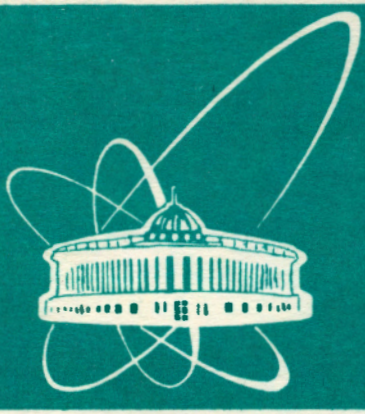

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ZERO MODES OF FIRST CLASS SECONDARY CONSTRAINTS IN GAUGE THEORIES

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Introduction

There is a significant difference between the description of Yang–Mills field ground state and that of any other fields for which the conventional methods of quantum field theory work. The instability of the naive perturbation theory (for example, see ref. [1] and review [2]) is one of the crucial problems in application of the non-Abelian field theory to hadron physics.

Efforts to understand the problem of the QCD vacuum give rise to a huge number of speculations connected with the nontrivial topological properties of gauge fields [3-8]. We want to continue these attempts from a different point of view.

In this article , we would like to underline the significant role of zero modes of the secondary first–class constraints [9, 10] in the quantization procedure of gauge theories. We will demonstrate that the presence of zero–modes reflects a global structure of the initial gauge symmetry group. These zero modes can be also treated as some collective excitations of gauge fields. An example of that type zero mode is the Coleman electric field in the two–dimensional QED [11, 12, 13]. It is well known that the local $U(1)$ in the two–dimensional space time and the non–Abelian compact groups in four dimensions have the same topological properties. Therefore, before the consideration of the non–Abelian gauge theory we study a simple example, electrodynamics on a finite line and emphasize the nature of these zero modes as remaining quantum mechanical variables. With this experience we proceed to investigate the four-dimensional $SU(2)$ gauge model where in direct analogy with the previous example we introduce the same type residual variable describing the zero mode of the secondary constraint. On the basis of this,

we speculate on a possible role of these collective excitations for the Yang–Mills ground state and stable perturbation theory.

The paper will be organized as follows. In section 1 , we present a systematic analysis of electrodynamics in two-dimensional finite space time in the Lagrangian and Hamiltonian forms. Section 2 is devoted to the (1+3) dimensional $SU(2)$ Yang–Mills theory. We prove a no-go theorem about the local realization of the representation of a homotopy group without the collective mode, and show that the presence of a zero-mode of the first–class secondary constraint leads to another realization different from the "instanton" one [3-5].

1. Electrodynamics in the two – dimensional finite space – time

1.1. Zero mode of the Gauss equation

Let us start with the Abelian $U(1)$ gauge theory action in the two-dimensional finite space time

$$W[A^\mu] = \int\limits_{-\frac{1}{2}T}^{\frac{1}{2}T} dt \int\limits_{-\frac{1}{2}R}^{\frac{1}{2}R} dx \left(-\frac{1}{4}F^2{}_{\mu\nu}\right). \qquad (1)$$

In the (1+1) space time we have only electric tension E [1]

$$F_{01} := \dot{A}_1 - \partial A_0 = E. \qquad (2)$$

[1]Below in the text we will use the following notation $\dot{f} := \frac{\partial f}{\partial t}, \partial f := \frac{\partial f}{\partial x}$.

The action is invariant under the local gauge transformation

$$A_\mu(t,x) \rightarrow A'_\mu(t,x) = g(t,x)(A_\mu(t,x) + \frac{i\hbar}{e}\partial_\mu)g^{-1}(t,x), \qquad (3)$$

affected by an element of the gauge group

$$g(t,x_1) = \exp(\frac{i}{\hbar}\lambda(t,x)) \qquad (4)$$

with an arbitrary function $\lambda(t,x)$. (The constant $e/\hbar$ has the dimension of mass and $\lambda/\hbar$ is dimensionless.)

The Euler – Lagrange equations for the gauge field follows from the action (1) by varying $A_\mu(t,x)$

$$\partial^2 A_0(t,x) - \partial\dot{A}_1(t,x) = 0, \qquad (5)$$

$$\ddot{A}_1(t,x) - \partial\dot{A}_0(t,x) = 0. \qquad (6)$$

The Gauss equation (5) does not contain a time derivative of the time component A_0 and is considered as a constraint.

The general solution of the Gauss law (5) with respect to the time component can be represented as a sum of a general solution of the homogeneous equation

$$\partial^2\varphi_0(t,x) = 0\ ,\ \varphi_0(t,x) = c_1(t) + c_2(t)\,x,$$

and a particular solution of the inhomogeneous one constructed with the help of Green's function $G(x,x')$:

$$A_0(t,x) = \varphi_0(t,x) + \int\limits_{-\frac{1}{2}R}^{\frac{1}{2}R} dx' G(x\ ,x')\left(\partial\dot{A}_1(t,x')\right)\ , \qquad (7)$$

$$\partial^2 G(x\ ,x') = \delta(x-x'). \qquad (8)$$

To specify the zero mode $\varphi_0(t,x)$ and the Green function $G(x\ ,x')$, we need a boundary condition for the gauge field $A_\mu(t,x)$. The usually exploited boundary condition

$$A_\mu(t,x)\Big|_{x=\pm\frac{1}{2}R} = 0 \tag{9}$$

leads to the well-known result for the zero mode

$$\varphi_0(t,x) = 0$$

and the Green function

$$G(x,\ x') = \frac{1}{2}\mid x-x'\mid + \frac{(xx')}{R} - \frac{R}{4}.$$

Substitution of the solution (7) with these quantities into (6) leads to the identity

$$\ddot{A}_1(t.x) = \ddot{A}_1(t,x). \tag{10}$$

Thus , as have been expected , we do not get any restriction on $A_1(t.x)$. Due to the gauge invariance (3) only transversal components are dynamical. In the two-dimensional space we have only a longitudinal component. As a consequence , we obtain

$$E = 0.$$

Now let us suppose a more subtle than (9) condition

$$A_\mu(t,x)\Big|_{x=+\frac{1}{2}R} = A_\mu(t,x)\Big|_{x=-\frac{1}{2}R} + a_\mu(t) \tag{11}$$

with an arbitrary time- dependent vector $a_\mu(t)$ and

$$\dot{A}_1(t,x)\Big|_{x=+\frac{1}{2}R} = \dot{A}_1(t,x)\Big|_{x=-\frac{1}{2}R} \tag{12}$$

$$\partial A_0(t,x)\Big|_{x=+\frac{1}{2}R} = \partial A_0(t,x)\Big|_{x=-\frac{1}{2}R}. \tag{13}$$

These conditions mean for the physical quantity E the following:

$$E\Big|_{x=+\frac{1}{2}R} = E\Big|_{x=-\frac{1}{2}R}.$$

It is evident that for this case we have zero mode

$$\varphi_0(t,x) = c_1(t) + \frac{a_0(t)}{R}\,x.$$

Due to the presence of this zero mode of the operator ∂^2, we face the problem of the correct definition of Green's function with the boundary conditions

$$G(x,x')\Big|_{x=+\frac{1}{2}R} = G(x,x')\Big|_{x=-\frac{1}{2}R} \tag{14}$$

$$\partial G(x,x')\Big|_{x=+\frac{1}{2}R} = \partial G(x,x')\Big|_{x=-\frac{1}{2}R}. \tag{15}$$

To solve this problem , we exclude the zero mode from the Green function spectral representation

$$G(x\ ,x') = \sum_{n=-\infty,n\neq 0}^{\infty}\frac{1}{\lambda_n}u_n(x)u_n^*(x'), \tag{16}$$

where the function $u_n(x)$ is an eigenfunction of the one-dimensional Laplace operator with boundary conditions of the type (14), (15)

$$\partial^2 u_n(x) = \lambda_n u_n(x), \qquad u_n(x) = \frac{1}{\sqrt{R}}\exp(i\frac{2\pi n}{R}x),$$

with an eigenvalue $\lambda_n = -(\frac{2\pi n}{R})^2$.

The representation (16) leads to the following equation

$$\partial^2 G(x,x') = \sum_{n=-\infty,n\neq 0}^{\infty} u_n(x)u_n^*(x') = \delta(x-x') - \frac{1}{2R}$$

instead of the conventional one (8). Nevertheless , it is easy to check the representation (7) with this solution of the Gauss equation (5) because A_1 satisfies (12). The explicit form of the Green function (16) is

$$G(x-x') \;=\; \frac{1}{2}\mid x-x'\mid -\frac{(x-x')^2}{2R}-\frac{R}{12}.$$

After substitution of the solution (7) into the equation for $A_1(x)$ instead of identity (10) we get

$$\partial\dot{\varphi}_0(t,x) \;-\; \frac{1}{R}\int\limits_{-\frac{1}{2}R}^{\frac{1}{2}R} dx'\ddot{A}_1(t,x') \;=\; 0. \qquad (17)$$

This is the crucial point. We obtain the remaining variable depending on the zero mode and on the functional $N_L[A_1]$

$$N_L[A_1] \;=\; \frac{e}{2\pi\hbar}\int\limits_{-\frac{1}{2}R}^{\frac{1}{2}R} dx'A_1(t,x'). \qquad (18)$$

It is useful to introduce the following notation for remaining variable

$$N_T(t) \;=\; N_0(t) \;+\; N_L[A_1], \qquad (19)$$

where

$$N_0(t) \;=\; \frac{eR}{2\pi\hbar}\int\limits_{-\frac{1}{2}T}^{t} dt'\partial\varphi_0(t',x). \qquad (20)$$

In terms of $N_T(t)$ eq.(17) has a simple form

$$\ddot{N}_T(t) \;=\; 0 \qquad (21)$$

and for electric tension we get

$$E \;=\; \frac{2\pi\hbar}{R}\dot{N}_T(t).$$

It is easy to see that the new variable $N_T(t)$ is connected with the two dimensional topological invariant , the Pontryagin number functional $\nu[A]$

$$\nu[A] \;=\; \frac{e}{4\pi\hbar}\int d^2x\epsilon_{\mu\nu}F^{\mu\nu} \;=\; \frac{e}{2\pi\hbar}\int\limits_{-\frac{1}{2}T}^{\frac{1}{2}T} dt\int\limits_{-\frac{1}{2}R}^{\frac{1}{2}R} dx(\dot{A}_1-\partial A_0)$$

in a simple way

$$\nu[A]\Big|_{\text{constraint}} \;=\; N_T(\frac{T}{2}) - N_T(-\frac{T}{2}). \qquad (22)$$

Now, we can see the double role of the remaining variable $N_T(t)$ in different gauges. In the gauge $A_1 \;=\; 0$ it looks like a zero mode of the Gauss equation for electric tension

$$\partial^2 A_0(t,x) \;=\; 0.$$

In the temporal gauge $A_0 \;=\; 0 \quad , N_T \;=\; N_L[A_1]$ is the topological variable which transforms under the residual stationary gauge transformations

$$A_1(t,x) \;\rightarrow\; A_1'(t,x) = g(x)(A_1(x_1)+\frac{i\hbar}{e}\partial)g^{-1}(x), \quad g(x) = \exp\,(\frac{i}{\hbar}\lambda(x)\,)$$

in the following way:

$$N_L[A_1'] \;=\; N_L[A_1] \;+\; \frac{\hbar}{2\pi}\left[\lambda(\frac{R}{2}) \;-\; \lambda(-\frac{R}{2})\right]. \qquad (23)$$

Recall that the functions $g(x)$ represent maps of the line $(-R/2,\ R/2)$ with the identified ends $g(\frac{R}{2}) \;=\; g(-\frac{R}{2})$ onto the $U(1)$-group space. All these maps are split into the classes characterized by the integer index n pointing out how many times the closed line turns around the $U(1)$-space circle. The quantity $g_{(n\neq 0)}$ is called the large gauge transformation; while $g_{(n=0)}$, the small one.

The factor–space of all stationary gauge transformations with respect to the small ones G/G_0 coincides with the homotopy group of all one–dimensional closed paths on the $U(1)$–circle

$$\Pi_1(U(1)) = Z, \tag{24}$$

where Z is the group of integers. The new variable $N_T(t)$ is invariant under the small gauge transformations and changes by an integer under large transformations

$$N_T(t) \to N_T + n. \tag{25}$$

The invariance of the theory under the large gauge transformation means that the points N_T, $N_T + n$ are physically identical. The configurations $N_T = 0$ and $N_T = 1$ are the same; so the manifold $\{N_T\}$ is a circle of the length of unity.

Thus, the explicit solution of the constrained equation (5) leads to some reduced theory, effective action for which can be obtained after substitution of the solution (7) into initial action (1)

$$W[A_\mu]\Big|_{\text{constraint}} = W^{Red}[N_T(t)] = \int_{-\frac{T}{2}}^{\frac{T}{2}} dt \left(\frac{1}{2}\dot{N}_T(t)^2 I\right). \tag{26}$$

The gauge field theory in the 1+1 space time reduces to a simple mechanical system . The reduced action with the definition of the manifold $\{N_T\}$ describes a plane rotator with the mass $I = \frac{2\pi}{eR}$.

To get a physical consequence of the new variable , we should quantize our reduced theory. Thus , the next goal is the consideration of this zero mode in the Hamiltonian form of gauge dynamics.

1.2. Zero modes in the Hamiltonian approach. Primary and secondary reduction

It is easy to get the Hamiltonian form of the reduced theory starting from the reduced action (26)

$$W^{Red} = \int_{-T/2}^{T/2} dt \left[P\dot{N}_T - \frac{1}{2} P^2 I^{-1}\right], \tag{27}$$

where P is canonically conjugate to N_T. For the generalization to the non-Abelian case , it is useful to obtain this action and to elucidate the status of zero modes in the canonical Hamiltonian scheme.

For our purpose it is convenient to use first order formalism action

$$W_I[E, A^\mu] = \int_0^T dt \int_{-\frac{1}{2}R}^{\frac{1}{2}R} dx \left(E(\dot{A}_1 - \partial A_0) - \frac{1}{2}E^2\right), \tag{28}$$

where the time component A_0 plays the role of the Lagrange factor. The momentum canonically conjugate to $A_0(t,x)$ is equal to zero

$$\pi_0(t,x) = \frac{\partial \mathcal{L}}{\partial \dot{A}^0(t,x)} = 0. \tag{29}$$

Equation (28) is the primary constraint and its Poisson bracket with the canonical Hamiltonian $\mathcal{H}$ leads to a secondary constraint for the electric tension

$$\{\pi_0,\ \mathcal{H}\} = \partial E = 0. \tag{30}$$

These two constraints according to Dirac's definition [9] , form the first-class ones. Equations (29) and (30) mean that the time component is not physical and can be removed from phase space by the gauge transformation. The removal of nonphysical components by explicit solving of primary and secondary constraints will be called the *primary and secondary reductions* respectively. To remove the time component , it is

enough to choose the gauge $A_0 = 0$. The primary reduction of the action (28) gives

$$W_I^{Red} = \int d^2x \left[E\dot{A}_1 - \frac{1}{2}E^2 \right] \tag{31}$$

with the Gauss law

$$\partial E = 0.$$

This first-class secondary constraint should be accompanied by the second -class constraint (gauge condition). Let us choose the gauge

$$\partial \dot{A}_1 = 0. \tag{32}$$

This gauge is distinguished by the equation of motion following from the action (31)

$$E = \dot{A}_1 \tag{33}$$

and by the constraint (30) .

The secondary reduction of the Hamiltonian scheme action (28) evidently coincides with the action (26) obtained from the Lagrangian approach. To verify this , let us write down the explicit solution of constraints (30) and (32) in the form

$$E = P\frac{e}{2\pi\hbar} + E^T, \tag{34}$$

$$A_1 = g(N_0)(A_1^T + \frac{i\hbar}{e}\partial)g^{-1}(N_0), \tag{35}$$

where the gauge transformation

$$g(N_0) = \exp\left\{ i\frac{2\pi x}{R} N_0(t) \right\} \tag{36}$$

and A^T and E^T are the transversal variables equal to zero in the two-dimensional case.

Thus, a global subgroup of gauge symmetry leads to collective excitation P of the type of a zero mode of the first class secondary constraint (30) which is the remaining longitudinal part of the gauge field momentum. This zero mode is accompanied by the zero mode of the "radiation" gauge, which is well known as the Gribov ambiguity [14].

1.3. Quantization of the reduced theory

For quantum description of the reduced theory we will use a fixed time Schrödinger representation.The canonical variables $\hat{N}_T(t), \hat{P}(t)$ are fixed at time t^*

$$\hat{N}_T = \hat{N}_T(t^*), \quad \hat{P} = \hat{P}(t^*)$$

and satisfy the commutation relation

$$i\left[\hat{P}, \hat{N}_T\right] = \hbar. \tag{37}$$

The stationary Schrödinger equation

$$H\Psi_\epsilon = \epsilon\Psi_\epsilon, \tag{38}$$

should be complected by the constraint of identification of the points N, $N+1$ on the circle

$$\Psi_\epsilon(N+1) = e^{i\theta}\Psi_\epsilon(N) \qquad (0 \le \theta \le \frac{\pi}{2}). \tag{39}$$

The solution of these equations is the Bloch plane wave

$$\Psi_\epsilon = e^{\frac{i}{\hbar}PN}, \quad P = (2\pi k + \theta)\hbar, \tag{40}$$

where k is the number of the Brillouin zone. The spectra of the electric tension and the energy have the following forms

$$E = e\left(k + \frac{\theta}{2\pi}\right) \tag{41}$$

$$\epsilon = RE^2/2 \tag{42}$$

which coincide for the ground state ($k = 0$) with the Coleman constant electric tension [11]. The nonzero tension (or the collective persistent current in the functional space) appears here as a pure quantum effect (of the type of the Josephson one) due to the jump of the phase of the wave function (40).

There is another way of describing this zero mode where it is represented as the functional of the local variable [4,5]. However, different (equivalent for the considered model) ways of introducing the zero mode lead to different results in the non-Abelian theory. The reader can see this fact in the next section.

2. The $SU(2)$ Yang–Mills Theory in Four Dimensions

2.1. Primary reduction

We start with primary reduction of the Yang - Mills theory with the local $SU(2)$ group

$$W^{Red} = \int d^4x \left[E_i^a \dot{A}_i^a - \frac{1}{2}\left(E_i^{a2} + B_i^{a2}\right)\right], \tag{43}$$

where the electric tension E_i^a satisfies the secondary constraints

$$\nabla_i^{ab} E_i^b = \left(\delta^{ab}\partial_i + e\epsilon^{acb} A_i^c\right) E_i^b = 0, \tag{44}$$

and magnetic tension B_i^a

$$B_i^a = \epsilon_{ijk}\left(\partial_j A_k^a + \frac{e}{2}\epsilon^{abc} A_j^b A_k^c\right). \tag{45}$$

Below we will use the gauge

$$\nabla^{ab}{}_i(A)\dot{A}_i^b = 0 \tag{46}$$

which is consistent with the equations of motion

$$E_i^a = \dot{A}_i^a. \tag{47}$$

This theory has the topological nontrivial gauge symmetry group [3—8]

$$\hat{A}_\mu \rightarrow \hat{A}_\mu^g = g(\hat{A}_\mu + \partial_\mu)\, g^{-1};\ \ \hat{A}_\mu = \frac{e\tau^a A_\mu^a}{2i}. \tag{48}$$

All stationary transformations with the boundary conditions

$$\lim\, g\ (\vec{x}) = 1 \quad |\,\vec{x}\,| \rightarrow \infty \tag{49}$$

represent the manifold of three–dimensional closed paths on the three–dimensional sphere SU(2), and can be split into the classes characterized by the integer index of a map (n) of the space $\{\vec{x}\}$ into the SU(2) group space:

$$n = \frac{1}{24\pi^2}\int d^3x\, \epsilon^{ijk} tr\left[\hat{V}_i\,\hat{V}_j\,\hat{V}_k\right];\ \ \hat{V}_i = g\,\partial_i g^{-1}. \tag{50}$$

As in eq.(24), we can speak here about the homotopy group

$$\Pi_3(SU(2)) = Z. \tag{51}$$

There is a topological variable $N_L[A]$

$$N_L[A] = \frac{e^2}{16\pi^2} \int d^3x \epsilon_{ijk} \left(A_i^a \, \partial_j \, A_k^a + \frac{1}{3} \, \epsilon^{abc} \, A_i^a \, A_j^b \, A_k^c\right) \qquad (52)$$

which realizes the representation of the homotopy group [6]

$$N_L[A^g] \; = \; N_L[A] \; + \; n. \qquad (53)$$

2.2. No-Go Theorem for the Local Quantum Representation of the Homotopy Group

Apart from the experience obtained from the above considered two-dimensional theory there is one more mathematical argument in favor of the existence of the independent collective variable N_T of the type of (19).

The exact formulation of the problem of quantization of the Yang–Mills theory with the nontrivial homotopy group is given in refs. [4, 5] and consists in solving of the set of equations

$$H_L \Psi_\epsilon \; = \; \epsilon \Psi_\epsilon \qquad (54)$$

$$\nabla_i E_i \; \Psi_\epsilon \; = \; 0 \qquad (55)$$

$$T_L \Psi_\epsilon \; = \; e^{i\theta} \; \Psi_\epsilon. \qquad (56)$$

A first equation is the stationary Schrödinger equation with the Hamiltonian

$$H_L[A, E] = \int d^3x \frac{1}{2} \left(E_i^{a2} + B_i^{a2}\right), \qquad (57)$$

Eq.(55) reflects the invariance of the theory under the small gauge transformations, and Eq.(56) describes the covariant properties of the wave function under a large gauge transformation. The topological shift operator T_L has the form

$$T_L = \exp\left\{\frac{d}{dN_L[A]}\right\}, \qquad (58)$$

where N_L is the functional (51). This form is justified in refs.[4, 5] by representing the solution of (54)–(56) in the form of the Bloch wave function

$$\Psi_\epsilon(N_L, A^T) = e^{\frac{i}{\hbar} P \, N_L} \; \Psi_\epsilon(A^T)$$

and by the exact nonphysical solution with energy $\epsilon \; = \; 0$

$$\Psi_o = \exp\left\{\pm \frac{8\pi^2}{e^2} \, N_L[A]\right\}.$$

No-go theorem: **There are no physical solutions of equations** (54)–(56) .

The Proof: It is easy to check that the operators H_L, T_L do not commute

$$[H_L, T_L] \neq 0; \quad [[H_L[H_L, \, T]] \neq 0;$$

therefore they cannot have a complete system of physical eigenstates.

In the two- dimensional Abelian case , this local realization works due to the absence of a transversal variable. There is only a plane wave excitation. In the three- dimensional case , these transversal variables describe the oscillator- like local excitations in the Schrödinger equation

due to the magnetic field potential while (56) means that the wave function is simultaneously a plane wave, which is impossible.

2.3. Secondary Reduction

As we have seen in two-dimensional case, there is another way to get a nontrivial physical representation of the homotopy group (51). For this goal it is sufficient to introduce an independent collective topological variable N_0, which describes the Gribov ambiguity of the " motion equation gauge" (46)

$$\hat{A}_i = \hat{A}_i^{g_{N_0}} = g_{N_0}\left(\hat{A}_i^T + \partial_i\right) {g_{N_0}}^{-1}, \tag{59}$$

and its conjugate momentum

$$\hat{E}_i = g_{N_0}\left[\hat{E}_i^T + P_0 I_B {I_\Phi}^{-1}\nabla_i (A^T)\hat{\Phi}_0\right] {g_{N_0}}^{-1} \tag{60}$$

as a zero mode of the first-class secondary constraint (44). Here Φ_0 is the zero eigenfunction of the Gauss constraint (44)

$$\nabla_i^{ab}(A^T)\,\nabla_i^{bc}(A^T)\,{\Phi_0}^c = 0 \tag{61}$$

$$g_{N_0} = T\exp\left(\hat{\Phi}_0 N_0(t) {I_B}^{-1}\right) \tag{62}$$

I_B , I_Φ are the following surface integrals:

$$\begin{aligned} I_B &= \int d^3x(\nabla_i\Phi)^a \bar{B}_i^a \equiv \int d^3x\partial_i(\bar{B}^a\Phi^a);\ \ \bar{B} = \frac{e^2}{8\pi^2}B_i^a \\ I_\Phi &= \int d^3x(\nabla_i\Phi)^a(\nabla_i\Phi)^a = \frac{1}{2}\int d^3x\partial_i^2(\Phi^a)^2. \end{aligned} \tag{63}$$

Note that the new variables E^T and A^T satisfy the same constraints (44),(46) while the topological variable (50) and action (43) acquire additional terms

$$N_T[A^T, N_0] = N_L[A^{T g_{N_0}}] = N_L[A^T] + N_0 + \text{Inv.term} \tag{64}$$

$$W^{Red}[A,E] = W^{Red}[A^T, E^T] + \int\limits_{\frac{-T}{2}}^{\frac{T}{2}} dt\left[P\dot{N} - \frac{1}{2}P^2 I_\Phi {I_B}^{-2}\right] \tag{65}$$

Eq.(64) is defined within a term invariant under large gauge transformations; Eq.(65) is just the secondary reduction action. Let us consider the simplest case when the surface integrals (63) are time independent. We choose them as

$$I_\phi = \frac{2(2\pi)^3}{\mu e^2}, \quad I_B = 1 \tag{66}$$

with μ being the parameter of the mass dimension. Emphasize that this condition means a slow increase in the fields at spatial infinity. An example of fields like those is the well known Prasad- Somerfield solution [15] of the Bogomolny equation

$$\nabla_i^{ac}(A_{asympt}){\Phi_0}^c = \frac{2\pi}{\mu}B_i^a(A_{asympt}), \tag{67}$$

where

$$\begin{aligned} {A_{asympt}}_i^a &= \frac{1}{e}\epsilon^{abi}m^e\left[\frac{\mu}{\sinh(\mu r)} - \frac{1}{r}\right];\ \ m^l = \frac{x^l}{r};\ \ r = \mid \vec{x} \mid \\ (\Phi^a)_0 &= \frac{2\pi}{e}m^a\left[\mu\coth(\mu r) - \frac{1}{r}\right]. \end{aligned} \tag{68}$$

For these fields , the invariant term in Eq.(64) has the following form:

$$\text{Inv.term} = -\frac{\sin(2\pi N_0)}{2\pi}. \tag{69}$$

Thus , we get the non-Abelian analog of the Coleman electric field in the (1+1) QED.

2.4. The global representation of the homotopy group

From the reduced action (65) we get the following Hamiltonian:

$$H_{Red}[P, E, A^T, E^T] = \frac{1}{2I_\Phi}\hat{P}^2 + H_L[A^T, E^T], \qquad (70)$$

where H_L is defined by (57). In this case , the Schrödinger equation

$$H_{Red}\ \Psi_\epsilon = \epsilon\Psi_\epsilon \qquad (71)$$

admits the factorization of the wave function on the plane wave describing the topological collective motion, and oscillator like part depending on transversal variables

$$\Psi_\epsilon(N_0,\ A^T) = e^{\frac{i}{\hbar}PN_0}\ \Psi_L[A^T]. \qquad (72)$$

Thus , the representation of homotopy group is realized as

$$T_G\Psi_\epsilon = e^{i\theta}\Psi_\epsilon;\ \ T_G = \exp\left(\frac{i}{\hbar}\hat{P}\right) = \exp\left(\frac{d}{dN_0}\right). \qquad (73)$$

Recall that P has a discrete spectrum

$$P = (2\pi k + \theta)\ \hbar.$$

The oscillator like part of the wave function $\Psi_L[A^T]$ is described by the Hamiltonian H_L.

It is useful to separate the stationary asymptotic part of the transversal variable A_{asympt} and the quasiparticle excitations with the zero boundary conditions

$$\hat{A}^T(x_0, \vec{x}) = \hat{A}_{asympt}(\vec{x}) + \hat{a}^T(x_0; \vec{x}). \qquad (74)$$

In the "homogeneous" approximation, if we neglect quasiparticles $\hat{a}^T(x_0; \vec{x})$, we get from (65) the following effective action

$$W_{\text{eff}} = W^{Red}[A_{asympt}, E^T = 0] = \int\limits_{\frac{-T}{2}}^{\frac{T}{2}} dt \left[\frac{1}{2}\ P^2 I_\Phi I_B^{-2} - \int d^3x \hat{B}^2\right]. \qquad (75)$$

For the Prasad-Somerfield asymptotic field there are values of the coupling constant

$$\frac{e^2}{4\pi} = 1/(k + \frac{0}{2\pi}) \qquad (76)$$

for which the effective collective action (75) is equal to zero.

We want to emphasize the attractive peculiarities of the considered global realization, zero action and stability of perturbation theory under small deformations [16, 17]. This is just the main difference from instanton contributions witch are suppressed by the action factors.

Conclusion

We have discussed the mathematical and physical arguments in the favour of the introduction of the zero modes of the first-class secondary constraints in gauge theories.

It is shown that the reduced action approach allows us to take explicitly into account the zero modes including the Gribov ambiguity mode and clarifies their double role as independent variable or the winding number functional. To reproduce this result in the Dirac Hamiltonian approach , it is useful to consider the procedures of primary and secondary reductions. The primary reduction has been introduced by Dirac to conserve the uncertainty principle for gauge field components included

in the phase space (see discussion of eq.(2.28) in Dirac's Lectures [9]). This reduction is equivalent to the choice of a temporal gauge. The secondary reduction consists in the fixation of the remaining ambiguity due the presence of stationary gauge transformations, generated by the Gauss constraint. For this purpose , we explicitly solve the Gauss constraint and the additional gauge condition that does not contradict the equation of motion. At this step , to reproduce the result of the Lagrangian method , it is necessary to introduce the *zero mode of the first-class secondary constraint* together with the zero mode of the "motion equation gauge " (second- class constraint). These two modes are considered as canonically conjugate variables, the winding number and its momentum. The introduction of independent modes allows us to consistently describe the representation of the homotopy group and to construct the quantum theory with the effective finite action for the local excitation.

In the Yang-Mills theory the zero- mode dynamics is realized in the presence of a stationary condensate of the type of the Prasad-Sommerfield "bag" [7,15,18] . Perturbation theory around this condensate is stable unlike the conventional one. This situation is very similar to the theory of gravity ,where the metric excitation of the type of the Friedmann expansion leads to the stabilization of the Universe. This metric excitation is also the zero mode sector of secondary constraint in the theory of gravity [19].

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Нулевые моды вторичных связей первого рода
в калибровочных теориях

Методом редуцированного фазового пространства изучаются нулевые моды вторичных связей первого рода в двумерной электродинамике и теории Янга — Миллса в контексте проблемы стабильного вакуума. Сравнивается описание этих мод в дираковском гамильтоновом подходе и выявляется их связь с топологической структурой калибровочной группы. В рамках «редуцированного» квантования строится новая реализация группы гомотопии в неабелевой теории, где роль стабильного неабелевого вакуума с конечным действием выполняет решение Прасада — Зоммерфельда.

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Zero Modes of First Class Secondary Constraints
in Gauge Theories

Zero modes of first class secondary constraints in the two-dimensional electrodynamics and the four-dimensional SU(2) Yang-Mills theory are considered by the method of reduced phase space quantization in the context of the problem of a stable vacuum. We compare the description of these modes in the Dirac extended method and reveal their connection with the topological structure of the gauge symmetry group. Within the framework of the «reduced» quantization we construct a new global realization of the homotopy group representation in the Yang-Mills theory, where the role of the stable vacuum with a finite action plays the Prasad-Sommerfeld solution.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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