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ON THE CRITICAL BEHAVIOUR OF (2 + 1)-DIMENSIONAL QED

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Quantum Electrodynamics in 2+1 dimensions (QED_3) has acquired increasing attention [1]-[6] because of its similarities to (3+1) dimensional QCD. A number of investigations have been performed for the study of dynamical chiral symmetry breaking in QED_3 and very different results have been obtained. Using the leading order (LO) in the $1/N$ expansion of the Schwinger-Dyson (SD) equation, Appelquist et al. [1] showed that the theory exhibits a critical behaviour as the number N of fermion flavours approaches $N_c = 32/\pi^2$; that is, a fermion mass is dynamically generated only for $N < N_c$. On the contrary, Pennington and collaborators [2], adopting a more general non-perturbative approach to the SD equations, found that the dynamically generated fermion mass decreases exponentially with N, vanishing only as $N \to \infty$. This conclusion was supported also by Pisarski [3] by the use of the other methods. On the other hand, an alternative non-perturbative study by Atkinson et al. [4] suggested that chiral symmetry is unbroken at sufficiently large N. The theory has also been simulated on the lattice [5, 6]. Remarkably, the conclusions of Ref. [5] are in the agreement with the existence of a critical N as predicted in the analysis of Ref. [1] while the second paper [6] contains the opposite results.

Because the critical value N_c is not large, the contribution of the higher orders in the $1/N$ expansion can be essential and may lead to better understanding of the problem. The purpose of this work is to include the $1/N$ correction to Appelquist et al. [1] LO result.

The Lagrangian of massless QED_3 with N flavours is

$$L = \overline{\Psi}(i\hat{\partial} - e\hat{A})\Psi - \frac{1}{4}F^2_{\mu\nu},$$

where Ψ is taken to be a four component complex spinor. In massless case, which we are considering, the model contains infrared divergences, which can be canceled when the model is analysed in a $1/N$ expansion [7, 8]. Since the theory is massless the mass scale is the dimensional coupling constant $a = Ne^2/8$ which is kept fixed as $N \to \infty$.

In the four component case, we can introduce the matrices γ_3 and γ_5 which anticommutate with γ_0, γ_1, and γ_2. Then the massless case is invariant under transformations $\Psi \to exp(i\alpha_1\gamma_3)\Psi$ and $\Psi \to exp(i\alpha_2\gamma_5)\Psi$. Together with the identity matrix and $[\gamma_3, \gamma_5]$ we have $U(2)$ symmetry for each spinor and the full global "chiral" symmetry is $U(2N)$. A mass term will break this symmetry to $U(N) * U(N)$. The dynamical generation of such a mass will be considered here. It is also possible to include a parity-nonconserving mass (see, for example, [9]), however we will not considere this possibility here.

Following Ref. [1] we study the solution of the SD equation. The inverse fermion propagator has the form

$$S^{-1}(p) = -[1 + A(p)]\,(\hat{p} + \Sigma(p))\,,$$

where $A(p)$ is the wave-function renormalizable coefficient and $\Sigma(p)$ is a dynamical, parity-conserving mass taken to be the same for all the fermions.

The SD equation has the form

$$\Sigma(p) = \frac{2a}{N} Tr \int \frac{d^3k}{(2\pi)^3} \frac{\gamma^\mu D_{\mu\nu}(p-k)\,[1 + A(k)]\,(\hat{k} + \Sigma(k))\Gamma^\nu(p,k)}{[1 + A(k)]^2\,(k^2 + \Sigma^2(k))}, \qquad (1)$$

where[2]

$$D_{\mu\nu}(p) = \frac{g_{\mu\nu} - (1-\xi)p_\mu p_\nu/p^2}{p^2[1+\Pi(p)]}$$

is the photon propagator and $\Gamma^\nu(p,k)$ is the vertex function.

1. The LO approximations in the $1/N$ expansion are

$$A(p) = 0, \Pi(p) = a/\mid p \mid, \text{ and } \Gamma^\nu(p,k) = \gamma^\nu,$$

where we neglect the fermion mass in the calculation of $\Pi(p)$. The gap equation is

$$\Sigma(p) = \frac{8a(2+\xi)}{N} Tr \int \frac{d^3k}{(2\pi)^3} \frac{\Sigma(k)}{k^2[(p-k)^2 + a \mid p-k \mid]}, \tag{2}$$

where we ignore the term $\Sigma^2(k)$ in the denominator of r.h.s., as this has been done in Ref. [1].

Following Ref. [1], we set

$$\Sigma(k) = (k^2)^\alpha \tag{3}$$

One can see, that for large a the r.h.s. of eq.(2) together with condition (3) (and the contributions of higher orders also) can be calculated by the standard rules for massless diagrams of the perturbation theory (see, for example, [12]). Thus, we have for large a

$$1 = \frac{2+\xi}{\beta L}$$

with $\beta = (-\alpha)(\alpha+1/2)$ and $L \equiv \pi^2 N$, or

$$\alpha_\pm = (-1 \pm [1 - 16(2+\xi)/L]^{1/2})/4 \tag{4}$$

We reproduce the solution given by Appelquist et al in Ref. [1]. Their analysis yields a critical number of fermions $N_c = 16(2+\xi)/\pi^2 \approx 1.62(2+\xi)$ (i.e. $L_c = 16(2+\xi)$), such that for $N > N_c$ $\Sigma(p) = 0$ and

$$\Sigma(0) \simeq exp[-2\pi/(N/N_c - 1)^{1/2}]$$

for $N < N_c$. Thus, chiral-symmetry breaking occurs when α becomes complex, that is for $N < N_c$.

2. The next-to-leading order (NLO) approximation has been included early by Nash in Ref. [10]. He studied the solution of SD equation assuming the following form for $\beta(L)$

$$\beta(L) = \frac{d(\xi)}{L}\left(1 - \frac{b(\xi)}{L}\right) \tag{5}$$

and found the weakening of the gauge dependence of the LO result. The NLO result has been found in the Feynman gauge:

$$d(1) = 8/3 \approx 2.67, \quad b(1) \approx 7.81$$

[2]Following the Ref. [10] we introduce a nonlocal gauge-fixing term. The detailed analysis of this possibility has been given in Ref. [11].

Note that the weak gauge dependence of the result (5) is connected with its form, where NLO contributions were sought as $1/N$ correction to the LO result. This phenomenon is similar to the absence of the dynamical mass (i.e. gauge-independent terms) when we use only the $1/N$ expansion without SD equation (see ref. [1, 2]).

We calculate exactly the contributions of the NLO Feynman integrals (see Fig.1 from Ref. [10]) using the rules for the calculation of massless diagrams. The results have the cumbersome form (they contain two- and three-sum terms) and will be given in the separate publication. Here we analyse only simplified form, which contains only the terms $\sim (-\alpha)^{-k}$ and $\sim (\alpha+1/2)^{-k}$ $(k = 1, 2, 3)$ from the series. These terms are most important in the neighbourhood of the critical point N_c. We get the following equation

$$1 = \frac{(2+\xi)}{\beta L} + [f(\xi) + \beta\varphi(\xi)]\frac{1}{(\beta L)^2}, \tag{6}$$

where $f(\xi) = 4(1-\xi)/3 - \xi^2$, $\varphi(\xi) = 176/9 - 4\pi^2 - (16/3)\xi + 4\xi^2$

If we take the condition (5) as the solution of eq. (6), we get weak gauge dependence of the values of parameters d and b as well as N_c obtained from (5) for $\beta = \beta_c \equiv 1/16$:

$$d(\xi) = (2.53, 2.62, 2.67, 2.62), \quad b(\xi) = (6.52, 7.19, 8.24, 9.51)$$

$$N_c(\xi) = (3.27, 3.32, 3.19, 2.77) \text{ for } \xi = (0.0, 0.3, 0.7, 1.0),$$

respectively. Note that for $\xi = 2/3$ and $\xi = -2$ the solution (5) of eq. (6) is exact.

Let us get the exact critical value N_c from eq. (6). Supposing $\alpha = \alpha_c \equiv -1/4$ we obtain the critical values in the following form

$$N_{c,\pm} = \frac{8}{\pi^2}\left[(2+\xi) \pm ((2+\xi)^2 + 4f(\xi) + \varphi(\xi)/4)^{1/2}\right], \tag{7}$$

i.e.

$$N_{c,+}(\xi) = (3.31, 3.35, 3.09, 2.81), \quad N_{c,-}(\xi) = (-0.07, 0.38, 1.29, 1.88)$$

for $\xi = (0.0, 0.3, 0.7, 0.9)$, respectively.

Notice the intriguing fact that follows from eq.(7). The addition of $1/N$ correction leads to the occurrence of the second critical point (for $0.05 \leq \xi \leq 0.95$) such that for $N < N_{c,-}$ the chiral symmetry does not break. The dynamical mass generation exists in the interval between the critical points $N_{c,-}$ and $N_{c,+}$. For $\xi \geq 0.95$ this interval disappears and the chiral symmetry breaking is absent. For small values of gauge parameter ξ ($\xi \leq 0.05$) new critical point does not occur. We note also that the use of the exact solution of SD equation in the first two orders can change only weakly the numerical values of critical points $N_{c,+}$ and $N_{c,-}$ but not qualitative picture of the decrease of the interval, where the dynamical mass generates, with the increase of the value of ξ.

The solution of the eq.(6) is

$$\beta_\pm = \frac{1}{2L}\left[2 + \xi + \frac{\varphi(\xi)}{L} \pm ((2+\xi)^2 + 4f(\xi) + 2(2+\xi)\frac{\varphi(\xi)}{L} + \frac{\varphi^2(\xi)}{L^2})^{1/2}\right]$$

has the simple form in Landau gauge

$$\beta_\pm(\xi = 0) = \frac{1}{L}\left[1 + \frac{\varphi(0)}{2L} \pm \sqrt{7/3}\left(1 + \frac{3}{14}\frac{\varphi(0)}{L}\right)\left(1 + \frac{\frac{3}{49}\varphi^2(0)/L^2}{\left(1 + \frac{3}{14}\frac{\varphi(0)}{L}\right)^2}\right)^{1/2}\right], \tag{8}$$

where the last term in r.h.s. of eq.(8) is very small for $L \sim L_c$. Leaving it out we get the following equation for β_+ :

$$\beta_+(\xi = 0) \approx 1 + \sqrt{7/3}\frac{1}{L} + \left(1 + \sqrt{3/7}\varphi(o)\right)\frac{1}{2L^2} \approx \frac{2.52}{L}\left(1 - \frac{6.52}{L}\right),$$

which has coefficients are close to those from the paper [10].

Resume. We included $O(1/N^2)$ terms into SD equation exactly and found the strong gauge dependence of the result which in general has not the form of eq.(5) (see eq. (8)). Hence, the addition of $1/N$ correction does not lead to the essential improvement in our understanding of dynamical chiral symmetry breaking. However, we note that in Landau gauge, where Ward identities can be satisfied in the case of free vertex (see [13]), the inclusion of $O(1/N^2)$ terms slightly changes only quantitative (but not qualitative) properties of the LO results. Thus our analysis gives further evidence in favour of the solution has been given by Appelquist et al. [1]. Hence, it seems that the $1/N$ expansion of the kernel of SD equation describes reliably the critical behaviour of the theory.

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