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BOHR'S QUANTIZATION RULE IN THE WORLD OF RESONANCES OF ELEMENTARY PARTICLES

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1. Introduction

In earlier papers [1-5] we outlined an approach to describe resonance spectra of hadronic resonances having strong 2-particle decay modes. This approach was surprisingly successful in reproducing the known resonances. The invariant masses of unknown hadronic resonances were also predicted. Moreover, it allowed us to get rather good estimates of widths for those resonance families (including dibaryons), where the interaction potential between their decay products is more-or-less well known.

This approach was based on some very general key points: (1) every hadronic resonance can be treated as a radiating system confined in the coordinate space within a region with characteristic size r_0=0.86 fm; (2) when this system has a non-negligible 2-particle decay mode, it can be considered as a corresponding binary particle system in final stage of its life; 3) for such a system the classical resonance (eigen-frequency) condition is valid for existence of eigenwaves in an open radiating resonator (antenna) with the effective size r_0:

$$Pr_0 = (n + \gamma), \tag{1}$$

Here P is the asymptotic momentum (i.e. the momentum measured in experiment) of decay products taken in the rest frame of the resonance, n is an integer positive number and $0 \leq \gamma \leq 1$ is a number which depends on the boundary conditions for a given degree of freedom and on the type of a dynamical equation for the resonating system. In refs.[1-5] the value of γ=1/2 was chosen according to the Heisenberg uncertainty relation in its minimal version$Pr_0 \geq 1/2$ and n=0, 1, 2,... or γ=0 and n=l=1, 2, 3,.... The latter case can be considered as the well-known Bohr-Sommerfeld orbital quantization

$$Pr_0 = l. \tag{2}$$

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Note that the wavelength $\lambda_1 = 2\pi/P$ corresponding to the first resonance ($n = 1$ for $\gamma = 0$ and $n = 0$ for $\gamma = 1/2$) must be of order of r_0 which can be seen from dimensional considerations (see for details ref. [2]), but on the other hand, in deriving the Bohr-Sommerfeld quantization rule for the bound states the wavelength λ_1 must be smaller than the characteristic size of the considered system, as is well known. In the papers [1–5], in order to calculate invariant resonance masses for given pairs of decay product (meson+meson, baryon+meson, baryon+baryon), a mass formula which incorporates above-mentioned key points was used with the same **fixed** value of the parameter r_0. The question is now, is this parameter really an universal one. We have all the time used the asymptotic values of the momenta in the resonance condition, neglecting possible interaction between the decay products. We suspect that it is due to considering "shape" resonances (which carry properties characteristic for the interplay between the effective size and wavelength of the system) that our consideration must be independent on a particular form of the interaction.

Thus the suggested mass formula should give almost the same mass of a resonance for every of its 2-particle decay modes (and exactly the same mass within a full coupled-channels treatment) without changing the parameter r_0, if the approach is self-consistent. It is worthwhile to note that multiparticle decays can be considered as a chain of binary decays: the 2-particle decay of a "primary" resonance into two clusters, further these clusters again decay into 2-particles and so on. This is consistent with the observation that multiparticle production processes proceed mainly through resonance production. Therefore the multiparticle decay can be treated as a tree-like phenomena where the intermediate resonances play an essential role. It indicates a way how to use the suggested approach in studies of multiparticle decays of resonances. Our approach was applied to selected resonances covering low and also high invariant masses (including bottomonium) as well with low decay momenta in the rest frame of the resonance; some unknown resonances and decay modes were predicted.

This approach predicts more resonances than are observed hitherto. The question is: which of the predicted resonances exist in nature? It is evident that some of them might be forbidden by selection rules. Therefore some criteria have to exist which limits the number of resonances. Some examples of our predictions and some comparison with recent data were given in refs. [1–5] in which very exciting correlations between the calculated results and experimental data were obtained. The parameter r_0=0.86 fm was in refs. [1–5] associated with the first Bohr orbital or with the confinement radius **which is nearly the same for all hadron and dibaryon resonances within the experimental accuracy.**

The quantization condition (1) of the asymptotic momenta for resonating system was obtained in the cited papers in a heuristic way. Here we would like to derive equation (1) from general quantum mechanical arguments starting from the well-known R (P) -matrix theory of the resonance reactions. The aim of this article is to get some common properties of the resonating system, having waves with the wavelength of order r_0 or $Pr_0 \approx l + 1/2$ well localized near its surface. This phenomenon is in a full analogy (in the correspondence principle sense) with the "whispering gallery" phenomenon in acoustics, which was first observed by Rayleigh [6] in 1910 year, with the open radiating resonators in classical electrodynamics[7], with the rainbow and glory effects also (ref. [16]). The same phenomenon was observed in the consideration of the

"stadium billiard" problem in classical mechanics (ref. [14, 15, 29]). It is interesting to mention, that in nuclear physics the significant non-uniformity of the distribution of single-particle energies (gross shells, properties of magic nuclei, non-sphericity of nuclei, gross structure of resonances in the optical model and etc.) is a result of semiclassical quantization of motion along many-dimensional closed orbits (see refs. [8, 9] for details and bibliography).

The question of what are underlying reasons for such surface localization is out of the scope of this paper; still it is worthwhile to note that it is effects of refraction of inner waves which are responsible for emergence of the localized surface-like waves in the examples mentioned above. Therefore we would like exploit the wave nature of particles at low energies when their Lui de Broglie wavelenghts are of the same order as the radius of strong interactions. Our general physical conception of resonances is as follows: it is the periodic motion and refraction of waves in the restricted region of space which are responsible for the creation of resonances in any resonating system.

2. Quantization of the asymptotic momenta of resonances

The asymptotic quantization condition (1) can be obtained by applying the R-matrix [10] or, equivalently, the P-matrix formalism to particle reactions [11–13]. According these papers, one can assume that the resonating system having several two-particle decay channels is free at relative separation $r \geq r_0$ in the center of mass, hence the logarithmic radial derivative of the internal wave functions can be introduced:

$$(\frac{r}{u_{in}}\frac{du_{in}}{dr})_{r=r_0} = f \equiv \frac{1}{R} \equiv P, \tag{3}$$

which should be calculated in the framework of some modern quark models.

For simplicity, let us consider only the systems with one dominating open channel. As was mentioned in the previous section, the decay of hadronic resonances can be considered in a full analogy with open classical electrodynamic resonators[7] and the mathematical formalism given in this excellent monograph can be used. Therefore the boundary conditions for the emitted waves must be written as follows (*the conditions of radiation*):

$$(\frac{r}{h_l^{(1)}(Pr)}\frac{dh_l^{(1)}(Pr)}{dr})_{r=r_0} = f, \tag{4}$$

where $h_l^{(1)}(Pr)$ are the spherical Rikkati-Hankel functions which are equal to $exp[i(Pr - l\pi/2)]$ at $Pr \gg 1$.

Rikkati-Hankel functions can be expressed via the Aery functions [28] at large positive τ values:

$$h_l^{(1)}(x) = -i(\frac{\tau}{sh^2\eta})^{\frac{1}{4}}\omega(\tau), \quad \frac{dh_l^{(1)}(x)}{dx} \equiv h_l^{(1)'}(x) = i(\frac{sh^2\eta}{\tau})^{\frac{1}{4}}\omega'(\tau), \tag{5}$$

where

$$\omega(\tau) = u(\tau) + iv(\tau), \quad \omega'(\tau) = u'(\tau) + iv'(\tau), \tag{6}$$
$$u(\tau) = \tau^{-\frac{1}{4}}e^{\frac{2}{3}\tau^{3/2}}, \quad u'(\tau) = \tau^{\frac{1}{4}}e^{\frac{2}{3}\tau^{3/2}},$$

$$v(\tau) = \frac{1}{2}\tau^{-\frac{1}{4}} e^{-\frac{2}{3}\tau^{3/2}}, \quad v'(\tau) = -\frac{1}{2}\tau^{\frac{1}{4}} e^{-\frac{2}{3}\tau^{3/2}}, \tag{7}$$

$$\frac{2}{3}\tau^{3/2} = (l+1/2)(\eta - th\eta), \quad ch\eta = \frac{l+1/2}{x}, \quad x = Pr_0. \tag{8}$$

Ignoring the imaginary part of the ω and ω', taking into account eq.(3) and demanding $X \geq 0$ one obtains:

$$\frac{h_l^{(1)'}(x)}{h_l^{(1)}(x)} = -sh\eta, \quad sh\eta = -\frac{f}{Pr_0} \equiv X, \tag{9}$$

Using eqs. (8) and (10), we find

$$Pr_0 = \frac{l+1/2}{\sqrt{1+X^2}}. \tag{10}$$

Finally, we note that X=0 for the well isolated resonances by definition [10]; hence eq.(10) becomes

$$Pr_0 = l + 1/2. \tag{11}$$

This is the resonance condition for asymptotic decay momentum P which is valid for large positive value of τ. Note that the quantization of asymptotic momenta is not typical for the standard quantum mechanics but is rather common in the physics of open resonators. The strong agreement between our calculated asymptotic decay momenta of particle resonances and experimental data mentioned above indicates an existence of a common base in physics of open classical electrodynamical resonators and particle resonances.

Considering the special case

$$Pr_0 \approx l + 1/2, \quad |\tau| \ll \nu^2, \tag{12}$$

one obtains with use of eq.(5):

$$h_l^{(1)}(x) = -i(\nu)^{\frac{1}{2}}\omega(\tau), \quad h_l^{(1)'}(x) = i(\nu)^{-\frac{1}{2}}\omega'(\tau), \tag{13}$$

where

$$Pr_0 = l + 1/2 - \nu\tau, \quad \nu = (\frac{l+1/2}{2})^{1/3}. \tag{14}$$

Using eqs.(3), (4) and (13), a characteristic equation for eigenvalue P can be obtained:

$$\omega'(\tau) - \nu X\omega(\tau) = 0. \tag{15}$$

From the asymptotic expressions (6) we have

$$\frac{\omega'(\tau)}{\omega(\tau)} = \sqrt{\tau}[1 - \frac{i}{2}e^{-\frac{4}{3}\tau^{3/2}}][1 + \frac{i}{2}e^{-\frac{4}{3}\tau^{3/2}}]^{-1} \approx \sqrt{\tau}[1 - ie^{-\frac{4}{3}\tau^{3/2}}]. \tag{16}$$

It follows from (15) and (16), that the solution of (15), in first approximation on $\nu X \equiv g$ at the condition $g > 0$, is equal to

$$\tau_0 = g^2, \quad \tau_0^{3/2} = g^3, \tag{17}$$

This solution can be taken as a basic one if $g \gg 1$, then in the second approximation

$$\tau_1 = g^2[1 + 2ie^{-\frac{4}{3}g^3}]. \tag{18}$$

The imaginary part of τ is exponentially small. That leads to the peculiar eigen-wave $(Pr > l + 1/2)$

$$exp\left(\frac{2}{3}[\frac{Pr - l - 1/2}{\nu}]^{3/2}\right) / (Pr - l - 1/2)^{1/4}, \tag{19}$$

localized on the surface at $r = r_0$; its eigen-frequency is:

$$Pr_0 = (l + 1/2)(1 - \frac{1}{2}X^2), \tag{20}$$

which occurs at conditions

$$|\tau_0| \ll \nu^2, X \ll 1. \tag{21}$$

Such surface waves localized at $r = r_0$ have exponentially small absorption in full analogy with the waves in the "whispering gallery". This phenomenon is very close to the phenomenon of the full refraction of the waves on the boundary separating two media with different refraction properties. Rainbow effects [16] and open resonators [7] can be considered as another examples of such kind. It means that nuclear and hadronic resonances have the same physical origin: emergence of well-localized surface waves with wavelengths of order r_0.

The method of R-matrix allows to estimate widths of resonances as well; it is known that (for details see [18]):

$$\Gamma = \frac{2\hbar^2}{Mr_0^2}Pr_0, \quad \text{or} \quad \Gamma = \frac{2\hbar^2}{2Mr_0^2} \tag{22}$$

for l=0 and $Pr_0 = 1/2$ according eq.(11); or

$$\Gamma = \frac{2\hbar^2}{Mr_0^2}Pr_0 v_l(Pr_0)\frac{2l-1}{2l+1}, \tag{23}$$

for $l > 0$ and $Pr_0 < l^{1/2}$. Here $v_l(Pr_0) = |h_l^{(1)}(Pr_0)|^{-2}$ is the "penetration factor". Strictly speaking, the condition (20) is inconsistent with $Pr_0 < l^{1/2}$, still (23) is in practice valid at list approximately. One can see that the widths of resonances having low decaying momenta depends only upon quantities r_0, P and M. Roughly speaking, $\Gamma \propto l$ and $\Gamma \propto M^{-1}$.

3. Resonances decaying with small momenta

Let us consider a hadronic resonance as a binary system and use the results of the previous sections. According to [1, 2], the invariant mass of the resonance at its peak can be written as follows:

$$m_n(R) = \sqrt{m_1^2 + P^2} + \sqrt{m_2^2 + P^2} + \Delta m_n = \sqrt{m_1^2 + (\frac{n+\gamma}{r_0})^2} + \sqrt{m_2^2 + (\frac{n+\gamma}{r_0})^2} + \Delta m_n, \tag{24}$$

where R labels the resonance, while the indices 1 and 2 refer to the constituents 1 and 2 observed in the 2-particle decay of the resonance $R \to 1+2$ respectively. As already mentioned, the "main" quantum number n is equal to 0, 1, 2... while γ is equal to 0 or 1/2, so $n+1/2$=0, 1/2, 1, 3/2, ... The parameter $r_0 = 0.86$ fm is fixed in all calculations presented below and in refs.[1-5] as well.

Formula (24) describes the gross structure of the resonance spectrum with reasonable accuracy because of the relation $\Delta m_n < \Gamma$ which is valid in all investigated cases of strong decays $R \to 1+2$. The leading term of the mass formula describes only the "center of gravity" position of the corresponding multiplets and thus the gross structure of the hadron and dibaryon resonances. The fine structure in each multiplet is determined by residual interactions and corresponding quantum numbers which are not contained in the approach [1, 2]. Therefore the condition $\Delta m_n < \Gamma$ is to be considered as an empirical fact.

Neglecting the last term in (24) and subtracting m_1+m_2, we obtain under the conditions $m_1^2 > [(n+\gamma)/r_0]^2$ and $m_2^2 > [(n+\gamma)/r_0]^2$ that the "excitation energy" E_n is

$$E_n(R) = \sqrt{m_1^2 + (\frac{n+\gamma}{r_0})^2} + \sqrt{m_2^2 + (\frac{n+\gamma}{r_0})^2} - m_1 - m_2 \approx \frac{1}{2m_{12}}(\frac{n+\gamma}{r_0})^2, \qquad (25)$$

where $m_{12} = m_1 m_2/(m_1+m_2)$. This expression is completely the same as the well known formula for the rotational energy of a diatomic molecule [19] in quasiclassical approach. Indeed, the quantity $m_{12}r_0^2$ plays a role of a moment of inertia of a molecule while $n+\gamma$ (if γ=1/2) is a quasiclassical analog of total angular momentum of the molecule. This gives us an extremely useful tool for interpretation of the spectra of heavy resonances. We will return to this point somewhere.

If $m_1^2 < [(n+\gamma)/r_0]^2$ and $m_2^2 < [(n+\gamma)/r_0]^2$, then

$$E_n(R) \approx 2\frac{n+\gamma}{r_0}, \qquad (26)$$

which is in full analogy with the formula for vibrational energy of nuclei within the molecule.

Thus, the Lorenz-invariant mass formula (24) obtained from the resonance condition using Heisenberg uncertainty relation, contains two limiting cases: 1) the rotational spectra and 2) the vibrational spectra.

It is well-known in nuclear physics that pure elementary states (say, rotational, vibrational etc.) are model concepts in nuclei, and are only approximately realized for the ground and low-lying parts of spectra in nuclei having large spectroscopic factors (branching ratios, see for details, ref.[18]). Such states played a decisive role in the development of modern nuclear physics. Similar situation could take place in particle physics.

Let us restrict ourselves by resonances with large branching ratios for decays in two-clusters and small values of the decay momenta P in the rest frame of the decaying resonances. These restrictions correspond to conditions of validity of the relations obtained in section 2. In calculating the invariant masses for clusters consisting of N physical particles, the formula (24) can be used in the following way: (i) the invariant mass for two particles in their lowest state is to be calculated; (ii) a third particle

mass is to be combined with the obtained value thus giving the invariant mass for the cluster (again in the lowest state for the three particles) and so on. Some results of such calculations are presented below. All masses and widths are given in MeV, momenta in MeV/c. When references to the experimental data are not quoted, they are taken from ref. [17]. We choose here only well-established (according to the compilation[17]) resonances except the case of dipion system.

Table 1

The invariant masses of ground state resonances ($n+\gamma$=1/2)

Decay modes	Fraction Γ_i/Γ	P(exp)	P(theor)	m(theor)	$n+\gamma$
$(\pi^+\pi^-)$, m=388±2 MeV, Γ=11±8 MeV [20]					
$\pi^+\pi^-$			115	361	1/2
$\eta(547)$ $I^G(J^{PC}) = 0^+(0^{-+})$, $\Gamma = 1.19$ keV					
$\pi^+\pi^-\pi^0$	23.6%		115	556	1/2
$3\pi^0$	38.9%		115	549	1/2
$\eta'(958)$ $I^G(J^{PC}) = 0^+(0^{-+})$, $\Gamma = 0.198$ MeV					
$\pi^+\pi^-\eta$	44.1%		115	938	1/2
$\pi^0\pi^0\eta$	20.6%		115	933	1/2
$\phi(1020)$ $I^G(J^{PC}) = 0^-(0^{--})$, $\Gamma = 4.43$ MeV					
K^+K^-	49.1%	127	115	1014	1/2
$K_L^0K_S^0$	34.4%	110	115	1007	1/2
$K_1(1270)$ $I^G(J^P) = \frac{1}{2}(1^+)$, $\Gamma = 90 \pm 20$ MeV					
$K\rho$	42%	71	115	1286	1/2
$D^*(2010)^\pm$ $I^G(J^P) = \frac{1}{2}(1^-)$, $\Gamma < 1.1$ MeV					
$D^0\pi^+$	55%	40	115	2049	1/2
$D^+\pi^0$	27%	39	115	2050	1/2
$D^*(2010)^0$ $I^G(J^P) = \frac{1}{2}(1^-)$, $\Gamma < 2.1$ MeV					
$D^0\pi^0$	55%	44	115	2045	1/2
$D_{s1}(2536)^\pm$ $I^G(J^P) = 0(1^+)$, $\Gamma < 4.6$ MeV					
$D^*(2010)^+K^0$	seen	153	115	2524	1/2
N(1440) P_{11} $I^G(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, $\Gamma = 350$ MeV					
$\Delta\pi$	20-30%	143	115	1418	1/2
N(1535) S_{11} $I^G(J^P) = \frac{1}{2}(\frac{1}{2}^-)$, $\Gamma = 150$ MeV					
Nη	30-50%	182	115	1505	1/2

N(1720) P_{13} $I^G(J^P) = \frac{1}{2}(\frac{3}{2}^+)$, $\Gamma = 150$ MeV

$N(1440)\rho$	25-75%	104	115	1724	1/2

$\Delta(1700)$ D_{33} $I^G(J^P) = \frac{1}{2}(\frac{3}{2}^-)$, $\Gamma = 300$ MeV

$N\rho$	30-50%		115	1724	1/2

$\Lambda(1116)$ $I^G(J^P) = 0(\frac{1}{2}^+)$, $\tau = 2.632 * 10^{-10}$s

$p\pi^-$	64.1%	102	115	1125	1/2
$n\pi^0$	35.7%	104	115	1123	1/2

$\Lambda(1405)$ S_{01} $I^G(J^P) = 0(\frac{1}{2}^-)$, $\Gamma = 50$MeV

$\Sigma\pi$	100%	152	115	1376	1/2

$\Lambda(1670)$ S_{01} $I^G(J^P) = 0(\frac{1}{2}^-)$, $\Gamma = 35$MeV

$\Lambda\eta$	15-35%	64	115	1681	1/2

$\Sigma(1750)$ S_{11} $I^G(J^P) = 1(\frac{1}{2}^-)$, $\Gamma = 90$MeV

$\Sigma\eta$	15-55%	81	115	1753	1/2

$\Xi(1321)$ $I^G(J^P) = \frac{1}{2}(\frac{1}{2}^+)$, $\tau = 1.639 * 10^{-10}$s

$\Lambda\pi$	100%	139	115	1302	1/2

$\Xi(1530)$ P_{13} $I^G(J^P) = \frac{1}{2}(\frac{3}{2}^+)$, $\Gamma = 10$ MeV

$\Xi\pi$	100%	152	115	1480	1/2

$\Sigma_c(2455)$ $I^G(J^P) = 1(\frac{1}{2}^+)$

$\Lambda_c^+\pi$	100	93	115	2468	1/2

One can see from table 1 that the calculations reproduce the experimental data (the invariant masses of resonances and decay momenta of two clusters in the rest frame of the resonance) with rather high accuracy which increases with increasing invariant mass of the resonance. This means that the suggested clustering effects and the shape resonance (in terminology of Landau and Lifshitz characteristic waves) seem to be adequate for the physical content of the particle resonances. It is worthwhile to note that the calculations reproduce the masses of the stable particles as well. We expect that the most stable resonances have minimal total angular momenta (which is equal to Pr_0=1/2) and zero value of the orbital part. The quantization of the angular momenta leads to the quantization of the Lorentz-invariant mass. The most stable state (independent of the nature of the state) of the rotating system is the ground state which has the minimal allowed angular momentum Pr_0=1/2, minimal invariant masses and also minimal decay momentum of the clusters. (Below such resonances will be referred as the ground state resonances to.) We would like to point out that one cuts out the low momenta of registered particles in modern high energy experiments. That

means that in such cases one looses signals from the very interesting low momentum resonances. As one can see from the table 1, the cut should be made at a rather small value of P (say, lower than 100 MeV/c).

We used the same value of the parameter r_0 (r_0=0.86 fm) for all resonances with remarkable success in description of the existing experimental data. This indicates on a fundamental role of the parameter r_0 in particle physics.

Here we described a well established resonances [17] having large values of the two-particle branching ratios. Our approach gives a method for calculating and prediction of invariant masses of resonating clusters, that may consist of N-particles of different physical nature, as we demonstrated above. Predictions for the resonance production of clusters slightly above threshold are important due to the scarf information about them.

Let us consider the dipion system. Our model predicts the ground state resonance for dipion system at 361 MeV. It is important to note that this meson cannot be accounted for by the quark model. The search of this meson gave no well-established results up to now, although there were several experiments in which some evidences or indications for this object were obtained. Nevertheless the common opinion is that its very existence cannot be considered to be finally established. Below some references on experimental data without discussions are given. Dipion resonance was claimed in ref. [20] at $m(\pi\pi) = 388 \pm 2$ MeV with the width $\Gamma = 11 \pm 8$ MeV, in ref. [21] at 371 MeV. The narrow resonance structure with width of about 5-7 MeV in the excitation function of pion production by protons at proton energies near 350 MeV has been reported on cooper target in different laboratories during last years (see for references and discussions paper [22]). This resonance decays mainly by emitting two pions.

The way which was chosen to present the decay channels for the decaying resonances [17] indicates a physical language for the explanation of the decomposition over resonating clusters of different type. Therefore heavy resonating clusters again consist of resonating clusters of smaller invariant mass and so on. This multicluster nature of resonances must display itself in a resonance production at lowest invariant mass (nearly above threshold) for different clusters. Indeed such type significant enhancement of events has been observed [23-26] in the mass spectrum from $\Upsilon(3S) \to \pi\pi\Upsilon(1S)$ (the invariant dipion mass is equal to 340-440 MeV).

It is interesting to note that the invariant masses of resonances decaying into three particles (say, $\eta \to \pi^+\pi^-\pi^0$ and $\eta' \to \pi^+\pi^-\eta$, see table 1) were calculated in the following way. First, we calculate the dipion invariant mass in the ground state ($m(\pi^+\pi^-)$=361 MeV) according to the formula (24) than we exploit again this formula using $m(\pi^+\pi^-)$=361 MeV and mass of third particle. The agreement observed between the calculated results and experimental data indicates the way of experimental research of the dipion (or any pair particles) invariant mass near the resonance threshold. This can be done for example, using the followings reactions: $\eta \to \pi^+\pi^-\pi^0$ or $\eta' \to \pi^+\pi^-\eta$ etc.

4. Bohr orbital quantization with l=1

In the previous section we discussed the "radial quantization" of resonances. It is well-known in the nuclear physics, that the phenomena of pure states is very seldom.

Nevertheless one can speak about dominant channels having large spectroscopic factors (see for details ref. [18]) and these channels play a prominent role in description of investigated states. Let us restrict ourselves with resonances which we can interpret as candidates for Bohr's orbital quantization with l=1.

Table 2

The invariant masses of resonances with l=1

Decay modes	Fraction $\frac{\Gamma_i}{\Gamma}$	P(exp)	P(theor)	m(theor)	$n+\gamma$
$f_1(1285)$ $I^G(J^{PC}) = 0^+(1^{++})$, $\Gamma = 24 \pm 3$ MeV					
$a_0(980)\pi$	37%	233	229	1275	1
$f_0(1590)$ $I^G(J^{PC}) = 0^+(0^{++})$, $\Gamma = 175 \pm 19$ MeV					
$\eta\eta'(958)$	dominant	241	229	1578	1
$\psi(3770)$ $I^G(J^{PC}) = ?^?(1^{--})$, $\Gamma = 23.6$ MeV					
$D\overline{D}$	dominant	242	229	3766	1
$\psi(4040)$ $I^G(J^{PC}) = ?^?(1^{--})$, $\Gamma = 52 \pm 10$ MeV					
$D^0\overline{D}^0$	seen	777	803	4059	3+1/2
$D^*(2010)^0\overline{D}^0$	seen	577	574	4040	2+1/2
$D^*(2010)^0\overline{D}(2010)^0$	seen	228	229	4040	1
$\Delta(1232)$ P_{33} $I^G(J^P) = \frac{3}{2}(\frac{3}{2}^+)$, $\Gamma = 120$ MeV					
$N\pi$	99.3-99.5%	227	229	1234	1
$\Lambda(1520)$ D_{03} $I^G(J^P) = 0(\frac{3}{2}^-)$, $\Gamma = 15.6$MeV					
$N\overline{K}$	45%	244	229	1510	1
$\Sigma(1385)$ P_{13} $I^G(J^P) = 1(\frac{3}{2}^+)$, $\Gamma = 40$MeV					
$\Sigma\pi$	12%	127	115	1375	1/2
$\Lambda\pi$	88%	208	229	1407	1
$\Omega^-(1672)$ $I^G(J^P) = 0(\frac{3}{2}^+)$, $\tau = 0.822 * 10^{-10}$s					
ΛK^-	67.8%	211	229	1683	1
$\Xi(1690)$ $I^G(J^P) = \frac{1}{2}(?^?)$, $\Gamma < 50$MeV					
$\Lambda\overline{K}$	seen	240	229	1683	1

One can see from table 2 that calculations made for this section describe the experimental data much better then in previous section. Strikingly high accuracy was obtained for the Δ-isobar and $\psi(3770)$, $\psi(4040)$ mesons in particular.

We can calculate widths in the framework of proposed approach. However such calculations can put only upper limit of the resonance width corresponding to the so called "single particle" limit which ignores interactions between different channels, phase space etc. The $\Delta(1232)$-isobar is a good candidate for the "single particle" resonance, so our calculation gives Γ=126 MeV while its experimental value is equal to 120 MeV. We calculated also the P_{33} πp phase shift; the agreement with the experimental data[31] is surprisingly good (see Fig.1).

In the framework of our approach the quark degrees of freedom do not appears explicitly in the mass formula (24). This fact can be easily explained, because inverse R-matrix vanishes at the resonance by definition. We are in need of quark models to calculate the spectroscopic factors, reduced width, selection rules etc. for correct evaluation of resonances widths and Δm_n in the mass formula (24).

5. Diproton and proton-antiproton resonances

It was shown in the previous sections that the same quantization rule and the same mass formula can be used for resonances decaying as via strong as through weak interactions (the description of invariant masses has about the same order of accuracy in all cases: see Tables 1 and 2; for example $\Xi(1690)$ and Ω^- in Table 2). Other examples can be found in ref. [30]; the narrow enhancements have been observed at almost the same masses (3060 ± 5 (stat.) ± 20(syst.) MeV) in the invariant mass spectra of the different final states with different strangeness: ($\Lambda\overline{p}\pi^\pm$, $\Lambda\overline{p}\pi\pi^\pm$) and ($\overline{\Lambda}p\pi^\pm$, $\overline{\Lambda}p\pi\pi^\pm$). The invariant masses of resonances in our model are independent on strangeness. Therefore, the systems $\overline{\Lambda}p\pi^\pm$, $\Lambda\overline{p}\pi^\pm$ and $\Lambda p\pi^\pm$ must have almost the same invariant masses. The predicted mass for these systems in our model is equal to 3030 MeV which is close to the one reported in ref.[30]: 3060 ± 5(stat.) ± 20(syst.) MeV.

Let us to return to the systems (pp) and ($p\overline{p}$) which must have approximately the same masses according to above-mentioned arguments. Of course, their widths can be different because they are determined by the number of opened channels and by the particular form of the interaction potential between the resonance decay products[1-4].

The present status of the diproton resonances was discussed by Yu.Troyan in his review paper[32] where he analysed experimental evidences on existence of 15 resonance diproton states in the region of invariant masses up to 2300 MeV. Other details about the diproton problem one can find also, in refs.[20,33-35] and reviews [32, 36, 37]. The situation is rather controversial; here we use the experimental data coming from Dubna collaboration [20, 33]. The peculiar property of the diproton resonances is their small widths ($\Gamma \leq$10 MeV) which stimulates rather exotic explanations beyond the traditional picture.

Below we present results of our calculations for diproton resonances[4] and recent experimental data[33]. One can see very exciting correlations between the calculated results and experimental data [33]; our estimations[4] of their width are made within rather traditional approach based on the analogy with the α-decay of atomic nuclei. It is interesting to note that the diproton mass spectrum have a "rational-like" behaviour.

It is worthwhile to mention here, that results of rather large number of experiments can be interpreted as indications on broad dibaryon resonances at masses $\sqrt{s} \approx$ 2.4, 2.7 and 2.9 GeV (see for details review [44]).

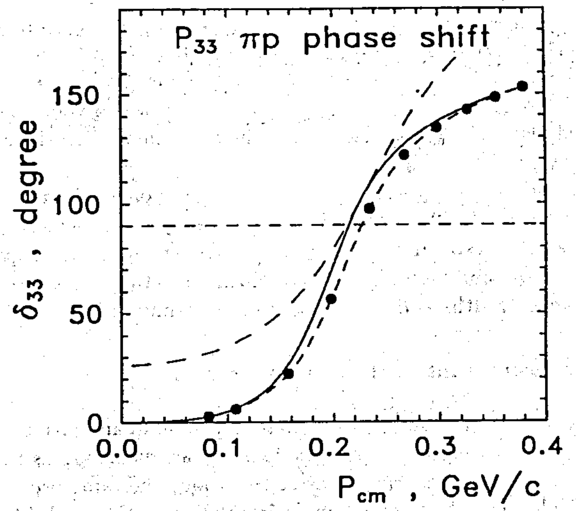

Fig.1 πp scattering P_{33} phase shift calculated in our approach. Solid line: the phase shift is calculated with centrifugal barrier taken into account[45]; lond dashes - calculation without centrifugal barrier. Full circles connected with short dashes - data taken from ref.[31].

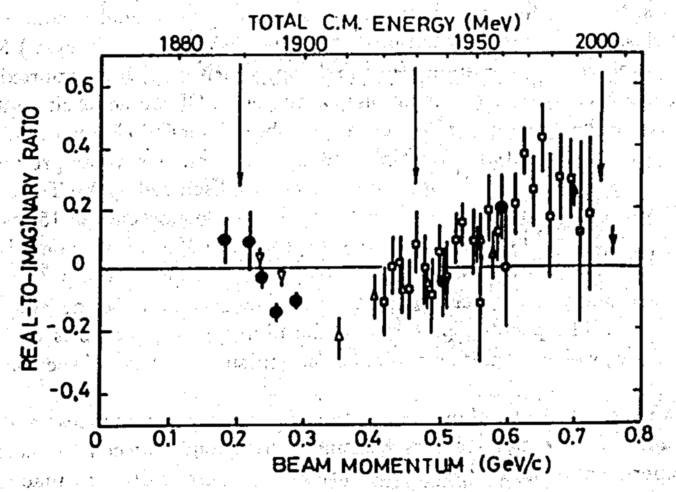

Fig.2 The real-to-imaginary ratio, ρ, of the forward elastic $p\bar{p}$ scattering as a function of the beam momentum. Experimental data are taken from compilation [41], Vertical arrows show our predictions of the invariant masses for $(p\bar{p})$ system.

Table 3.
Spectrum of the invariant masses and widths for the diproton resonances. Experimental data are taken from ref.[33]

	$n+\gamma$	1/2	1	3/2	2	5/2	3	7/2	4	9/2	5
m	theory	1890	1932	1998	2088	2198	2326	2468	2623	2788	2961
	exp	1886	1937	1999	2087	2172					
Γ	theory	4	9	12	17	22					
	exp	4±1	5±2	5±4	4^{+7}_{-4}	0+3					

The history of narrow $p\bar{p}$ resonances is dramatic (for details see review[38]). Shapiro in his review article[40] pointed on 6 possible resonances with small widths ($\Gamma \leq 20$ MeV). We would like to remark that the narrow resonance ($\Gamma \approx 10$ MeV) with mass 1936 MeV (what is equal to the diproton resonance mass 1937 in ref.[33]) was reported in ref.[39] . This result was confirmed in a number of papers but further this resonance was not observed in the $p\bar{p}$ cross section experiments. Nevertheless we think that the question of existence of narrow resonances in the $p\bar{p}$ system is not closed completely and requires further investigations.

In connection with the question of narrow $p\bar{p}$ resonances we think that an interesting information can be extracted from data on the real part of the forward elastic $p\bar{p}$ scattering amplitude. It is well-known that zeroes of ρ (the ratio of real to imaginary parts of the forward elastic scattering amplitude) are correlated, under some conditions, with resonances in the corresponding system. The most famous example is the Δ resonance in the πN scattering. Experimental data on the ρ for $p\bar{p}$ elastic scattering amplitude are plotted in Fig.2 taken from the ref.[41]. There are at least two (perhaps three) zeroes in the momentum interval $0 < P_L < 0.8$ MeV/c or in the total c.m. energy interval $1800 < \sqrt{s} < 2020$ MeV. One can see that our predictions of the invariant masses for $(p\bar{p})$ system and experimental data [33] for (pp) system are correlated with these zeroes of the ρ for $(p\bar{p})$ system. At the same time the data[41] for S, P and D phase shifts have very large error bars and do not display the resonance-like behaviour. It is worthwhile to mention here, that authors of ref. [42] predicted a rather wide ($\Gamma \approx 46$ MeV) resonance in S-wave at invariant mass 1942 MeV. This prediction was argued from the unusual behaviour of $\rho(P_L)$. Therefore we can conclude that is necessary to carry out new more precise experiments on measurements of ρ and σ_{tot} in the interval of laboratory momenta of $0 < P_L < 0.8$ MeV/c.

More or less well established broad $p\bar{p}$ resonances were discussed in review[38]. In 1970 several broad $p\bar{p}$ resonances have been observed in ref.[43]: two resonances in the I=1 channel with masses 2190 (Γ=85 MeV) and 2350 MeV (Γ=140 MeV), one resonance in the I=0 channel with mass 2375 MeV (Γ=190 MeV). Similar structures were observed later by different groups (for references see review[38]). Our approach predicts resonances at masses 2198 and 2326 MeV; this should be compared with structure at 2172 MeV in the data of ref.[33] for pp system. Therefore one can see exciting correlations between experimental data for the (pp) and $(p\bar{p})$ systems and with the theoretical results for this region of invariant masses. It would be a crucial test of our approach, which predicts a similarity of gross structure in the invariant mass distributions in (pp) and ($p\bar{p}$) systems or analogous *particle-particle and particle-antiparticle systems.*

6. Conclusion

The quantization of the asymptotic values of momenta is carried out for the elementary particle resonances using R-matrix or equivalently P-matrix formalism of resonance reactions. The asymptotic momenta are convenient for the comparison of the theoretical results with experimental data because one measures the asymptotic values of momenta in resonance decay. Physical origin of hadronic resonances is due to the emergence of the well localized surface-like waves with wavelength of order of the strong interaction radius. This localization results from refraction effects of inner waves on the boundary of nuclear matter. This is the common property of the chaotic motion, where some states have regions of high amplitude, called "scars", near certain classical periodic orbits (see review paper [29] and references therein).

The Balmer-like mass formula obtained from the first principles in refs. [1-5] was applied for systematic analysis of gross structure of all known hadronic resonances starting from dipion and ending with charmed hadronic resonances. The accuracy of the mass formula is surprisingly high and unusual for this branch of physics. It means that equation (24) could be useful at least for prediction and estimation of the invariant masses of unknown resonances. This observation requires further systematic investigations. We can only say that the correspondence principle between old classical and quantum theories played an outstanding role in the interpretation of the results, and this "correspondence" allows one to go even into fine details.

The upper "single-particle" limit of the resonance widths can be estimated in the suggested approach. More accurate calculations have to be done taking into account the quark degrees of freedom of resonances say within the R or P-matrix formalism. The $\Delta(1232)$-isobar is a good candidate for the "single particle" resonance; in this case we obtained rather good agreement of the calculated width and the phase shift with the experimental data.

The parameter r_0 in the mass formula (24) is the same (at least we used the same value) for all resonances considered here and in refs. [1-5] and plays the role of an elementary "size" for the resonating radiating system. This parameter r_0 and the corresponding minimal decay momentum of the resonances, determined by the minimal allowed angular momentum $Pr_0=1/2$, determine the minimal allowed Lorenz-invariant mass of the resonances. Further the resonance condition gives the quantization of the mass for resonating system.

In the approach presented here, the problem of diproton resonances is strongly correlated with the problem of resonances in the $p\bar{p}$ system: their masses must be almost the same while widths can be rather different. We think that experimental confirmation of such correlation would be decisive for both of these problems.

Our model predict the similarity of the gross structure in the invariant mass distribution in any pair of a particle-particle and particle-antiparticle system; it seems that this similarity does not contradict with the existing experimental data.

Finally, all arguments given in this paper and refs. [1-5] bring us to the conclusion that the gross structure and also fine details of the resonance spectra can be understood in full analogy with the modern nuclear structure concepts.

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Квантование по Бору в мире
резонансов элементарных частиц

На основе общих квантово-механических соображений проведено квантование для асимптотических величин импульсов продуктов распада адронных резонансов. Используя полученное условие квантования, удалось найти массовую формулу для адронных резонансов. Полученные спектры имеют структуру, напоминающую структуру серии Бальмера. Проводится сравнение вычисленных спектров с экспериментальными данными; найденная массовая формула может быть применена для предсказаний новых возможных резонансов и для их поиска.

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Bohr's Quantization Rule in the World of Resonances
of Elementary Particles

Quantization based on general quantum mechanical arguments is carried out for asymptotic values of momenta of decay products of hadronic resonances. Mass formula for hadronic resonances is obtained with making use of the above-mentioned quantization conditions. Calculated spectra having a structure similar to that of the Balmer series are compared with the experimental data; the corresponding mass formula can be used for predictions of new possible resonances and their searches.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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