form in Eqs. (3) and (5) means

$$[A,B]_{-q} \equiv AB - qBA\,. \tag{7}$$

The q and ρ are any *real*-valued parameter. Their reality is conditioned by the hermiticity of the observables which have the q-form. For example, the Hamiltonians and charges are written as

$$\mathcal{H}_{scal.} = -\rho^{-1}\int d^3x\{[\partial_t\varphi(x),\partial_t\varphi^+(x)]_{-q} + [\nabla\varphi(x),\nabla\varphi^+(x)]_{-q}$$
$$+m^2[\varphi(x),\varphi^+(x)]_{-q}\} + const., \tag{8}$$

$$\mathcal{Q}_{scal.} = ie\rho^{-1}\int d^3x\{[\varphi(x), \partial_t\varphi^+(x)]_{-q} - [\partial_t\varphi(x), \varphi^+(x)]_{-q}\} + const., \quad (9)$$

and

$$\mathcal{H}_{Dir.} = -\rho^{-1}\int d^3x[(-i\gamma\cdot\nabla + m)_{\alpha\beta}\psi_\beta(x), \tilde{\psi}_\alpha(x)]_{-q} + const., \quad (10)$$

$$\mathcal{Q}_{Dir.} = e\rho^{-1}\int d^3x\gamma^0_{\alpha\beta}[\psi_\beta(x), \tilde{\psi}_\alpha(x)]_{-q} + const. \quad (11)$$

We can readily verify that our theory is self-consistent. Indeed, the substitution of the Hamiltonian (8) or (10) into the Heisenberg equation

$$-i\partial_t\psi(x) = [\mathcal{H}, \psi(x)]_- \quad (12)$$

gives the (free) Klein–Gordon[3] or Dirac equation due to the commutation relations (3) or (5) and the projective properties of functions (4) and (6) (see, for example, [16]).

Further, these relations (3) or (5) ensure the locality of any observables taken in the q-form. For instance, for the Dirac field we have[4]

$$\begin{aligned}[][\psi_\alpha(x), \tilde{\psi}_\beta(x)]_{-q}, [\psi_\mu(y), \tilde{\psi}_\nu(y)]_{-q}]_- &= -i\rho S_{\mu\beta}(y-x)[\psi_\alpha(x), \tilde{\psi}_\nu(y)]_{-q} + \\ &+ i\rho S_{\alpha\nu}(x-y)[\psi_\mu(y), \tilde{\psi}_\beta(x)]_{-q}. \quad (13)\end{aligned}$$

The right-hand side of this relation vanishes when x and y are separated by the space-like interval.

However, eqs. (3) and (5) are not invariant under the charge-conjugation transformation

$$\varphi(x) \to \varphi_c(x) = \varphi^+(x), \quad \varphi^+(x) \to \varphi_c^+(x) = \varphi(x), \quad (14)$$

$$\psi(x) \to \psi_c(x) = C\tilde{\psi}^T(x), \quad \tilde{\psi}(x) \to \tilde{\psi}_c(x) = [C^{-1}\psi(x)]^T, \quad (15)$$

where ψ^T is the transposed (bi)spinor and C is the charge conjugation matrix. The theory remains C-invariant only in the case $q^2 = 1$ and this is just the Green paraquantization scheme [1]. Thus, in the general case $q^2 \neq 1$ our theory is not valid for the Hermitian field: for the truly neutral scalar field when $\varphi^+(x) = \varphi(x)$ and for the Majorana neutral field when $\psi_c(x) = \psi(x)$. For them we have only the Green quantization.

[3]For the scalar field it is necessary to consider the Heisenberg equation (12) both for the field $\varphi(x)$ and for its canonically conjugate momentum $\pi(x) = \partial_t\varphi^+(x)$.

[4]For derivation of this relation one ought to make use of the general identity

$$[[A,B]_\varepsilon, [C,D]_\eta]_- \equiv [[[A,B]_\varepsilon, C]_-, D]_\eta + [C, [[A,B]_\varepsilon, D]_-]_\eta,$$

where ε and η are any numbers. In eq. (13) we use also the Hermitian-conjugate relation of eq. (5).

On the other hand, our theory remains P-invariant under the space- reflection. But it is not invariant under the anti-unitary time-inversion transformation. As any local theory, our theory possesses the CPT-invariance. Thus, in the general case particles are replaced by antiparticles and vice versa under the time inversion. The only Green quantization remains invariant under C, P and T transformations separately.

Now we can determine commutation relations for the creation and annihilation operators of particles and antiparticles in given one-particle states (for instance, with the definite momentum $\mathbf{k}$ and spin-state σ). We label these states by the indices r, r', r'' for particles and by s, s', s'' for antiparticles. For simplification we propose these states to be discrete (We can assume that the system is placed into a finite volume.).

The substitution of decompositions (1) and (2) and their Hermitian conjugate expressions into eqs. (3) and (5), respectively, gives the set of formula:

$$[[a_{\mathbf{r}}, a^+_{\mathbf{r}'}]_{-q}, a_{\mathbf{r}''}]_- = \rho\delta_{\mathbf{r}'\mathbf{r}''}a_{\mathbf{r}}, \tag{16}$$
$$[[a_{\mathbf{r}}, a^+_{\mathbf{r}'}]_{-q}, a^+_{\mathbf{r}''}]_- = -\rho\delta_{\mathbf{r}\mathbf{r}''}a^+_{\mathbf{r}'}, \tag{17}$$
$$[[a_{\mathbf{r}}, b_{\mathbf{s}}]_{-q}, a_{\mathbf{r}'}]_- = 0, \tag{18}$$
$$[[b^+_{\mathbf{s}}, a^+_{\mathbf{r}}]_{-q}, a^+_{\mathbf{r}'}]_- = 0, \tag{19}$$
$$[[b^+_{\mathbf{s}}, a^+_{\mathbf{r}}]_{-q}, a_{\mathbf{r}'}]_- = \rho\delta_{\mathbf{r}\mathbf{r}'}b^+_{\mathbf{s}}, \tag{20}$$
$$[[a_{\mathbf{r}}, b_{\mathbf{s}}]_{-q}, a^+_{\mathbf{r}'}]_- = -\rho\delta_{\mathbf{r}\mathbf{r}'}b_{\mathbf{s}}, \tag{21}$$
$$[[b^+_{\mathbf{s}}, b_{\mathbf{s}'}]_{-q}, a_{\mathbf{r}}]_- = 0, \tag{22}$$
$$[[b^+_{\mathbf{s}}, b_{\mathbf{s}'}]_{-q}, a^+_{\mathbf{r}}]_- = 0, \tag{23}$$
$$[[a_{\mathbf{r}}, a^+_{\mathbf{r}'}]_{-q}, b^+_{\mathbf{s}}]_- = 0, \tag{24}$$
$$[[a_{\mathbf{r}}, a^+_{\mathbf{r}'}]_{-q}, b_{\mathbf{s}}]_- = 0, \tag{25}$$
$$[[a_{\mathbf{r}}, b_{\mathbf{s}}]_{-q}, b^+_{\mathbf{s}'}]_- = \mp\rho\delta_{\mathbf{s}\mathbf{s}'}a_{\mathbf{r}}, \tag{26}$$
$$[[b^+_{\mathbf{s}}, a^+_{\mathbf{r}}]_{-q}, b_{\mathbf{s}'}]_- = \pm\rho\delta_{\mathbf{s}\mathbf{s}'}a^+_{\mathbf{r}}, \tag{27}$$
$$[[b^+_{\mathbf{s}}, a^+_{\mathbf{r}}]_{-q}, b^+_{\mathbf{s}'}]_- = 0, \tag{28}$$
$$[[a_{\mathbf{r}}, b_{\mathbf{s}}]_{-q}, b_{\mathbf{s}'}]_- = 0, \tag{29}$$
$$[[b^+_{\mathbf{s}}, b_{\mathbf{s}'}]_{-q}, b^+_{\mathbf{s}''}]_- = \mp\rho\delta_{\mathbf{s}'\mathbf{s}''}b^+_{\mathbf{s}}, \tag{30}$$
$$[[b^+_{\mathbf{s}}, b_{\mathbf{s}'}]_{-q}, b_{\mathbf{s}''}]_- = \pm\rho\delta_{\mathbf{s}\mathbf{s}''}b_{\mathbf{s}'}. \tag{31}$$

In eqs. (26, 27, 30, 31) the up and down signs correspond to the scalar and spinor case, respectively. These signs determine the correct connection of the spin with parastatistics: para-Bose statistics for a scalar field and para-Fermi statistics for a spinor field.

Finally, the substitution of decompositions (1) and (2) into expressions (8) and (10) for the Hamiltonian and (9) and (11) for charges gives the following uniform expressions

$$\mathcal{H} = \sum_{\sigma} \int d^3k E_{\mathbf{k}}(N_{\sigma\mathbf{k}} + N^c_{\sigma\mathbf{k}}) \tag{32}$$

$$Q = \sum_{\sigma} \int d^3k (N_{\sigma\mathbf{k}} - N^c_{\sigma\mathbf{k}}) \tag{33}$$

where $N_{\sigma\mathbf{k}}$ and $N^c_{\sigma\mathbf{k}}$ are the number operators for particles and antiparticles

$$N_{\sigma\mathbf{k}} = -\rho^{-1}[a_{\sigma\mathbf{k}}, a^+_{\sigma\mathbf{k}}]_{-q} + const, \tag{34}$$

$$N^c_{\sigma\mathbf{k}} = \mp\rho^{-1}[b^+_{\sigma\mathbf{k}}, b_{\sigma\mathbf{k}}]_{-q} + const. \tag{35}$$

Due to eqs. (16)–(31) they obey the required equations

$$[N_r, a_{r'}]_- = -\delta_{rr'}a_{r'}, \tag{36}$$

$$[N^c_s, b_{s'}]_- = -\delta_{ss'}b_{s'}, \tag{37}$$

$$[N_r, N^c_s]_- = 0. \tag{38}$$

After all, only these properties make it possible to consider operators a, a^+ and b, b^+ as annihilation and creation operators for particles and antiparticles, respectively. That was a starting point for H. S. Green [1] in his formulation of a generalization of the usual quantization scheme.

3 The Fock representation of the q-deformed Green relations

First we consider the Fock representation only for particles, i. e. we consider only relations (16) and (17).

This construction has been fulfilled in paper [8], and here we briefly formulate general results of this investigation.

As usual, the Fock representation is defined by the requirement of the existence of a *unique* vacuum vector $\mid 0 >$ such that

$$a_r \mid 0 >= 0 \qquad \text{for all one-particle states r .} \tag{39}$$

Then the following relation

$$a_r a^+_{r'} \mid 0 >= p\delta_{rr'} \mid 0 >, \tag{40}$$

also holds, where p is any numeral parameter. The proof of this relation is performed just in the same manner as it was done by Greenberg and Messiah [17] for the Green quantization.

The basis vectors of the Fock representation are obtained by applying all monomials in the creation operators to the vacuum vector. Herein vectors obtained with different orders of these operators are independent of each other.

In this representation the action of a^+ on basis vectors merely adds one particle to the number of initial particles. The action of an operator a on the basis vectors can be calculated via eq. (17); we can move the a-operator to the right vacuum according to eq. (17) and then make use of eqs. (39) or (40). In the general case we obtain

$$a_r a^+_{r_1} a^+_{r_2} ... a^+_{r_n} \mid 0 >= p\delta_{rr_1} a^+_{r_2} ... a^+_{r_N} \mid 0 > + \sum_{k=2}^{n} \delta_{rr_k} [q^{k-2}(qp-\rho) a^+_{r_1} ... a^+_{r_{k-1}}$$
$$-\rho \sum_{l=1}^{k-2} q^{k-l-2} a^+_{r_1} ... a^+_{r_{k-l-2}} a^+_{r_{k-l}} ... a^+_{r_{k-1}} a^+_{r_{k-l-1}}] a^+_{r_{k+1}} ... a^+_{r_n} \mid 0 > . \quad (41)$$

The general state vector with a non-fixed number of particles is written in the form

$$\mid \Psi >= \Psi_o \mid 0 > + \sum_{n=1}^{\infty} \sum_{r_1,...,r_n} \Psi^{(n)}(r_1,...,r_n) a^+_{r_1} ... a^+_{r_n} \mid 0 >, \quad (42)$$

We do not propose any symmetry properties of amplitudes $\Psi^{(n)}(r_1,...,r_n)$ from the outset. For projections of vector (42) on basis vectors we have

$$< 0 \mid \Psi >= \Psi_o, \quad (43)$$
$$< 0 \mid a_{r_1} \mid \Psi >= p\Psi^{(1)}(r_1), \quad (44)$$
$$< 0 \mid a_{r_2} a_{r_1} \mid \Psi >= p^2\Psi^{(2)}(r_1,r_2) + p(qp-\rho)\Psi^{(2)}(r_2,r_1), \quad (45)$$
$$< 0 \mid a_{r_3} a_{r_2} a_{r_1} \mid \Psi >= p^3\Psi^{(3)}(r_1,r_2,r_3) + p^2(qp-\rho)\Psi^{(3)}(r_1,r_3,r_2) +$$
$$+p^2(qp-\rho)\Psi^{(3)}(r_2,r_1,r_3) + p(qp-\rho)^2\Psi^{(3)}(r_3,r_1,r_2) +$$
$$+p[q(qp-\rho)^2 - p\rho]\Psi^{(3)}(r_3,r_2,r_1) + p(qp-\rho)^2\Psi^{(3)}(r_2,r_3,r_1) \quad (46)$$

etc.

We see, the connection between amplitudes and projections is not so simple as it is for the usual quantization with commutators or anticommutators. For the construction of orthogonal combinations of projections we ought to form basis vectors of irreducible representations of the permutation groups S_n $(n = 2,3,...)$ of their arguments. We shall consider those combinations later.

From (44) we can deduced the norm of one-particle vector

$$\parallel \Psi^{(1)} \parallel^2 = p \sum_r \mid \Psi^{(1)}(r) \mid^2 . \quad (47)$$

Its being positive definite means the reality and positiveness of the parameter p

$$p^* = p > 0. \tag{48}$$

(Here the star denotes the complex conjugate.)

Further, the following theorem holds [4, 8]: *assume that p takes finite real number values. Then the squared norms of state vectors being positive definite in the Fock space implies that the number of particles in a symmetric or in an antisymmetric state cannot exceed a certain number M and the parameter q takes only three admissible values*[5]

$$q = 0, \quad \pm 1. \tag{49}$$

Cases $q = \pm 1$ correspond to the Green quantization [1, 17]. Herein the case $q = 1$ corresponds to the restriction for the number of particles in symmetrical states (para-Fermi statistics), and the case $q = -1$ corresponds to the same restriction for antisymmetrical states (para-Bose statistics).

In these cases there is the connection between parameters

$$\rho = 2pq/M. \tag{50}$$

Of course, we always can renormelize operators

$$a_r \to \sqrt{p} a_r. \tag{51}$$

So, p is a free parameter and we can choose this parameter to be equal to the order of parastatistics

$$p = M. \tag{52}$$

Then, instead of eq. (50) we have

$$\rho = 2q. \tag{53}$$

Thus, the parameter ρ introduced into the initial relations (3) and (5) and, respectively, into eqs. (16) and (17) takes the positive value $\rho = 2$ for the para-Fermi statistics ($q = 1$) and the negative value $\rho = -2$ for the para-Bose statistics ($q = -1$). As we indicated before, the Green quantization is charge-symmetrical.

[5] In paper [8] this theorem was formulated somewhat differently: the restriction that the number of particles in a symmetric or in an antisymmetric state cannot exceed a certain number M has been proposed as the point of departure. In paper [4] it was shown for the simplest case $M = 2$ (i. e. within a special parametrization for p, q and ρ [3]) that this restriction is in fact a consequence of a more general property of the positive definiteness of state vector norms. This result can be extended to the cases of para-Bose and para-Fermi statistics of arbitrary order. I am intending to publish the proof of this general theorem elsewhere.

Another allowed value of q: $q = 0$ corresponds to the new charge-asymmetrical paraquantization [14, 15][6]. In this case the connection between parameters is

$$\rho = \pm p/M. \tag{54}$$

If the relation (52) holds again, then the parameter ρ takes the values

$$\rho = \pm 1, \tag{55}$$

and now its sign determines the kind of statistics: para-Fermi for $\rho = 1$ and para-Bose for $\rho = -1$.

In this case projections have a simple form

$$< 0 \mid a_{r_n} a_{r_{n-1}} ... a_{r_2} a_{r_1} \mid \Psi >= \sum_{\mathcal{P} \in S_n} (-\lambda)^{\mathcal{N}(\mathcal{P})} p^{n-\mathcal{N}(\mathcal{P})} \Psi^{(n)}(r_{\mathcal{P}1}, r_{\mathcal{P}2}, ..., r_{\mathcal{P}n}), \tag{56}$$

where $\mathcal{P}$ is an arbitrary permutation of indices $1, 2, ..., n$, and $\mathcal{N}(\mathcal{P})$ is a *minimal* number of transpositions of two indices which are necessary for the restoration of their normal order. It is easy to prove that the combination (56) satisfies the requirement of parastatistics of order p: its symmetrical combination in the case $\rho = 1$ or antisymmetrical combination in the case $\rho = -1$ for $p + 1$ particles vanish automatically. We can consider these combinations (56) as a natural generalizations of usual antisymmetrical and symmetrical combinations for Fermi and Bose statistics, respectively, which correspond to $p = 1$.

It may not be out of place to mention that the combinations (56) do not obey any other symmetry relations in contrast with the Green quantization [1, 17].

In both admissible events $q^2 = 1$ and $q = 0$ a continuous transition from one parastatistics to another parastatistics of different order is forbidden. In particular, a small violation of Fermi or Bose statistics corresponding to $p = 1$ is not possible in the framework of parastatistics of finite orders.

We should notice that the above theorem [4, 8] can be proved whether or not antiparticles are included into the theory. In reality, it holds for both relativistic and nonrelativistic theories. The crucial property which one uses is the requirement that the squares of state vector norms be positive.

Nevertheless, the inclusion of antiparticles into consideration permits us to establish the correct connection between spins and parastatistics: particles with integer spins obey para-Bose statistics and particles with half-integer

[6] In paper [8] this case was omitted.

spins obey para-Fermi statistics[7]. For the scalar and Dirac fields which we consider this theorem may be proved by means of the requirement for the vacuum state

$$b_s \mid 0 >= 0 \quad \text{for any antiparticle state s,} \tag{57}$$

and the requirement of positiveness of squared state vector norms for both particles and antiparticles. The important ingredients of this proof are (A) the difference of signs in the right-hand side of eqs. (30) and (31) for these fields and (B) the above-discussed connection between signs of the parameter p in eqs. (53) and (55) and the kind of parastatistics.

Remark that the description of antiparticles within the Green paraquantization and in the paraquantization corresponding to $q = 0$ is somewhat different. In the former case the relation analogous to eq. (40) takes place for antiparticles too

$$b_s b_{s'}^+ \mid 0. = p_c \delta_{ss'} \mid 0 > \tag{58}$$

and the order of antiparticle parastatistics p_c coincides with the order of particle parastatistics p.

But in the case $q = 0$ there is another relation

$$b_s b_{s'}^+ \mid 0. = \delta_{ss'} \mid 0 > \tag{59}$$

Nevertheless, the orders of parastatistics for particles and antiparticles coincide again because the antiparticles may occur only in the particle-antiparticle pairs (for more details see [14, 15]). The number of particles in the initial state may be arbitrary. We cannot assert that this peculiarity of the new paraquantization scheme is able to explain the particle-antiparticle asymmetry of the world but it does not contradict this asymmetry.

4 The uniqueness of the infinite statistics

Heretofore we considered only parastatistics of finite order. Now we include into consideration our main intention: the infinite statistics.

We shall interpret the infinite statistics as a limit case of parastatistics when their orders go to infinity.

Our previous theorem has been proved under the assumption of the finiteness of the parameter p. However, it is not valid in the infinite limit. We ought to consider this limit again from the outset.

[7]The connection between spins and the kinds of parastatistics has been established in paper [18]. However, the authors made use of a special representation for the Green parafield, the so called Green ansatz: the sum of ordinary bosonic or fermionic fields which obey anomalous (contrary) mutual commutation relations. This representation is not convenient in the case $q = 0$.

Again at the first step we restrict our consideration only to the particles. Taking into account the limit $p \to \infty$ we keep in eq. (41) only terms proportional to p. Then instead of eq. (41) we get

$$a_r a^+_{r_1} \dots a^+_{r_n} \mid 0 >= p \sum_{k=1}^{n} \delta_{r r_k} q^{k-1} a^+_{r_1} \dots a^+_{r_{k-1}} a^+_{r_{k+1}} \dots a^+_{r_n} \mid 0 > . \tag{60}$$

It is easy to prove that the following ***bilinear*** relations take place within this representation

$$a_r a^+_{r'} - q a^+_{r'} a_r = p \delta_{r r'}. \tag{61}$$

Evidently, we can accomplish the renormalization of operators (51) at once and get rid of the infinitely rising parameter p. Then we arrive at the q-deformed bilinear commutation relations

$$a_r a^+_{r'} - q a^+_{r'} a_r = \delta_{r r'}. \tag{62}$$

As it was mentioned above, these relations were used by Greenberg [12] for the formulation of small violation of ordinary Bose or Fermi statistics.

Now the right-hand sides of eqs. (16) and (17) vanish due to eq. (62). It means that the left-hand sides of these equations vanish too, i. e. $\rho = 0$, which corresponds to the limit $\rho/p \to 0$ in the limit $p \to \infty$ at a finite ρ.

Using eq. (62) we have for a projection of an arbitrary vector

$$< 0 \mid a_{r_n} a_{r_{n-1}} \dots a_{r_2} a_{r_1} \mid \Psi >= \sum_{\mathcal{P} \in S_n} q^{\mathcal{M}(\mathcal{P})} \Psi^{(n)}(r_{\mathcal{P}1}, r_{\mathcal{P}2}, \dots, r_{\mathcal{P}n}), \tag{63}$$

where $\mathcal{M}(\mathcal{P})$ is the minimal number of *successive* (neighboring) transpositions of two indices necessary for the restoration of their normal order (cf. eq.(56) where the question is the minimal number of *any* transpositions). The particular cases can be obtained from eqs. (45) and (46) in the limit $p \to \infty$ and operator renormalization (51).

Surely, the cases $q = 1$ and $q = -1$ correspond to ordinary Bose and Fermi statistics. But if $q^2 \neq 1$, then we can compose from expression (63) any combinations which are the basis vectors of irreducible representations of the permutation group including symmetrical and antisymmetrical functions in any number of particle states. Such statistics without any restrictions on the allowed Young schemes are called ***infinite statistics***. Each of them is defined by its parameter q.

Nevertheless, the requirement of positive definiteness of state vector norms imposes a definite restriction on possible values of the parameter q even in the case of infinite statistics but not so strict as for finite statistics.

First, consider the symmetrical or antisymmetrical vector of two particles

$$| \Psi_\lambda > = \sum_{r_1,r_2} \Psi_\lambda(r_1,r_2) a^+_{r_1} a^+_{r_2} \mid 0 >, \qquad (64)$$

where the function $\Psi_\lambda(r_1,r_2)$ is λ-symmetrical

$$\Psi_\lambda(r_2,r_1) = \lambda \Psi_\lambda(r_1,r_2) \quad \text{and} \quad \lambda = \pm 1. \qquad (65)$$

The norms of such vectors can be calculated by means of eq. (63) (at $n = 2$). We obtain

$$\| \Psi_\lambda \|^2 \equiv < \Psi_\lambda \mid \Psi_\lambda > = (1 + \lambda q) \sum_{r_1,r_2} \mid \Psi_\lambda(r_1,r_2) \mid^2 . \qquad (66)$$

Their positiveness requires

$$-1 \leq q \leq 1. \qquad (67)$$

It can be proved that the squared norms of *all* vectors made by limits of polynomials of the creation operators a^+ are strictly positive within the interval (67) [12]. As q approaches ± 1, the symmetric or antisymmetric combinations are more heavily weighted and for q very close to $1(-1)$ the theory becomes very close to the Bose (Fermi) statistics. Thus, we can, as Greenberg [12] proposed, consistently formulate "small violations" of Bose or Fermi statistics in the framework of such a "quonic" theory.

However, let us to include antiparticles into our consideration. Herein we eliminate the special case $q = 0$ from the outset. Then we can copy eqs. (30) and (31) for antiparticles as

$$[[b_s, b^+_{s'}]_{-q}, b_{s''}]_- = \rho_c \delta_{s's''} b_s, \qquad (68)$$

$$[[b_s, b^+_{s'}]_{-q}, b^+_{s''}]_- = -\rho_c \delta_{ss''} b^+_{s'}. \qquad (69)$$

where

$$q_c = 1/q \quad \text{and} \quad \rho_c = \mp \rho / q. \qquad (70)$$

These relations are utterly analogous to eqs. (16) and (17) for particles, and we can arrange the whole our consideration for the case of antiparticles.

Again we propose the relation (57) for the vacuum state and obtain the relation (58). If p_c has a finite value (irrespective of either p is a finite or an infinite parameter) then our theorem is valid for antiparticles too, and we have only three admissible values for $q_c = 0, \pm 1$. We eliminate the case $q_c = 0$ from our consideration [8]. The values $q_c = \pm 1$ correspond to $q = \pm 1$. In these cases $p_c = p$ [17].

[8] This case corresponds to the limit $q \to \infty$ when the ratio ρ/q is finite (i. e. $\rho \to \infty$ too). In this case relations (16) and (17) can be transformed to the case $q = 0$ by exchanging operators: $a \leftrightarrow a^+$.

Thus, it remains to consider the case of the infinite limit: $p_c \to \infty$ (and $p \to \infty$). In this case we obtain, as in the previous particle case, the limiting bilinear commutation relations for the antiparticle operators

$$b_s b_{s'}^+ - q_c b_{s'}^+ b_s = \delta_{ss'}. \tag{71}$$

where we have already performed the renormalization $b_s \to \sqrt{p_c} b_s$.

Then we obtain the restriction on q_c analogous to eq. (67)

$$-1 \leq q_c \leq 1, \tag{72}$$

or due to eq. (70) we have

$$-1 \leq 1/q \leq 1. \tag{73}$$

The compatibility of conditions (67) and (73) means $q^2 = 1$. Thus we have only $q = \pm 1$, i. e. the usual Bose and Fermi statistics.

Therefore we have the only possibility for the infinite statistics

$$q = 0. \tag{74}$$

This scheme has been considered in [13-15] in more detail. According to eq. (62) at $q = 0$ the particle operators satisfy very simple relations

$$a_r a_{r'}^+ = \delta_{rr'}. \tag{75}$$

Equation (63) also transforms into the direct expression

$$< 0 \mid a_{r_n} a_{r_{n-1}} ... a_{r_2} a_{r_1} \mid \Psi >= \Psi^{(n)}(r_1, r_2, ..., r_n), \tag{76}$$

where functions $\Psi^{(n)}(r_1, r_2, ..., r_n)$ have no any symmetry properties. We can conclude that these functions describe different particles obeying the classical Maxwell-Boltzmann statistics [13, 14].

In this case the behavior of antiparticles is very peculiar [14, 15][9]. The condition (59) is valid for them and the parameter p_c does not appear at all. Therefore we are not in need of the renormalization of the type (51) for antiparticles and the antiparticle operators satisfy the initial trilinear commutation relations (68) and (69) whereas the particle operators obey the bilinear relations (75). As a result, the relations (3) and (5) do not hold for fields, and the theory becomes nonlocal in accordance with the Fredenhagen theorem [11].

[9]Remark that our description of antiparticles in the limit $p \to \infty$ is somewhat different from the Greenberg's one [13].

5 Conclusion

In paper [4, 5, 8] it has been shown that it is impossible to construct a local free field theory for a small violation of Fermi or Bose statistics. The proof of this theorem does not require the existence of antiparticles. So it is valid within the nonrelativistic theory as well. The positiveness of squares of state vector norms plays a crucial role in this proof.

However, as Greenberg has shown, the nonrelativistic theory based on the q-deformed bilinear commutation relations admits a small deflection from the usual Fermi or Bose statistics without an infraction of the norm positiveness if values of the deformation parameter q are limited within the open interval $-1 < q < 1$.

However, we prove in this paper that the relativistic version of this theory obtained as an infinite limit of the q-deformed Green's trilinear commutation relations also rejects such possibility of small violation of usual statistics. In this proof the crucial role belongs to the existence of antiparticles. The same result has been obtained in papers [19, 20] within the q-deformed *bilinear* commutation relations for relativistic fields.

Of course, we can, as Greenberg has proposed [12], unify the particle and antiparticle operators obeying the same q-deformed bilinear commutation relations into a united field. However, this field does not obey any commutation relations. Then, the question arises: why must we suppose the coincidence of values of the deformation parameter q for particles and antiparticles which are independent from the outset? In contrast, the procedure of derivation of the commutation relations for the particle and antiparticle operators from the trilinear or bilinear commutation relations for a common relativistic field, in my opinion, seems a more natural. As we have seen, the commutation relations for them are different in the limit of infinite statistics for the case $q = 0$. Meanwhile, the theory remains CPT-invariant even in this case [14].

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