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ON THE BEHAVIOUR OF DIS STRUCTURE FUNCTION RATIO $R(x, Q^2)$ AT SMALL x

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For the experimental studies of hadron-hadron proceses on new powerful **LHC** and **SSC** colliders,it is necessary to know in detail the values of parton (quarks and gluon) distributions (PD) of nucleon, especially at small x. The basic information on quark structure of nucleon is extracted from the process of deep inelastic lepton-hadron scattering (DIS). Its differential cross-section has the following form:

$$\frac{d^2\sigma}{dxdy} \propto \left[(1+(1-y)^2) - \frac{y^2/2}{1+R(x,Q^2)}\right] F_2(x,Q^2)$$

where F_2 and $F_L \equiv \frac{R}{1+R}F_2$ are the transverse and longitudinal structure functions (SF), respectively. The ratio $R(x,Q^2)$ is a good QCD characteristic because it equals to zero in parton model. Moreover, the value of SF F_2, whose data is usually deduced from experiment, depends essentially on the corresponding values of R. We note that the value of the ratio R is very important also in the case of polarized SF which are deduced from experimentally measured asymmetry of cross-sections of polarized leptons and polarized nucleons.

The modern DIS experimental data (see [1] for review) are not reasonably accurate to determine $R(x,Q^2)$. Moreover, at small x the data for SF F_L is absent at all yet. The theoretical predictions (see [2], for example) in the leading (LO) and next-to-leading (NLO) of perturbation theory (PT), are strongly different. It couses some doubt to apply PT in this region.

In the present letter we are studying the behavior of $R(x,Q^2)$ at small x using the method (see [3]) of replacement of Mellin convolution by the usual integral. Moreover, we will use both usual PT and Grunberg's method of effective charges [4]. Note that in the first two orders of PT the latter method coincides with scheme-invariant (SI) PT [5, 6].

1. Assuming the Regge-like behaviour for gluon PD[2] $g(x,Q^2) = x^{-\delta}\tilde{g}(x,Q^2)$ and singlet quark one $s(x,Q^2) = x^{-\delta}\tilde{s}(x,Q^2) = g(x)/\rho(x)^3$, we get the following equation for SF F_2 and F_L

$$\begin{aligned} F_1(x) &= s(x) + O(x) \quad (F_1 = F_2 - F_L) \\ F_L(x) &= \alpha(Q^2) \sum_{p=s,g} B_L^p\, p(x) + O(x), \end{aligned} \tag{1}$$

where $B_k^p(k=1,L)$ are one-loop coefficients of Wilson expansion for the first ($n=1$) SF moments.

Hereafter we use two rather strong hypotheses. We assume the similar behaviour for $g(x)$ and $s(x)$ at small x, that is confirmed by numerical solution of Gribov-Lipatov-Altarelli-Parisi equation, which has been given in [7], and neglect the nonsinglet part. Hence, the function ρ is independent on x. Moreover, we suppose that ρ is also independent on Q^2, i.e. $\rho = const$. These hypotheses allow us to simplify the calculation essentially and perform the SI analysis of $R(x,Q^2)$ at small x.

Note that the hypothesis about the weak dependence of ρ on x and Q^2 is confirmed by PD parametrizations. However, different parametrizations result in essential different values of ρ. Hence, we will not use the fixed value for ρ in our analysis.

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[2] we use PD multiplied by x and do not separate out their Q^2 dependence

[3] We restrict our consideration to the case $\delta = 0$ corresponding to standard pomeron. The case $\delta \sim (1/2)$ is not very interesting because there the addition of the NLO leads only to a small change (see [2]) of the LO predictions

From eqs.(1), using the above hypotheses and the exact values of Wilson coefficients, we get

$$R = \alpha(Q^2)\left[B_L^g\,\rho + B_L^s\right] = \frac{4}{3}\alpha(Q^2)\left[f\rho + 2\right] \tag{2}$$

2. In the NLO approximation eqs.(1) are changed to the following form

$$\begin{aligned} F_1(x) &= (1+\alpha(Q^2)B_1^s)\, s(x) + \alpha(Q^2)B_1^g g(x) + O(x) = \\ &\left(1 - \frac{8}{3}\alpha(Q^2)\right) s(x) - \frac{2}{3} f\alpha(Q^2) g(x) + O(x) \\ F_L(x) &= \alpha(Q^2) \sum_{p=s,g} B_L^p\, (1+\alpha(Q^2)R_L^p)\, p(x) + O(x) = \\ &\frac{4}{3}\alpha(Q^2)\left[(1+\alpha(Q^2)R_L^g)\, f g(x) + (1+\alpha(Q^2)R_L^s)\, 2s(x)\right] + O(x), \end{aligned} \tag{3}$$

where the products $B_L^p R_L^p$ are the two-loop coefficients of Wilson expansion for the first ($n=1$) moment of longitudinal SF (see [9]), namely

$$R_L^g = -4\left[l(x) + \frac{5}{3}\right] \quad , \quad R_L^s = 8.46 - \frac{8f}{9}\left[l(x) + 5.64\right]$$

and $l(x) = ln(1/x) - \left[\Psi(\nu+1) + \gamma\right]$

Here $\Psi(x)$ and γ are the Eulierian Ψ-function and constant, respectively, and ν is the coefficient connected with $g(x)$ asymptotic at large x: $g(x) \sim (1-x)^\nu$. We use $\nu = 4$ in agreement with quarks count rules (see [8]).

From eq.(3) we have the following equation for $R(x)$

$$R = \frac{4}{3}\alpha(Q^2)\left[f\rho\,(1+\alpha(Q^2)R_L^g) + 2\,(1+\alpha(Q^2)R_L^s)\right] / \left[1 - \frac{2}{3}(4+f\rho)\alpha(Q^2)\right], \tag{4}$$

3. The SI equation for R can be easily obtained from eqs. (2) and (4) by the introduction of new SI effective coupling constant $a(x,Q^2)$, which contains the two-loop correction into its Λ parameter as follows

$$\Lambda = \Lambda_{\overline{MS}}^{f} exp\left(r/(2\beta_0)\right),$$

where

$$r = \left(f\rho\tilde{R}_L^g + 4\tilde{R}_L^g\right)/(f\rho+2), \tilde{R}_L^p = R_L^p + \frac{2}{3}(f\rho+4) \quad (p=g,s)$$

Thus, we have

$$R^{SI} = \frac{4}{3}a(x,Q^2)\left[f\rho + 2\right] \tag{5}$$

4. Let us study the predictions given by eqs. (2), (4) and (5). The LO of PT predicts the fixed value for ratio $R(x,Q^2)$. The two-loop correction is negative at small x and increases logarithmically for $x \to 0$. This behaviour agrees numerically with the predictions of papers [2, 10] for the longitudinal SF (the transverse SF is changed poorly when NLO is added). Hence, the standard PT is weakly applicable at very small x.

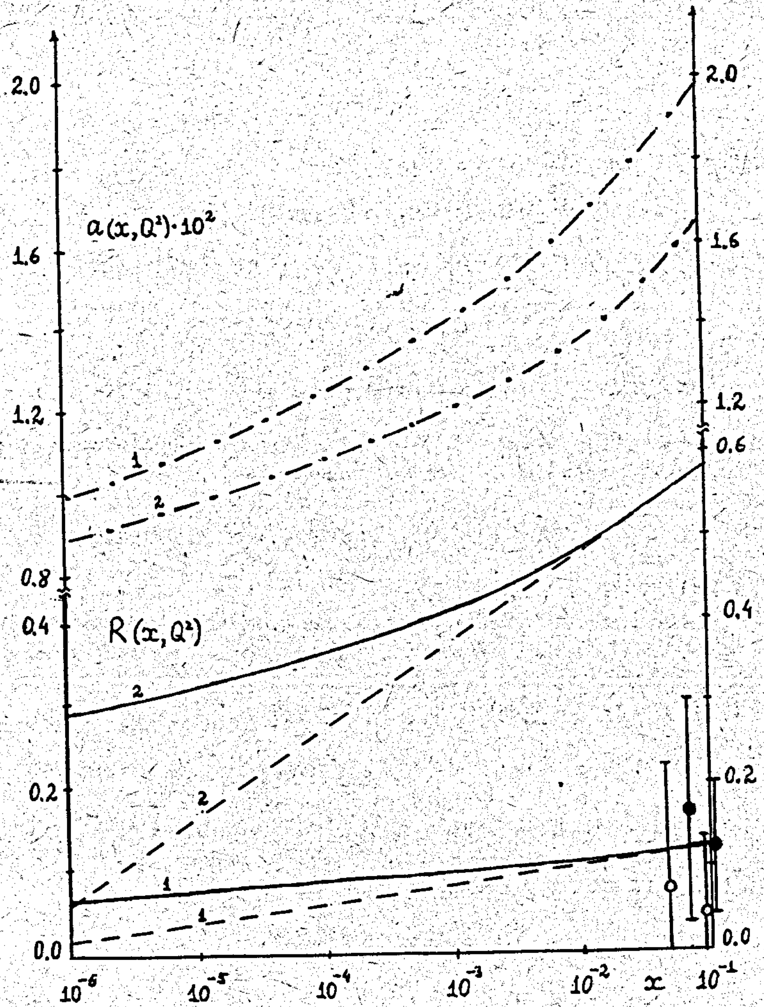

Figure 1. The effective coupling constant $a(x, Q^2)$ (dashen-dotten curve) and the ratio $R(x, Q^2)$ (the dashen and solid curves correspond to standard (i.e. eq.(4)) and SI PT (i.e. eq.(5)) results, respectively) are presented at $Q^2 = 10$ GeV^2. The symbols 1 and 2 correspond to the values 1 and 5 of ρ. EMC and BCDMS data is indicated by white and black cicles, respectively.

In the SI approach which is based on the first two orders of PT, we have no any discrepances : the variable $R(x, Q^2)$ tends to zero as $(ln(1/x))^{-1}$ when $x \to 0$. Indeed, in twice logarithmic approximation, the effective coupling constant $a(x)$ has the following form

$$a(x) = \overline{a}(x)\left[1 + \frac{\beta_1}{\beta_0}\overline{a}(x) ln(\overline{a}(x)/\beta_0)\right],$$

where

$$\overline{a}(x) = \left(\beta_0 ln(Q^2/\Lambda^2_{\overline{MS}}) + 4b ln(1/x) - \frac{4}{3}c\right)^{-1}$$

$$b = \frac{\rho + 4/9}{\rho + 2/f}, \qquad c = \frac{17}{4} + \frac{f\rho}{2} + \frac{\rho + 4.75 - 8.19/f}{\rho + 2/f}$$

Thus, in SI approach we do not get negative values for ratio R at any values of x. Moreover, the effective coupling constant decreases logarithmically at small x (see also Fig.1), hence, it has similar behaviour when $x \to 0$ and $Q^2 \to \infty$. This result is obtained in the first two orders of PT only. The values of higher orders of PT are unknown. Hovewer, if we construct SI PT following, for example, the authors of paper [6], where the effective coupling constant contains only NLO, then we get the new PT with decreasing coupling constant when $x \to 0$. So, the results given by this PT seem more reliable than in the standard case.

5. In conclusion, we analysed the behaviour of DIS ratio $R(x, Q^2)$ at small x in the first two orders of PT. We restricted ourselves to the case $g(x) \to const$, $s(x) \to const$ when $x \to 0$. The simple form for R was obtained . In standard PT the ratio R would be negative at a very small x ($x \approx 10^{-7}$) (see Fig.1). In SI approach the problem of the negative values for R does not appear. Moreover, the effective coupling constant decreases with the increase of Q^2 as well as the decrease of x (see Fig.1). This behaviour of the SI coupling constant gives some guarantee for the weak effect of the contribution of the higher orders corrections to the SI analysis results obtained by us.

As it is seen from Fig.1, the close to 1 values of ρ are favoured by both EMC [11] and BCDMS [12] data. We use here QCD parameter $\Lambda^{f=4}_{\overline{MS}} = 200$ MeV and experimental EMC points for $Q^2 = (12.5$ and $18)$ GeV^2 and BCDMS ones for $Q^2 = (15$ and $20)$ GeV^2, respectively (notice that larger values of Q^2 correspond to larger values of x). It is expected that an extended information about the ratio $R = \sigma_L/\sigma_T$ at $x < 10^{-2}$ and also on the examination of Regge-like behaviour of SF in this region, will be derived from experiments on colliders HERA and LEP*LHC.

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