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A.B.Pestov*

PHYSICAL MEANING OF CONFINEMENT

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*E-mail address: pestov@theor.jinrc.dubna.su

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1 Introduction

Despite prolonged and complicated experiments, free quarks have not been observed though it is commonly accepted that quarks are true elementary particles like electrons. Experimentalists gradually came to the conclusion that the matter is not in the details of the experiments but rather in the fundamental properties of the matter search for which was made under many different assumptions. For instance, it is hypothesized that the quark confinement can be explained by topological methods that have recently found still more wide use in physics. Nevertheless, the most natural and reliable way to the problem of confinement should be looked for in the classical mechanics. The very deep reason for this is the well-known connection between the Hamiltonian formalism of classical mechanics and the mathematical formalism of quantum mechanics [1],[2]. As it has turned out, application to classical mechanics may really produce far-reaching consequences that are just the subject of our consideration.

2 Formulation of the Problem

The connection between geometry and physics in classical mechanics is realized by abstraction when a geometrical point, namely, a point of the Euclidean three-dimensional space, is made to correspond to a material point.In the classical relativistic mechanics this correspondence becomes still more important as it turns out that one cannot ascribe finite sizes to elementary particles [3]. It is just the electron that represents an extremely exact physical realization of this mathematical scheme. At present there are no experimental indications that would contradict the notion of electron being a true point-like particle. Thus, the top in special theory of relativity cannot be considered as a fundamental concept as this rotating object has finite sizes ad hoc. However, experiments on scattering of electrons by protons testify to the existence of point-like particles localized in a finite region of space. Comparing all these facts we may assume a possible connection between quarks and tops. The idea is to associate a geometrical point, an element of a certain space, with the notion of a top.If this problem has a solution, then it becomes possible to develop a general mathematical formalism like in the case of material points. As the above association cannot be made within the Minkowski geometry, we should consider other possible geometries. So, we should find such a space-time whose geometry is adequate to the notion of a top. This problem is essentially formulated under the assumption that the notion of top is as fundamental as the notion of a material point is.

As the top is finite in size, it is clear intuitively that quark-tops, being fundamental particles, are to be related to a fundamental constant with the dimension of length. Other suggestions and connections can be found in the Minkowski geometry, quantum mechanics, and the Lie group theory. Consider a conventional quantum-mechanical operator of the 4-momentum with components

$$P_0 = -i\hbar\frac{\partial}{\partial x^0}, \quad P_1 = -i\hbar\frac{\partial}{\partial x^1}, \quad P_2 = -i\hbar\frac{\partial}{\partial x^2}, \quad P_3 = -i\hbar\frac{\partial}{\partial x^3}, \quad (1)$$

where $x^0 = ct$, $x^1 = x$, $x^2 = y$, $x^3 = z$, and the group of translations of space-time. The group of translations is a finite continuous Lie group with the generators

$$X_0 = \frac{\partial}{\partial x^0}, \quad X_1 = \frac{\partial}{\partial x^1}, \quad X_2 = \frac{\partial}{\partial x^2}, \quad X_3 = \frac{\partial}{\partial x^3}. \quad (2)$$

Besides, the linear operators (2) are generators of a simply transitive group of transformations [4]. In our case it is an Abelian simply transitive group of transformations because

$$[X_a, X_b] = 0, \quad a, b = 0, 1, 2, 3.$$

The connection between(1) and (2) is obvious. If a simply transitive group is a group of transformations of a certain space, this space is called homogeneous, as this group can transform any point of the space to any a priori given point. To make further analysis more transparent, we present the general characteristic of homogeneous spaces.

As any vector field with components V^i can be associated with the linear operator $X = V^i\partial/\partial x^i$, the operator X is called the vector field. Then, let the vector fields

$$X_a = V^i_a\frac{\partial}{\partial x^i}, \quad a = 0, 1, 2, 3 \quad (3)$$

are generators of a simply transitive group of transformations acting on a four-dimensional space-time manifold M. In this case

$$[X_a, X_b] = f^c_{ab}X_c, \quad (4)$$

where f^c_{ab} are structure constants of the group. The vector fields V^i_a uniquely determine the system of covector fields V^a_i such that

$$V^i_a V^a_j = \delta^i_j, \quad V^a_i V^i_b = \delta^a_b. \quad (5)$$

The simply transitive group induces a natural integrable connection Γ on M with Christoffel symbols

$$\Gamma^i_{jk} = V^i_a \partial_j V^a_k \quad (6)$$

and a natural metrics of the Lorentz signature on M

$$g_{ij} = \eta_{ab}V^a_i V^b_j, \quad g^{ij} = \eta^{ab}V^i_a V^j_b, \quad (7)$$

where $\eta_{ab} = \eta^{ab} = \mathrm{diag}(1, -1, -1, -1)$. From (4) and (6) for the torsion tensor and torsion covector of the connection (6) we obtain

$$T^i_{jk} = \Gamma^i_{jk} - \Gamma^i_{kj} = -f^a_{bc}V^i_a V^b_j V^c_k, \quad T_i = T^k_{ki} = -f^a_{ab}V^b_i.$$

and from (4) and (7) we have

$$V^i_a V^j_{b;i} = (f^c_{ab} - \eta_{ad}f^d_{bc}\eta^{ce} - \eta_{bd}f^d_{ae}\eta^{ce})V^j_c, \quad (8)$$

where semicolon means the covariant derivative with respect to the Levi-Civita connection of the metrics (7) with the Christoffel symbols

$$\{^{\,i}_{jk}\} = \frac{1}{2}g^{il}(\partial_j g_{kl} + \partial_k g_{jl} - \partial_l g_{jk})$$

Knowing generators of the simply transitive group X_a we can find generators Y_a of the mutual simply transitive group by solving the equation $\nabla_i V^j - T^j_{ik}V^k = 0$, where ∇ is the covariant derivative with respect to the connection (6). For X_a, Y_a we have $[X_a, Y_b] = 0$, $a, b = 0, 1, 2, 3$.

The manifold M that admits a simply transitive group of transformations and has the metrics (7) will be called the homogeneous space-time. The Minkowski space-time is a particular case of homogeneous space-time manifolds. If a homogeneous space-time defined by a non-Abelian simply transitive group of transformations has a physical meaning, from (1) and (2) it follows that the operator

$$H = -i\hbar X_0 = -i\hbar V^i_0\frac{\partial}{\partial x^i}$$

will represent a new energy operator.

3 Hyperbolic Space-Time

Here we will define a homogeneous space-time that differs from the Minkowski space-time by geometrical and topological properties and show that a space- time manifold of that kind obeys all the required conditions and is of interest for the problem of confinement and its physical interpretation.

In the five-dimensional Minkowski space-time $M^5_{1,4}$ with Cartesian coordinates x^A (indices denoted by capital letters run over five values $0,1,2,3,4$) and metrics

$$ds^2 = \eta_{AB} dx^A dx^B = (dx^0)^2 - (dx^1)^2 - (dx^2)^2 - (dx^3)^2 - (dx^4)^2 \quad (9)$$

we will consider the one sheet hyperboloid H^4

$$\eta_{AB} x^A x^B = (x^0)^2 - (x^1)^2 - (x^2)^2 - (x^3)^2 - (x^4)^2 = -a^2, \quad (10)$$

where a is the radius of H^4, and prove that it is a homogeneous space-time.

We will use the scalar product $(X,Y) = \eta_{AB} U^A V^B$ for any vector fields $X = U^A \partial_A$ and $Y = V^A \partial_A$ on $M^5_{1,4}$. The vector fields

$$P_A = \delta^C_A \partial_C, \quad M_{AB} = (x_A \delta^C_B - x_B \delta^C_A)\partial_C,$$

where $x_A = \eta_{AB} x^B$, are generators of the Poincare group of the five-dimensional Minkowski space-time. All vectors fields M_{AB} are orthogonal to the radius-vector $R = x^C \partial_C$, whereas for vector fields P_A this is not the case. Expanding P_A in the direction of the radius-vector R and the one orthogonal to it, we obtain the vector fields

$$M_A = aP_A + \frac{1}{a}(R, P_A)R = (a\delta^C_A + \frac{1}{a} x_A x^C)\partial_C,$$

tangent to H^4, since from (10) it follows that $(R, M_A) = 0$ at every point of H^4. The vector fields M_A and M_{AB} are generators of the group of conformal transformations of H^4 because

$$[M_A, M_B] = -M_{AB}, \quad [M_A, M_{BC}] = \eta_{AB} M_C - \eta_{AC} M_B. \quad (11)$$

Let us now introduce the vector fields

$$X_0 = M_0, \quad X_1 = M_{14} + M_{23}, \quad X_2 = M_{24} + M_{31}, \quad X_3 = M_{34} + M_{12} \quad (12)$$

with components

$$\begin{aligned} X_0 &= (a + \tfrac{1}{a}x_0^2, \ \tfrac{1}{a}x_0 x^1, \ \tfrac{1}{a}x_0 x^2, \ \tfrac{1}{a}x_0 x^3, \ \tfrac{1}{a}x_0 x^4), \\ X_1 &= (0, \ -x_4, \ -x_3, \ x_2, \ x_1), \\ X_2 &= (0, \ x_3, \ -x_4, \ -x_1, \ x_2), \\ X_3 &= (0, \ -x_2, \ x_1, \ -x_4, \ x_3), \end{aligned}$$

It is not difficult to see that the vector fields $X_0, \ X_1, \ X_2, \ X_3$ are continuous and do not vanish at any point of H^4. As $(X_a, X_b) = 0$ for $a \neq b, \ a, b = 0,1,2,3$ and

$$(X_0, X_0) = -(X_1, X_1) = -(X_2, X_2) = -(X_3, X_3) = a^2 + x_0^2,$$

the vector fields $X_0, \ X_1, \ X_2, \ X_3$ are linearly independent at every point of H^4. From (11) it follows that

$$[X_0, X_i] = 0, \quad [X_i, X_j] = 2e_{ijk} X_k, \quad i,j,k = 1,2,3,$$

where e_{ijk} is the completly antisymmetric Levi-Civita symbol with $e_{123} = 1$. In this way, we have proved that the one sheet hyperboloid (10) admits a simply transitive group of transformations with the generators (12) having only the following nonzero structure constants

$$f^1_{23} = f^2_{31} = f^3_{12} = 2. \quad (13)$$

Therefore, we will supply H^4 with a metrics of the type (7) and thus transform H^4 into the hyperbolic space-time $H^4_{1,3}$. From (8) and (13) it follows that the vector field X_0 is absolutely parallel with respect to the Levi-Civita connection on $H^4_{1,3}$ induced by the vector fields (12). For comparison we note that the vector field $X_0 = \partial/\partial x^0$ defined in (2) is also absolutely parallel.

4 Dirac and Maxwell Equations in Hyperbolic Space-Time

We will here show that in the homogeneous space-time $H^4_{1,3}$ a geometrical point of the spatial cross section can be associated with a notion of top. From (10) it follows that the cross section of H^4 by the hyperplane $x^0 = 0$ is a three-dimensional sphere

$$(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = a^2 \quad (14)$$

in the four-dimensional Euclidean space. Let $R = (x^1, x^2, x^3, x^4)$ be the radius-vector of a point from S^3; then from (14) it follows that the velocity vector $v = \dot{R}$ is orthogonal to R, $(v, R) = x^1\dot{x}^1 + x^2\dot{x}^2 + x^3\dot{x}^3 + x^4\dot{x}^4 = 0$. Let us write the Newton equation, in homogeneous coordinates, describing the dynamics of a material point on the three-dimensional sphere (14)

$$m(\ddot{R} + \frac{1}{a^2}v^2 R) = F, \tag{15}$$

where $v^2 = (\dot{x}^1)^2 + (\dot{x}^2)^2 + (\dot{x}^3)^2 + (\dot{x}^4)^2$ and $(R, F) = 0$.. Parametrizing the sphere (14) with the Euler angles and using the expressions for components of the angular velocity in terms of the Euler angles [5], we can show that equation (15) is equivalent to the Euler equations for a spherical top [5] whose moment of inertia I is

$$I = ma^2.$$

Thus, it is shown that the notion of a top can be associated with the notion of a material point in the space of constant positive curvature.

In this connection, it is of interest to analyse the Dirac and Maxwell equations in the hyperbolic space-time. In accordance with (1)-(4) and (12) we write the Dirac equation in the homogeneous space-time in the form

$$\gamma^c D_c \psi = \mu\psi, \tag{16}$$

where

$$\gamma^a\gamma^b + \gamma^b\gamma^a = -2\eta^{ab},$$

$$D_c = X_c + \frac{iqa}{\hbar c}A_c - \frac{1}{2}f_c, \quad f_c = f^a_{ac}$$

q are the charge of a quark -top and A_c are components of the vector potential of the electromagnetic field. For the present we do not concretize the values of structure constants of a simply transitive group of transformations of the space-time $H^4_{1,3}$. As $[X_a, X_b] = f^c_{ab}X_c$, we have

$$[D_a, D_b] = f^c_{ab}D_c + \frac{iqa}{\hbar c}F_{ab},$$

where

$$F_{ab} = X_a A_b - X_b A_a - f^c_{ab}A_c \tag{17}$$

are components of the strength tensor of the electromagnetic field in the basis X_a. The Jacobi identity $[D_a[D_b, D_c]] + [D_b[D_c, D_a]] + [D_c[D_a, D_b]] = 0$ results in the first four Maxwell equations

$$X_a F_{bc} + X_b F_{ca} + X_c F_{ab} + f^d_{ab}F_{cd} + f^d_{bc}F_{ad} + f^d_{ca}F_{bd} = 0 \tag{18}$$

To establish the form of other four Maxwell equations, we set $\tilde{F}^{ab} = \frac{1}{2}e^{abcd}F_{cd}$, where e^{abcd} are components of the antisymmetric Levi-Civita unit tensor in the basis X_a. Then we can write equations (18) in the following equivalent form

$$X_a\tilde{F}^{ab} + f_a\tilde{F}^{ab} + \frac{1}{2}f^b_{ad}\tilde{F}^{ad} = 0 \tag{19}$$

By analogy, from (18) and (19) it follows that the remaining Maxwell equations are of the form

$$X_a F^{ab} + f_a F^{ab} + \frac{1}{2}f^b_{ad}F^{ad} = \frac{4\pi a}{c}j^b, \tag{20}$$

where j^b are components of the current vector in the basis X_a.

Now we will write the Maxwell equations in the three-dimensional vector form. As usual, we put

$$j^a = (c\rho, \vec{\mathrm{j}}), \quad A_a = (\varphi, \quad -\vec{\mathrm{A}}),$$

$$E_i = F_{0i}, \quad H_i = \frac{1}{2}e_{ijk}F^{jk}, \quad i, j, k = 1, 2, 3.$$

Then from (13) and (17) we obtain

$$\vec{\mathrm{E}} = -\nabla_0\vec{\mathrm{A}} - \nabla\varphi, \quad \vec{\mathrm{H}} = \mathrm{rot}\vec{\mathrm{A}} = \nabla \times \vec{\mathrm{A}} - 2\vec{\mathrm{A}}. \tag{21}$$

where

$$\nabla = (\nabla_1, \quad \nabla_2, \quad \nabla_3), \quad \nabla_0 = X_0, \quad \nabla_i = X_i, \quad i = 1, 2, 3.$$

Considering that $\mathrm{div}\vec{A} = \sum_{i=1}^{3}\nabla_i A_i$, we can write the Maxwell equations (18) and (20) in the familiar vector form

$$-\nabla_0\vec{\mathrm{H}} = \mathrm{rot}\vec{\mathrm{E}}, \quad \mathrm{div}\vec{\mathrm{H}} = 0, \quad \mathrm{rot}\vec{\mathrm{H}} = \nabla_0\vec{\mathrm{E}} + \frac{4\pi\mathrm{a}}{\mathrm{c}}\vec{\mathrm{j}}, \quad \mathrm{div}\vec{\mathrm{E}} = 4\pi\mathrm{a}\rho. \tag{22}$$

Making use of the commutation relations $[\nabla_i, \nabla_j] = 2e_{ijk}\nabla_k$, $i, j, k = 1, 2, 3$, it is not difficult to verify the identities

$$\text{divrot} = 0, \quad \text{rotgrad} = 0.$$

Besides,

$$\text{divgrad} = \Delta,$$

where Δ is the Laplacian on a three-dimensional sphere. Torsion or non- Abelian character of a simply transitive group of transformations manifests itself not only in the definition of the operator rot, (21), but also in the identity

$$(\text{rot} + 1)^2 = -\Delta + 1 + \text{graddiv}.$$

In accordance with the interpretation given above for the Newton equation (15) one might expect that the Dirac equation (16) describes the quantum- mechanical behavior of particles having essential properties of a top. To verify this, we will derive eigenvalues E of the Dirac Hamiltonian when there is no electromagnetic field, i.e. $F_{ab} = 0$. Quadrating equation (16) and using (13), we obtain the following equation for E

$$E^2\psi = m^2c^4\psi - \frac{c^2\hbar^2}{a^2}(\Delta + P)\psi, \tag{23}$$

where

$$P = \Sigma_1\nabla_1 + \Sigma_2\nabla_2 + \Sigma_3\nabla_3$$

and $\Sigma_i = \frac{1}{2}e_{ijk}\gamma^j\gamma^k$. The operator P has properties analogous to those of the operator rot. In particular,

$$(P + 1)^2 = -\Delta + 1 \tag{24}$$

To determine eigenvalues of the operator P, consider Hermitean operators acting in the space of solutions to the Dirac equation (16). Generators of the group mutual to the simply transitive group of transformations of the space-time $H^4_{1,3}$ are of the form

$$Y_0 = X_0, \quad Y_1 = M_{14} - M_{23}, \quad Y_2 = M_{24} - M_{31}, \quad Y_3 = M_{34} - M_{12},$$

which gives the three Hermitean operators

$$N_i = -\frac{i}{2}Y_i$$

analogous to the momentum operators. The other three operators

$$M_i = -\frac{i}{2}(\nabla_i - \Sigma_i) = -\frac{i}{2}(X_i - \Sigma_i) \tag{25}$$

are analogs of the operators angular momentum of an electron. From (25) it follows that the spin of a quark-top equals $\hbar/2$. We have

$$\vec{\mathrm{M}} \times \vec{\mathrm{M}} = i\vec{\mathrm{M}}, \quad \vec{\mathrm{N}} \times \vec{\mathrm{N}} = i\vec{\mathrm{N}}$$

and, besides,

$$2(M^2 + N^2) = -\Delta + (P + \frac{3}{2}), \quad 2(M^2 - N^2) = P + \frac{3}{2}. \tag{26}$$

As the eigenvalues of the operators M^2 and N^2 are known, from (23) - (26) we can derive the formula for energy levels

$$E^2 = m^2c^4 + n^2\frac{c^2\hbar^2}{a^2}, \tag{27}$$

where n are integers. From physical considerations it is clear that at large n and $a \gg \lambda$, where $\lambda = \hbar/mc$, the quantum top in its properties will approach the classical top. Indeed, in the limit of large a it follows from (27) that

$$E = mc^2 + \frac{L^2}{2I},$$

where $L = n\hbar$ is the angular momentum of the top and I is its moment of inertia. The latter relation is consistent with the classical formula

$$E = \frac{L^2}{2I}$$

for the energy of a top.

Now consider the Coulomb law for quark-tops. As it is known, the Coulomb potential for electrons can be derived as a solution of the equations of electrostatics invariant under the group of Euclidean motions including rotations and translations. We will look for the Coulomb potential for quarks in an analogous manner. From (21) and (22) it follows that for a constant electric field $\text{div}\vec{\mathrm{E}} = 4\pi a\rho$, $\vec{\mathrm{E}} = -\nabla\varphi$, and consequently, φ obeys the equation

$$\Delta\varphi = -4\pi a^2\rho. \tag{28}$$

The invariant of the group of rotations $O(4)$ on a three-dimensional sphere is either the arc length or the angle between radius-vectors,

$$X = (x^1, x^2, x^3, x^4), \quad Y = (y^1, y^2, y^3, y^4)$$

$$\cos\theta = \frac{1}{a^2}(x^1y^1 + x^2y^2 + x^3y^3 + x^4y^4).$$

Since

$$M_{ij}\cos\theta = (x^i\frac{\partial}{\partial x^j} - x^j\frac{\partial}{\partial x^i})\cos\theta = \frac{1}{a^2}(x^iy^j - x^jy^i),$$

setting in (24) $\rho = 0$, $\varphi = \varphi(z)$, where $z = \cos\theta$, we obtain the following equation for $\varphi(z)$

$$(1-z^2)\frac{d^2\varphi}{dz^2} - 3z\frac{d\varphi}{dz} = 0.$$

The general solution to this equation is of the form

$$\varphi(z) = c_1\frac{z}{\sqrt{1-z^2}} + c_2 = c_1\cot\theta + c_2,$$

where c_1 and c_2 are arbitrary constants.

Introduce the frame of reference with respect to which one of the charged quark-tops is at rest and has the coordinates $(0, 0, 0, -a)$. In this system consider a stereographic projection of the three-dimensional sphere $(x^1)^2 + (x^2)^2 + (x^3)^2 + (x^4)^2 = a^2$ from point $(0, 0, 0, a)$ onto the hyperplane $x^4 = 0$ with Cartesian coordinates x, y, z. We have

$$x^1 = x\frac{2a^2}{r^2+a^2}, \quad x^2 = y\frac{2a^2}{r^2+a^2},$$

$$x^3 = z\frac{2a^2}{r^2+a^2}, \quad x^4 = a\frac{r^2-a^2}{r^2+a^2},$$

where $r^2 = x^2 + y^2 + z^2$. It may be verified that in the coordinates x, y, z

$$\cot\theta = \frac{a}{2r} - \frac{r}{2a}$$

and, consequently, the Coulomb potential of quark -tops can be written in the form

$$\varphi(r) = q \quad (\frac{1}{r} - \frac{r}{a^2} - \frac{1}{a}), \qquad (29)$$

where q is the quark-top charge. As the potential (29) does coincide with the known Cornel potential [6],, one can, on the basis of first principles, explain a successful application of the latter for describing the charmonium [7].

5 Conclusion

1. As the basic wave equation describing the dynamics of quarks, we have suggested the modified Dirac equation (16) written here in homogeneous coordinates. The conclusion that quarks are described by the wave equation different from the conventional wave equation for electrons is quite natural. In fact, it would be strange if the description of so different particles were based on the same equation.

2. The physical meaning of the phenomenon called the confinement consists in that quarks possess properties of a top. This means, in particular, that the Cornel potential expresses the fundamental physical law.

3. As the kinematics of quark-tops differs from the kinematics of electrons, there are possible such decays of hadrons and nuclei in which the energy conservation law is fulfilled but the momentum is not conserved.

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Пестов А.Б. E2-93-425

Физический смысл конфайнмента

Развивается представление о кварках как о волчках, то есть частицах, которые кроме массы, заряда, спина характеризуются также моментом инерции. Установлено, что геометрические и теоретико-групповые методы позволяют непротиворечивым образом ввести взаимодействующие элементарные частицы такого рода. Для описания динамики кварков в качестве основного волнового уравнения предложено уравнение, которое отличается от уравнения Дирака для электронов. Изменение уравнения Дирака естественным образом приводит к изменению уравнений Максвелла. Показано, что корнельский потенциал выражает закон Кулона для кварков-волчков.

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Physical Meaning of Confinement

A notion is developed for quarks as tops, i.e. particles characterized not only by the mass, charge, spin but also by the moment of inertia. It is established that geometrical and field-theoretical methods allow a consistent introduction of interacting particles of that type. The main wave equation for describing the dynamics of quark-tops is proposed to be different from the Dirac equation for electrons. The change of the Dirac equation naturally leads to the change of Maxwell equations. It is shown that the Cornell potential expresses the Coulomb law for quark-tops.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

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