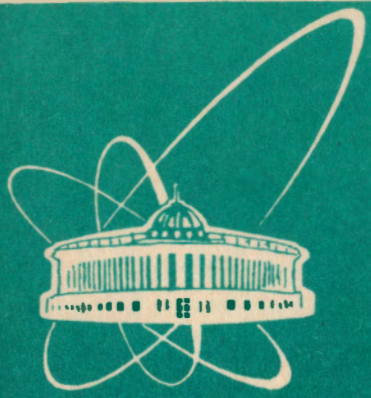

объединенный
институт
ядерных
исследований
дубна

E2-93-41

V.V.Burov, O.V.Deripaska[1], V.Y.Kamchatnova[1],
H.G.Miller[2], M.V.Rzjanin[3], K.V.Shitikova[1], G.D.Yen[2]

A MICROSCOPIC DESCRIPTION OF THE NEUTRON-RICH LITHIUM ISOTOPES

Submitted to «Journal of Physics G: Nuclear and Particle Physics»

[1]Institute of Nuclear Physics, Moscow State University,
Moscow, Russia
[2]Department of Physics, University of Pretoria,
Pretoria 0002, Republic of South Africa
[3]Russian Academy of Sciences, Science and Technical Centre
of Inspection and Employ of Physical Fields and Radiation,
Saratov 410030, Russia

1993

Exotic nuclei close to the neutron drip line are difficult to describe microscopically. The small binding energies and extended radial density distributions of neutron-rich nuclei such as the lithium isotopes, which are produced in radioactive beams [1, 2, 3, 4], are not correctly reproduced in either Hartree-Fock or shell model calculations [5, 6]. For this reason simple Gaussian parameterizations of the density distributions [6] have been used in order to describe the large experimentally observed reaction cross sections [1, 2, 3]. In the present work a fully microscopic calculation of the lithium isotopes has been performed in a basis of hyperspherical functions in which the symmetries have been properly taken into account [7, 8]. In this basis a better description of the asymptotic part of the wave functions is possible.

Unlike some previous theoretical attempts [9, 10] we shall attempt to provide a unified description of ^{6,7,8,9,11}Li rather than that of just a single isotope. No attempt will be made to parameterize the effective interaction used for each isotope, rather a simple parameterization for all the isotopes has been used. Furthermore in our treatment there is no inert core [9, 10] and all of the nucleons are properly antisymmetrized. Lastly because we make use of Jacobi coordinates no problems are encountered with the treatment of the center of mass [11].

In order to provide a unified description of all of the lithium isotopes we use the following group theoretical treatment. In Table 1 the spin and isospin of each of the isotopes are given. From knowledge of the total isospin

Table 1:

^{A}Li	$[f]$	J^π	L	S	T
6	[42]	1^+	0	1	0
7	[43]	$3/2^-$	1	1/2	1/2
8	[431]	2^+	2	0	1
9	[432]	$3/2^-$	1	1/2	3/2
11	[4322]	$3/2^-$	1	1/2	5/2

the symmetry of each isotope is determined. As can be seen from the table the corresponding Young diagram $[f]$ for each isotope exhibits a simple structure. In the same manner in which ^{9}Li is constructed from ^{7}Li plus two neutrons ^{11}Li is constructed from ^{9}Li plus two neutrons.

In the hyperspherical basis the wave function of nucleus A is expanded in terms of K-harmonic polynomials $|AK[f]\varepsilon LST\rangle$ in the following manner [7, 8]

$$\psi(1,2,3,\ldots,A) = \rho^{-\frac{1}{2}(3A-4)} \sum_{K\gamma} \chi_{K\gamma}(\rho)|AK\gamma\rangle\,, \tag{1}$$

where ρ is the hyperradius expressed in terms of Jacobi coordinates, $\gamma = [f]\varepsilon LST$ and ε represents the additional quantum numbers necessary to describe the state. In the hyperspherical basis the hyperradius is a collective variable which is related to the root-mean-square (rms) radius of the nucleus, $\rho^2 = A\, r^2_{\rm rms}$, i.e., to the mean nuclear density. Excitations of this degree of freedom correspond to the monopole oscillation of the nucleus as a whole. The density is therefore a dynamical variable.

The Schrödinger equation for the radial wave functions can be written as [7, 8]

$$\left\{\frac{d^2}{d\rho^2} - \frac{\mathcal{L}_K(\mathcal{L}_K+1)}{\rho^2} - \frac{2m}{\hbar^2}\left[E + W^{K\gamma}_{K\gamma}(\rho)\right]\right\}\chi_{K\gamma}(\rho) =$$
$$= \frac{2m}{\hbar^2} \sum_{K'\gamma'\neq K\gamma} W^{K'\gamma'}_{K\gamma}(\rho)\,\chi_{K'\gamma'}(\rho)\ , \tag{2}$$

where $\mathcal{L}_K = K + \frac{1}{2}(3A-6)$, and $W^{K'\gamma'}_{K\gamma}(\rho)$ are the matrix elements of the nucleon-nucleon interaction

$$V = \sum_{i<j}^{A} V(r_{ij}), \qquad V(r_{ij}) = f(r_{ij})W_{\sigma\rho} \tag{3}$$

which are expressed in terms of fractional parentage coefficients in the following manner

$$W^{K'\gamma'}_{K\gamma}(\rho) = \langle AK[f]\varepsilon LSTM_L M_S M_T|V|AK'[f]\varepsilon' L'S'T'M_{L'}M_{S'}M_{T'}\rangle =$$

$$= \frac{A(A-1)}{2} \sum_{K_2 f_2 \varepsilon_2 L_2} \langle AK[f]\varepsilon LST|(A-2)K_2[f_2]\varepsilon_2 L_2 S_2 T_2, \Lambda(L^1K^1); L_0S_0T_0\rangle$$

$$\times\langle AK'[f]\varepsilon' L'S'T'|(A-2)K_2[f_2]\varepsilon_2 L_2 S_2 T_2, \Lambda(L^1K^1); L_0S_0T_0\rangle$$
$$\times\langle S_0T_0|W_{\sigma\rho}|S_0T_0\rangle R^{KK'}_{K^1L_0}. \qquad (4)$$

Here

$$R^{KK'}_{K^1L_0} = \int d\theta_1 (\sin\theta_1)^{3A-7}(\cos\theta_1)^2 \mathcal{N}_{KK^1L_0}\mathcal{N}_{K'k^1L_0}$$
$$\times f(\rho\cos\theta_1)(\sin\theta_1)^{2K^1}(\cos\theta_1)^{2L_0} P^{K^1+\frac{1}{2}(3A-6)-1,L_0+\frac{1}{2}}_{K-K^1-L_0}\cos(2\theta_1)$$
$$\times P^{K^1+\frac{1}{2}(3A-6)-1,L_0+\frac{1}{2}}_{K'-K^1-L_0}\cos(2\theta_1), \qquad (5)$$

where $\mathcal{N}$ is a normalization constant and $P^L_{L'}(x)$ are Jacobi polynomials [7, 8].

It has already been pointed out [8] that the shape of the effective potential ($W^{K\gamma}_{K\gamma}(\rho)$ + centrifugal term) becomes broader for the higher-lying energy states. Nuclear properties such as the radial density distribution and the rms radius are thus functions of the excitation energy of the nucleus. The increase in size of the excited states takes place therefore automatically for the hyperspherical basis.

In the present work we adopted the $K_{\min}$ approximation in which all values of K greater than $K_{\min} = A-4$ are neglected. The success of this approximation arises from the fact that the centrifugal barrier reduces the contributions of configurations with K greater than $K_{\min}$ in the equations determining the hyperspherical wave functions [12]. We have also renormalized the Brink & Boeker B1 (BB1) potential [13] to the ground state energies and rms radii of ^{6}Li and ^{7}Li, and we call this renormalized potential RB1. The potential is parameterized with a sum of two Gaussians:

$$V(r) = \sum_{i=1}^{2} S_i[1 - m_i(1 - P_M)]\exp(-r^2/\mu_i^2), \qquad (6)$$

Table 2:

Name	S_1 [MeV]	m_1	μ_1 [fm]	S_2 [MeV]	m_2	μ_2 [fm]
BB1	-140.6	0.4864	1.4	389.5	-0.529	0.70
RB1	-120.0	0.4864	1.7	324.2	-0.529	0.85

where P_M is the Majorana exchange operator. In Table 2 we show the parameters used in BB1 and RB1 potentials. In order to fit the binding energies and rms radii of ^{6,7}Li, it was necessary to decrease the strength of the BB1 potential and increase its range. We obtain a good agreement with the experimental binding energies of ^{6,7}Li (see Fig. 1) and reproduce the experimental rms radii to within the experimental errors [14]. We have calculated the binding energies of the ground states, excitation energies of monopole excited states, rms radii, compressibilities and the radial density distributions of ^{6,7,8,9,11}Li with RB1 potential. In Fig. 1 we compare our RB1 results for the ground state binding energies with the experimental [14, 15] and BB1 results. Note that we qualitatively reproduce the variation in the binding energy as the neutron number increases. The discrepancies between our results and the experimental binding energies are similar on the average to those obtained in a large-scale shell model calculation [11]. Most importantly, the decrease in the slope of the binding

Table 3:

^{A}Li	$-E_0$ MeV	$r_{\rm rms}$ fm	$r_{rms}^{Bertsch}$ fm	r_{rms}^{exp} fm	$E_{\rm BM}$ MeV	$\mathcal{K}$ MeV
6	30.1	2.41	2.118	2.57(10)	17.5	42.9
7	38.0	2.45	2.139	2.41(10)	18.1	47.4
8	36.1	2.57	2.157		16.4	42.8
9	42.4	2.63	2.132		16.4	44.9
11	48.1	2.76	2.249		15.7	45.3

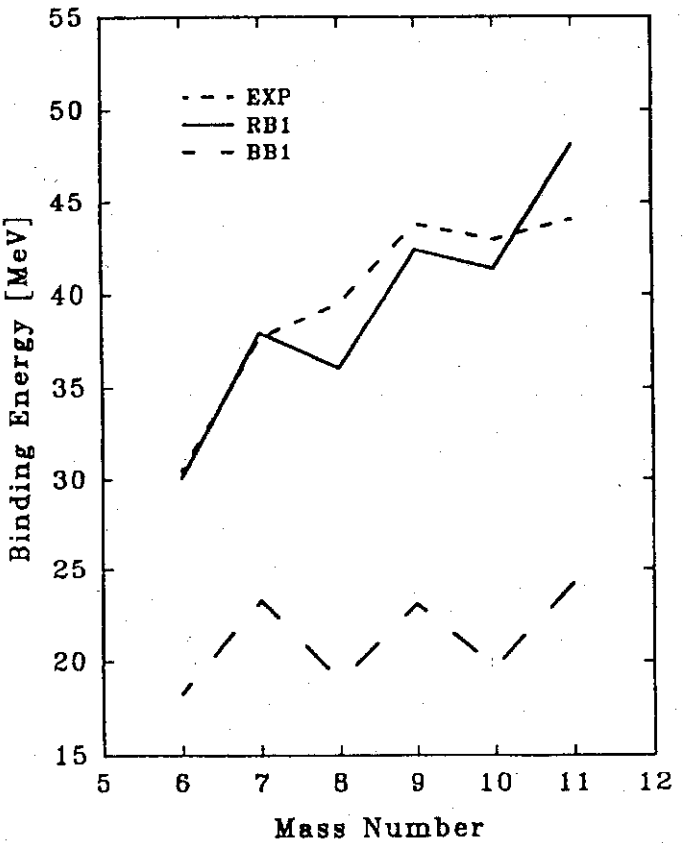

Figure 1: The comparison of the experimental binding energies (EXP) of ^{6,7,8,9,11}Li with those calculated using BB1 and RB1 potentials.

energy from $A = 7$ to 8 and from $A = 9$ is well represented in our results. In Table 3 we list our results of the ground state energies, rms radii (our, Bertsch [6] , and experimental), monopole excitation energies and compressibilities for all ^{6,7,8,9,11}Li isotopes with RB1 potential. The compressibility proposed by the hydrodynamical model is given as [16, 17, 18, 8]

$$\mathcal{K} = \frac{m}{\hbar^2} r_{\rm rms}^2 E_{\rm BM}^2, \tag{7}$$

where m is the nucleon mass, and $E_{\rm BM}$ is the monopole excitation energy.

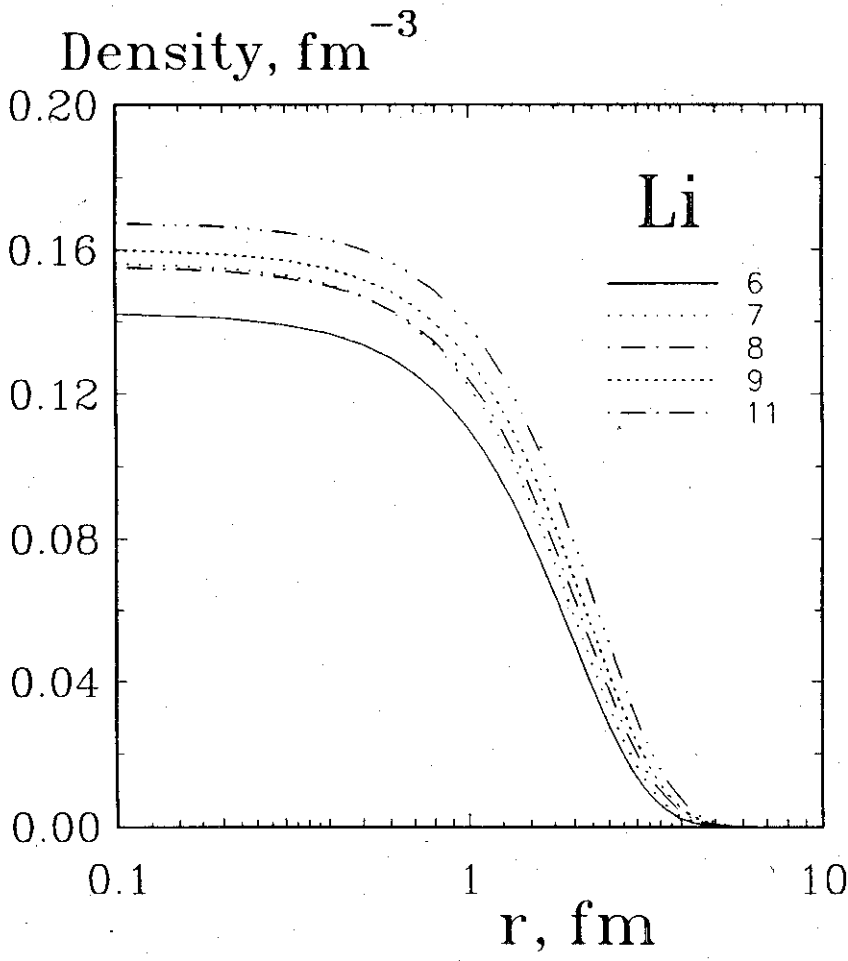

Figure 2: Radial density of Li

In Fig. 2 the radial density distributions of the isotopes of lithium are given. Increasing the neutron number leads to a more extended radial density distribution. Furthermore, in the hyperspherical basis we obtain an exponentially decreasing asymptotic behavior of the radial density distributions which differs from the results obtained in previous microscopic calculations in an oscillator basis [6].

With this radial density distributions of the isotopes of lithium we have calculated the form factors (see also ref.[23]). The expression for the elastic and inelastic form factors in the high energy approximation [19] has the form:

$$F_{ij} = 2\pi i q \sum_{\epsilon=\pm 1} \int_0^\infty \frac{G_{ij}(x,\epsilon)}{\tilde{q}^2(x,\epsilon)} exp\{i[qx\epsilon + \Phi(x,\epsilon)]\} n_{ij}(x) x \; dx, \qquad (8)$$

where the functions G, $\tilde{q}, \Phi$ take into account the distortion of electron wave with the Coulomb field of nucleus (see [19]). In the Born approximation $G = 1$, $\Phi = 0$, $\tilde{q} = q$. The formula (8) is correct for $qR \gg 1$, $V(0)/E \ll 1$, $E^* < E$, where $V(0)$ - the Coulomb potential in the centre of nucleus, E^* - loss of the energy of the electron. The cross section in this case takes the form:

$$\left(\frac{d\sigma}{d\Omega}\right)_{ij} = \left(\frac{d\sigma}{d\Omega}\right)_{Mott} f_{rec} \frac{2J_j + 1}{2J_i + 1} |F_{ij}|^2, \qquad (9)$$

where

$$\left(\frac{d\sigma}{d\Omega}\right)_{Mott} = \left(\frac{Ze}{2E}\right)^2 \frac{\cos^2 \frac{\Theta}{2}}{\sin^4 \frac{\Theta}{2}} \qquad (10)$$

- Mott cross section in the point nucleus Z,

$$f_{rec} = \left(1 + \frac{2E}{M} \sin^2 \frac{\Theta}{2}\right)^{-1} \qquad (11)$$

- the factor which takes into account the back movement of the nucleus; M - mass of nucleus; $J_{i,j}$ - spin of the initial (final) state of the nucleus.

In Fig.3 are given the calculation results of the form factors with the density distributions for ^{6}Li in comparison with the experimental results and the calculation results with symmetrized fermi-function [22]. It is seen that the hyperspherical approach gives a description of the form factor with the RB1 nucleon-nucleon potential at small transfer momenta. In Fig.4 are shown the theoretical form factors of the Li neutron-rich isotopes in the hyperspherical functions method. For comparison in Fig.5 are given form factors obtained with the density distributions [20]:

$$\rho(r) = \rho(0) exp\left(-\frac{r^2}{a^2}\right). \qquad (12)$$

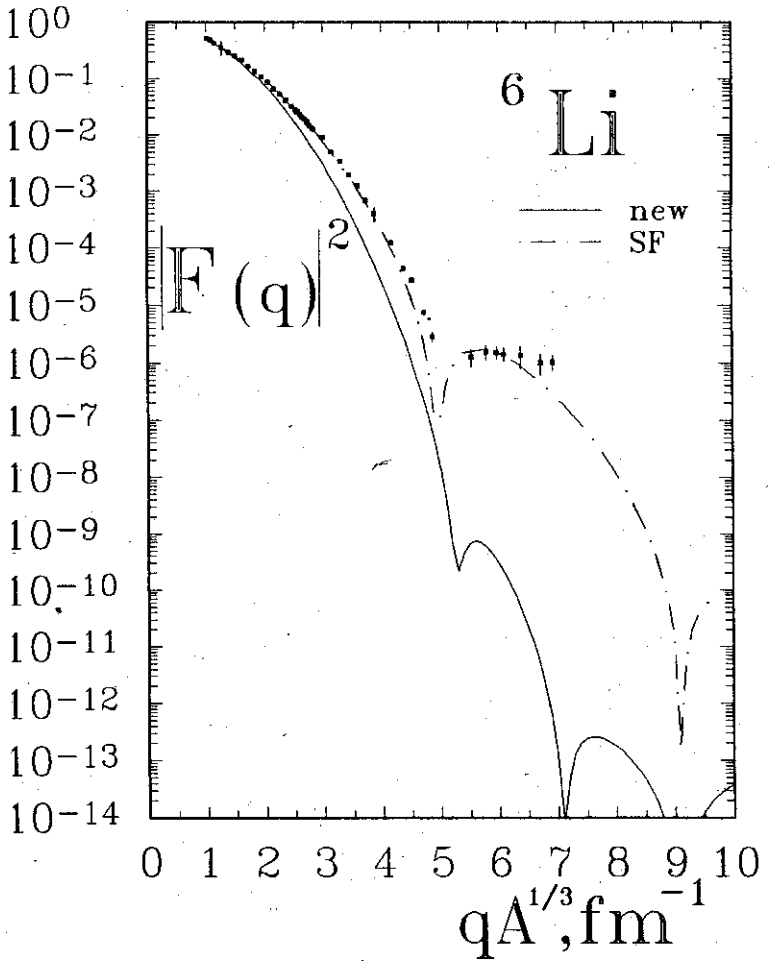

Figure 3: Form factors of ^{6}Li: new - this is our result, SF - symmetrized fermi-function

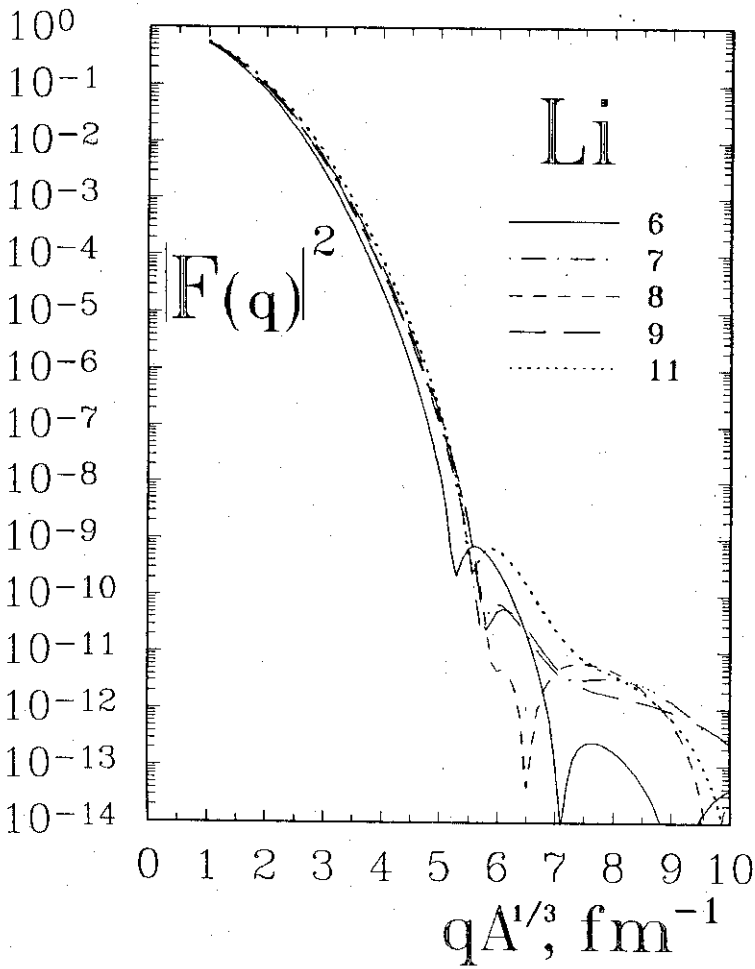

Figure 4: Form factors of Li

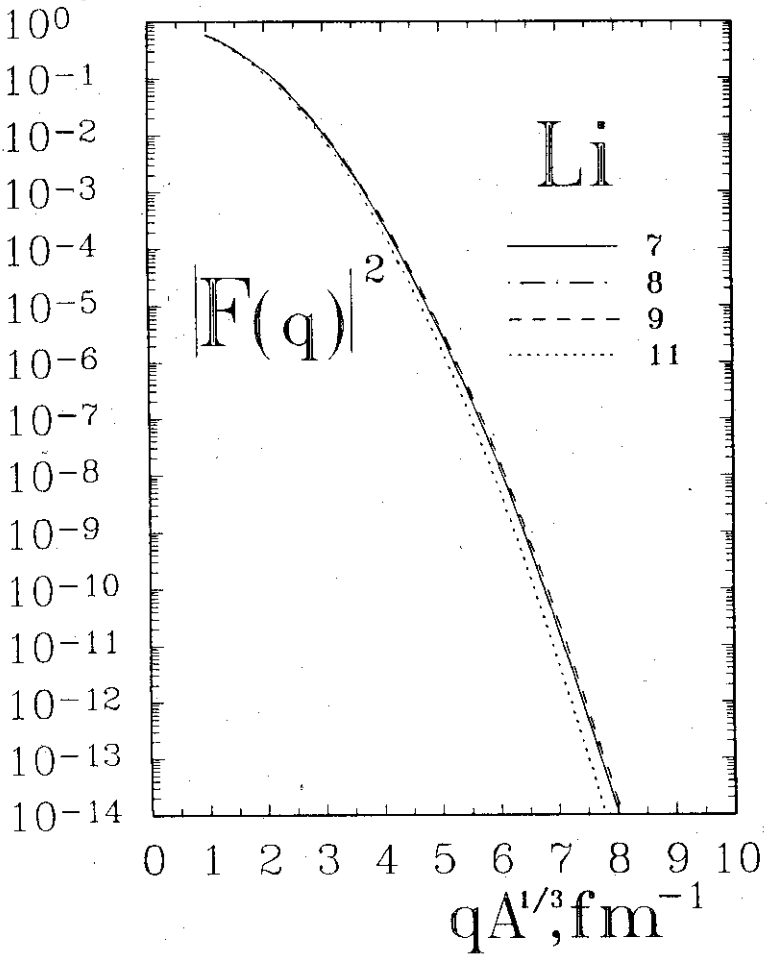

Figure 5: Form factors for density (12)

Table 4:

^{A}Li	a [fm]
7	1.797
8	1.885
9	1.952
11	2.175

Parameters of density distributions are represented in Table 4. For qualitative comparison in Fig.6 are given the calculation results of the $C0$ form factor with the density distributions for ^{7}Li in comparison with the new experimental results [21] (C0+C2) and the calculation results with density distribution [20] (C0).

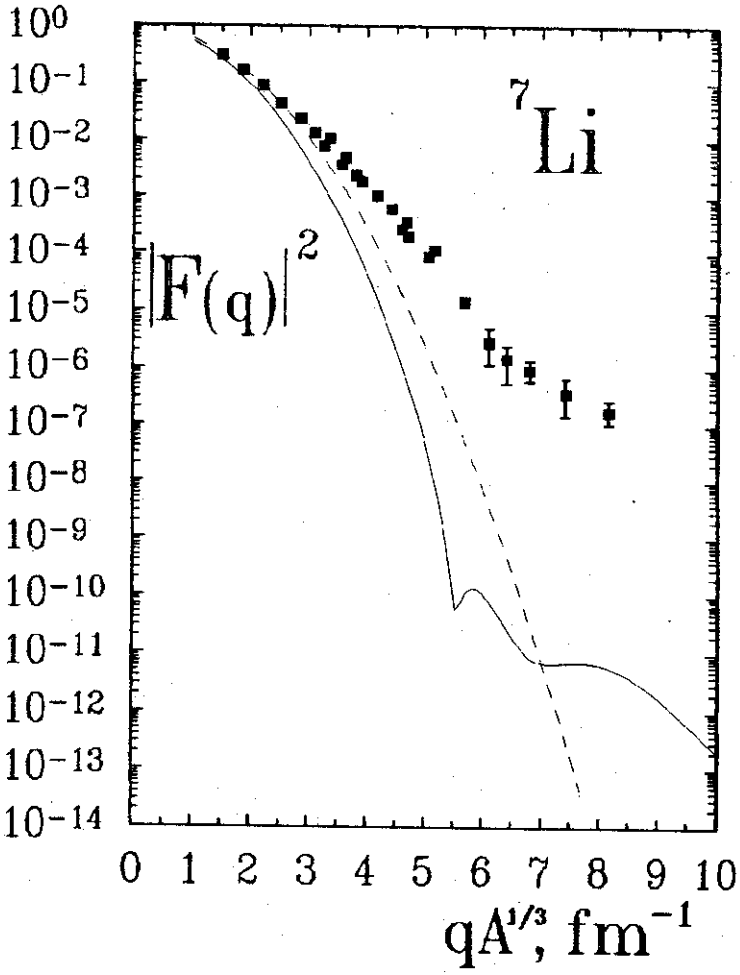

Figure 6: Form factors of ^{7}Li: solid - this is our result, dashed is result for density distribution (12)

Our results give the rich information about the structure of nuclei in contradiction with the results from [20].

In the present work we have provided a simple unified description of the isotopes of lithium. The Young diagram $[f]$ for each isotope has a simple structure which allows one to obtain a simple approximation for the wave functions of the different isotopes in a hyperspherical basis with a renormalized effective interaction. A good qualitative agreement with the variations of the mass number is obtained along with radial density distributions which have exponentially decreasing asymptotic behavior.

References

[1] Tanihata I, Hamagki H, Hashimoto O, Nagamiya S, Shida Y, Yoshikawa N, Yamakawa O, Sugimoto K, Kobayashi T, Greiner D E, Takahashi N and Nojiri Y 1985 *Phys. Lett.* **160B** 380

[2] Tanihata I, Hamagki H, Hashimoto O, Shida Y, Yoshikawa N, Sugimoto K, Yamakawa O, Kobayashi T and Takahashi N 1985 *Phys. Rev. Lett.* **55** 2676

[3] Kobayashi T, Yamakawa O, Omata K, Sugimoto K, Shimoda K, Takahashi N and Tanihata I 1988 *Phys. Rev. Lett.* **60** 2599

[4] Blank B, Gaimard J J, Geissel H, Schmidt K H, Stelzer H, Sümmerer K, Bazin D, Del Moral R, Dufour J P, Fleury A, Hubert F, Clerc H G and Steiner M 1991 *Z. Phys.* A **340** 41

[5] Sato H and Okuhara Y 1985 *Phys. Lett.* **162B** 217

[6] Bertsch G F, Brown B A and Sagawa H 1989 *Phys. Rev.* C **39** 1154

[7] Smirnov Y F and Shitikova K V 1977 *Particles and Nucleus* **8** 847

[8] Shitikova K V 1979 *Nucl. Phys.* A **331** 365

[9] Suzuki Y and Tosaka Y 1990 *Nucl. Phys.* A **517** 599

[10] Bertsch G F and Esbensen H 1991 *Ann. Phys.* **209** 327

[10] Bertsch G F and Esbensen H 1991 *Ann. Phys.* **209** 327

[11] Hoshino T, Sagawa H and Arima A 1990 *Nucl. Phys.* A **506**

[12] Strobel G L 1976 *Nucl. Phys.* A **271** 162

[13] Brink D M and Boeker E 1967 *Nucl. Phys.* A **91** 1

[14] Ajzenberg-Selove F 1988 *Nucl. Phys.* A **490** 1

[15] Ajzenberg-Selove F 1990 *Nucl. Phys.* A **506** 1

[16] Tassie L J 1956 *Austr. J. Phys.* **9** 407

[17] Walecka J D 1962 *Phys. Rev.* **126** 653

[18] Sotona M and Žofka J 1975 *Phys. Lett.* **57B** 27

[19] Lukyanov V K and Pol' Yu S 1974 *Particles and Nucleus* **5** 955

[20] Bauer W at al. march 1991 *"Pion production with radioactive nuclei"* **MSUCL-761**

[21] Lichtenshtadt J, Alster J, Moinester M A, Dubach J, Hicks R S, Peyerson G A and Kowalski S 1989 *Phys. Lett.* **219B** 394

[22] Burov V V, Eldyshev Yu V, Lukyanov V K and Pol' Yu S 1974 *JINR preprint, Dubna* **E4 8029**

[23] Burov V V, Dostovalov V N, Kaschiev M and Shitikova K V 1981 *J. Phys., G: Nucl. Phys.* **7** 137

Received by Publishing Department
on February 10, 1993.