объединенный
институт
ядерных
исследований
дубна

E2-93-394

A.V.Kotikov*

# GLUON DISTRIBUTION AS FUNCTION OF $F_L$ AND $F_2$ AT SMALL $x$

Submitted to «Ядерная физика»

*e-mail: KOTIKOV@LSHE7.JINR.DUBNA.SU

1993

## 1 Introduction

Recently the small-$x$ behaviour of the structure functions (SF) of deep inelastic scattering (DIS) was considered in connection with possibility of experimental studies on new powerful colliders **HERA** [1] and **LEP*LHC** [2]. Analysis of SF gives main information about the behaviour of parton (quarks and gluon) distributions (PD) of nucleon. The knowledge of PD is a basis for the study of other processes.

Let us introduce the standard parametrizations of singlet quark $s(x,Q^2)$ and gluon $g(x,Q^2)$ PD [2] (see, for example, [4])

$$\begin{aligned} s(x,Q^2) &= A_s x^{-\delta}(1-x)^{\nu_s}(1+\gamma_s x) \equiv x^{-\delta}\tilde{s}(x,Q^2) \\ g(x,Q^2) &= A_g x^{-\delta}(1-x)^{\nu_g}(1+\gamma_g x) \equiv x^{-\delta}\tilde{g}(x,Q^2), \end{aligned} \qquad (1)$$

with $Q^2$ dependent parameters on the r.h.s.. We use the similar small $x$ behaviour for gluon and sea quarks PD, that follows from the form of the kernel of Gribov- Lipatov-Altarelli-Parisi (GLAP) equation (see also recent fits of experimental data in [5] ).

The "conventional" choice is $\delta = 0$. It leads to nonsingular behaviour of PD (see $B_0$ fit in [6]) when $x \to 0$. Another value $\delta \sim \frac{1}{2}$ has been obtained in the papers [7] as the sum of leading powers of $\ln(1/x)$ in all orders of perturbation theory (PT) (see also $B_-$ fit in ref.[6]). Recent NMC data [8, 9] agree with small values of $\delta$ . This choice agrees with the present experimental data for pp and $\bar{\text{p}}$p total cross-sections (see [10, 11]) [3] and model of Landshoff and Nachtmann pomeron [12] with exchange of the pair of a nonperturbative gluons, yielding $\delta = 0.086$ . However, the preliminary data from **HERA**, which had been presented on **Moriond-93**, prefers $\delta \sim 0.3$ .Following to the above-written uncertainty of $\delta$ we shall not use its any fixed values.

In present paper we give the method to extract gluon distribution from the longitudinal and transverse SF of DIS. Our formulas are true for all orders of PT and agree in the leading order with the results of the paper [13].

## 2 The extraction of GD from SF of DIS

We consider the region $Q^2 \leq 150\ GeV^2$ , where ep collision can be represented by one photon exchange and electroweak interactions are neglected. Moreover, we suppose that the values of $Q^2$ are large considerably: $Q^2 \geq 15 GeV^2$ and think, that the higher twists contribution is small. Then, both the longitudinal and transverse SF of DIS are connected with PD as follows (hereafter index k marks 2 and L).

$$F_k(x) = \delta_s^k \sum_{f=s,g} C_k^f(x,\alpha) * f(x), \qquad (2)$$

where $f(x)*\varphi(x) \equiv \int_x^1 \frac{dy}{y} f(y)\varphi(\frac{x}{y})$ is a Mellin convolution, $C_k^f(x,\alpha)$ are the perturbatively calculated kernels, and $\delta_s^k$ are coefficients, which depend on process and the number of quarks $f$. For **ep** collision and $f$=4 we have $\delta_s^2 = \delta_s^L = 5/18 \equiv \delta_s$ .

[2] We use PD multiplied by $x$ and neglect the nonsingular quark distribution at small $x$. The full analysis can be found in ref. [3]

[3] In ref. [11] one shows that high energy $\bar{\text{p}}$p data have a linear lns behaviour.

As it is seen from Appendix, eq.(2) can be represented in the following form at small $x$

$$F_k(x) = \delta_s \sum_{f=s,g} C^f_{k,1+\delta}(\alpha)\ x^{-\delta}\ \tilde{f}(x/\xi^f_k(\delta)) + O(x), \qquad (3)$$

or

$$F_k(x) = \delta_s \sum_{f=s,g} C^f_{k,1+\delta}(\alpha) . \left(\xi^f_k(\delta)\right)^{-\delta} f(x/\xi^f_k(\delta)) + O(x), \qquad (4)$$

where

$$C^f_{k,\eta}(\alpha) = \int_0^1 dx\ x^{\eta-1}\ C^f_k(x,\alpha)$$

are the Wilson coefficients of the $\eta$th "moments" and $\xi^f_k(\delta) = C^f_{k,1+\delta}(\alpha)/C^f_{k,\delta}(\alpha)$ .

Thus, the PD and DIS structure functions are simply related to each other at small $x$ (for the leading twist of the Wilson expansion). The coefficients $C^f_{k,1+\delta}$ are the analytical continuation of Wilson coefficients to the noninteger values and, hence, they can be calculated in perturbative QCD in principle in any orders of PT. The method of the analytical expansion has been constructed in papers [14, 3].

Note that for the case $\tilde{f}(x) \approx const$ the form of SF from eqs.(3) and (4) coincides with that given in [14]-[17].

Because the Wilson coefficients are known only in first two orders of PT, we shall confine ourselves by their analysis.

The Wilson coefficients for longitudinal and transverse SF are well known in the leading (see [4]) and next-to leading (see [18]-[21]) [4] orders (LO and NLO) of PT. They have the following form:[5]

$$\begin{aligned} C^s_{2,\eta}(\alpha) &= 1 + \alpha B^s_{2,\eta} + O(\alpha^2), \quad C^g_{2,\eta}(\alpha) = \alpha B^g_{2,\eta} + O(\alpha^2) \\ C^f_{L,\eta}(\alpha) &= \alpha B^f_{L,\eta}(1 + \alpha R^f_{L,\eta}) + O(\alpha^3), \end{aligned} \qquad (5)$$

Note also that the coupling constants are different in LO and NLO of PT

$$\alpha_{LO}(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{LO})}$$

$$\alpha_{\overline{MS}}(Q^2) = \frac{1}{\beta_0 \ln(Q^2/\Lambda^2_{\overline{MS}})} \left[1 - \frac{\beta_1}{\beta_0^2} \frac{\ln\ln(Q^2/\Lambda^2_{\overline{MS}})}{\ln(Q^2/\Lambda^2_{\overline{MS}})}\right]$$

In what follows, if it does not mention spetially, this difference is not marked.

### 2.1 The LO of PT

Accounting for the form of Wilson coefficients 5, we have from eqs.(3) and (4)

$$\begin{aligned} F_2(x) &= \delta_s s(x) \\ F_L(x) &= \delta_s \alpha \sum_{f=s,g} B^f_{L,1+\delta} \left(\xi^f_k(\delta)\right)^{-\delta} f(x/\xi^f_L(\delta)) + O(x), \end{aligned} \qquad (6)$$

[4] In the papers [18, 19] the coefficient $R^f_{L,n}$ contains the mistake, which would be corrected in [21].

[5] Hereafter contrary to the standard one we use $\alpha(Q^2) = \alpha_s(Q^2)/4\pi$ .

From eqs.(6) we get for GD:

$$g(x) = \frac{(\xi^g_L(\delta))^\delta}{\delta_s B^g_{L,1+\delta}} \left[\frac{F_L(\xi^g_L(\delta)x)}{\alpha(Q^2)} - \frac{B^s_{L,1+\delta}}{(\xi^s_L(\delta))^\delta} F_2\left(\frac{\xi^g_L(\delta)}{(\xi^s_L(\delta)}x\right)\right], \tag{7}$$

The values of LO Wilson coefficients are

$$B^s_{L,1+\delta} = \frac{16}{3}\frac{1}{2+\delta} \quad \text{and} \quad \delta_s B^g_{L,1+\delta} = \frac{8e}{(2+\delta)(3+\delta)},$$

where $e = \sum_i e_i^2$ is the sum of squares of quark charges. Hence from eq.(7) we have

$$g(x) = \frac{3+\delta}{8e}\left[\frac{2+\delta}{\alpha(Q^2)}\left(\frac{1+\delta}{3+\delta}\right)^\delta F_L\left(\frac{1+\delta}{3+\delta}x\right) - \frac{16}{3}\left(\frac{2+\delta}{3+\delta}\right)^\delta F_2\left(\frac{2+\delta}{3+\delta}x\right)\right], \tag{8}$$

Note that eq.(8) coincides almost with the one from [13]. A slight difference in the $F_2$ contribution is not important at small $x$ (see [13]) and is the consequence of our choice of the singular form of the sea quark PD.

## 2.2. The LO & NLO of PT

In the NLO approximation we can connect SF and PD in the various ways. The more oftenly used schemes are DIS and $\overline{MS}$ ones. They lead to equal results for $g(x)$ (see [3]). It is not wonderful since GD is extracted from SF, which have physical values. The scheme dependence appears as $O(\alpha^2)$. Here for simplicity we will get this connection in DIS scheme only.

From eqs.(3)–(5) we get

$$\begin{aligned} F_2(x) &= \delta_s s(x) \\ F_L(x) &= \delta_s \alpha \sum_{f=s,g} B^f_{L,1+\delta}\left(1+\alpha(Q^2)\tilde{R}^f_{L,1+\delta}\right)\left(\xi^f_k(\delta)\right)^{-\delta} f(x/\xi^f_L(\delta)) + O(x), \end{aligned} \tag{9}$$

where $\tilde{R}^f_{L,\eta}$ are Wilson coefficients in DIS scheme, which are connected with the well-known ones in $\overline{MS}$ scheme (see [18]–[22]) by the following relations

$$\tilde{R}^s_{L,\eta} = R^s_{L,\eta} - B^s_{2,\eta}, \qquad \tilde{R}^g_{L,\eta} = R^g_{L,\eta} - \frac{B^s_{L,\eta}}{B^g_{L,\eta}} B^g_{2,\eta}, \tag{10}$$

and the variable $\tilde{\xi}^f_{L,\eta}$ is obtained from $\xi^f_{L,\eta}$ by the replacement $R^f_{L,\eta} \to \tilde{R}^f_{L,\eta}$. From eqs.(9) we get for GD

$$\begin{aligned} g(x) &= \frac{(\xi^g_L(\delta))^\delta}{\delta_s B^g_{L,1+\delta}(1+\alpha(Q^2)\tilde{R}^g_{L,1+\delta})}\left[\frac{F_L(\xi^g_L(\delta)x)}{\alpha(Q^2)}\right. \\ &- \left.\frac{B^s_{L,1+\delta}}{(\xi^s_L(\delta))^\delta}\left(1+\alpha(Q^2)\tilde{R}^s_{L,1+\delta}\right) F_2\left(\frac{\xi^g_L(\delta)}{(\xi^s_L(\delta)}x\right)\right], \end{aligned} \tag{11}$$



Using values of Wilson coefficients (see [18, 21]) we have

$$\begin{aligned} g(x) &= \frac{3+\delta}{8e}\frac{1}{1+\alpha(Q^2)\tilde{R}^g_{L,1+\delta}}\left[\frac{2+\delta}{\alpha(Q^2)}\left(\tilde{\xi}^g_L(\delta)\right)^\delta F_L\left(\tilde{\xi}^g_L(\delta)x\right) - \right. \\ &\left.\frac{16}{3}\left(1+\alpha(Q^2)\tilde{R}^s_{L,1+\delta}\right)\left(\overline{\xi}(\delta)\right)^\delta F_2\left(\bar{\xi}(\delta)x\right)\right], \end{aligned} \tag{12}$$

where

$$\tilde{\xi}^g_L(\delta) = \frac{1+\delta}{3+\delta}\left(1+\alpha(Q^2)[\tilde{R}^g_{L,1+\delta} - \tilde{R}^g_{L,\delta}]\right),$$

$$\overline{\xi}(\delta) = \frac{2+\delta}{3+\delta}\left(1+\alpha(Q^2)[\Phi_{1+\delta} - \Phi_\delta]\right), \qquad \Phi_\eta = \tilde{R}^g_{L,\eta} - \tilde{R}^s_{L,\eta}$$

The values of the coefficients $\tilde{R}^g_{L,\eta}$ and $\tilde{R}^s_{L,\eta}$ can be obtained by analytic continuation of integer arguments (see [14, 3]). As the coefficients have rather complicated form, we give here only approximate expressions

$$\begin{aligned} \tilde{R}^g_{L,1+\delta} &= -4\left[\tilde{\delta}^{-1} + 2 - 13,98\,\delta - 15.76\,\delta^2\right] \\ \tilde{R}^g_{L,\delta} &= -12\left[\overline{\delta}^{\,-2} + 1.28\,\overline{\delta}^{\,-1} - 10.26 - 18.70\,\delta\right] \\ \tilde{R}^s_{L,1+\delta} &= 8.46 + 38.86\,\delta - \frac{8f}{9}\left[\tilde{\delta}^{-1} + 5.64 - 0.25\,\delta\right] \\ \tilde{R}^s_{L,\delta} &= -\frac{25}{3}\left[\overline{\delta}^{\,-1} - 3.16\right] - 4f\left[\overline{\delta}^{\,-2} + 8.33\,\overline{\delta}^{\,-1} - 1.14\right] \end{aligned} \tag{13}$$

which coincide with the exact ones for $\delta = 0$ and $\delta = \frac{1}{2}$. The values of $\tilde{\delta}$ and $\overline{\delta}$, where

$$\begin{aligned} \tilde{\delta}^{-1} &= \delta^{-1}\left[1 - x^\delta\frac{\Gamma(1-\delta)\Gamma(1+\nu)}{\Gamma(1+\nu-\delta)}\right] \\ \overline{\delta}^{\,-1} &= \delta^{-1}\left[1 - x^\delta\frac{\Gamma(1-\delta)\Gamma(\nu)}{(1+\delta)\Gamma(\nu-\delta)}\right] \end{aligned} \tag{14}$$

are regular for $\delta = 0$, and

$$\begin{aligned} \tilde{\delta}^{-1} &= \ln\frac{1}{x} - (\Psi(1+\nu)+\gamma) \\ \overline{\delta}^{\,-1} &= \ln\frac{1}{x} - (\Psi(\nu)+\gamma-1) \end{aligned} \tag{15}$$

Here $\Psi(x)$ and $\gamma$ are the Eulierian $\Psi$-function and constant, respectively.

# 3 Discussion

The present investigation has been initiated by articles [23]–[25], in which the approximate relation [13]

$$\hat{g}(x) = \frac{5.9}{8e}\left[\frac{1}{\alpha(Q^2)}F_L(0.4x) - \frac{8}{3}F_2(0.8x)\right], \tag{16}$$

has been studied numerically. Using the Morfin and Tung parametrizations for GD from [26] the authors of [24] verified the validity of eq.(16). Knowing the exact values of one- and two-loop Wilson coefficients, they got the SF's values and returned to GD with help of eq.(16). They found that for singular form of $g(x)$ (i.e. $\delta \approx \frac{1}{2}$) eq.(16) leads to too high result already in the LO. The addition of NLO does not change practically the results for singular GD form. However, it leads to the very small values of nonsingular GD.

After we will give the explanation of these results, using the eqs.( 8) and ( 12) from the previous section[6].

## 3.1 The LO of PT

The eq.(8) is true for any $\delta \geq 0$ . For $\delta = \delta_1 \equiv 0.128$ and $\delta = \delta_2 \equiv 0.435$ , which have been found in ref.[25] for nonsingular and singular GD forms at $Q^2 = 50\ GeV^2$ , we have

$$g(x) = \frac{0.73}{e}\left[\frac{1}{\alpha(Q^2)} F_L(0.36x) - 2.39\ F_2(0.68x)\right] \quad \text{for } \delta = \delta_1$$
$$g(x) = \frac{0.72}{e}\left[\frac{1}{\alpha(Q^2)} F_L(0.42x) - 1.89\ F_2(0.71x)\right] \quad \text{for } \delta = \delta_2 \qquad (17)$$

The coefficients of eq.(16) are very close to the ones of eq.(17). However, it is not clear, how the difference of the arguments of the SF compensates the difference of the coefficients. Let us present a simple qualitative analysis.

Consider SF $F_L$ in the form $F_L = x^{-\delta}\varphi_L(x) + \ldots$ by analogy with PD in eq.(1). Moreover, we think that $\varphi_L(x) = const$ and neglect SF $F_2(x)$ [7]. In this approach the eqs.(16) and (17) have the following form

for $\delta = \delta_1$

$$\hat{g}(x) \approx g(x) = \frac{0.83}{e}\frac{1}{\alpha(Q^2)}\varphi_L\, x^{-\delta_1} \qquad (18)$$

for $\delta = \delta_2$

$$g(x) = \frac{1.05}{e}\frac{1}{\alpha(Q^2)}\varphi_L\, x^{-\delta_2} \qquad (19)$$

$$\hat{g}(x) = \frac{1.10}{e}\frac{1}{\alpha(Q^2)}\varphi_L\, x^{-\delta_2} \qquad (20)$$

It is clear seen, that the approximate eq.(16) reproduces the connection between $g(x)$ and SF for $\delta = \delta_1$ very well and gives a 5% excess for the correct result in case $\delta = \delta_2$ . The similar behaviour has been observed by authors of papers [23]–[25].

## 3.2 The LO & NLO of PT

Let us give the qualitative analysis of the results of eq.(12) by analogy with LO approximation. We use $\nu_g = 5$ and $\gamma_g = 0$ which are similar to the ones from [25] for $Q^2 = 50\ GeV^2$

Then, we have for GD

[6]Note that some explanation of their results was given by authors themselves in the paper [24]

[7]The analysis with nonzero values of $F_2(x)$ can be found in ref. [3]



for $\delta = \delta_1$

$$g(x) = \frac{0.83}{e}\frac{1}{\alpha(Q^2)}\frac{1}{1-5.09\alpha(Q^2)[\ln(1/x) - 0.06(\ln(1/x))^2 - 1.79]}\varphi_L\, x^{-\delta_1} \qquad (21)$$

for $\delta = \delta_2$

$$g(x) = \frac{1.05}{e}\frac{1}{\alpha(Q^2)}\frac{1}{1+19.06\alpha(Q^2)}\varphi_L\, x^{-\delta_2} \qquad (22)$$

As it is well seen, the NLO corrections for $\delta = \delta_1$ and $\delta = \delta_2$ are opposite in sign and, hence lead to different changes. This behaviour coincides qualitatively with Fig.1.

Let us estimate these changes. The variables $\alpha_{\overline{MS}}(Q^2)$ and $\alpha_{LO}(Q^2)$ for $\Lambda_{\overline{MS}} = 194MeV$ and $\Lambda_{LO} = 144MeV$ that have been found in [27], have the following values for $Q^2 = 50\ GeV^2$

$$\alpha_{\overline{MS}} = 1.33 \times 10^{-2} \quad \text{and} \quad \alpha_{LO} = 1.54 \times 10^{-2}$$

Hence, for $Q^2 = 50\ GeV^2$ and $x = \{10^{-4}, 10^{-3}, 10^{-2}, 10^{-1}\}$ the eqs.(17) can be represented in the form

$$g(x) = \frac{(1.26, 1.17, 1.07, 0.95)}{e}\frac{1}{\alpha_{LO}(Q^2)}\varphi_L\, x^{-\delta_1} \text{ for } \delta = \delta_1$$
$$g(x) = \frac{0.97}{e}\frac{1}{\alpha_{LO}(Q^2)}\varphi_L\, x^{-\delta_2} \text{ for } \delta = \delta_2$$

It is seen that the result for GD for $\delta = \delta_2$ is very similar to that from [24]. However in case $\delta = \delta_1$ we get the smaller values of GD than in [24]. This results from the fact that used in [24] Morfin-Tung parametrizations have the additional factor $\ln^{b(Q^2)}(1/x+1)$. At $Q^2 = 50\ GeV^2$ the values of $b(Q^2)$ are close to 0 and 1 for $\delta = \delta_2$ and $\delta = \delta_1$ , respectively. In principle, the addition of this term to parametrization could change the analysis given in Appendix. The full analysis including the logarithmic term was given in [3]. Here we note that all the results given in the present paper are not changed except of eqs.(14) and (15), which will take the following form

$$\tilde{\delta}^{-1} \to \left(1 + \frac{1}{ln(1/x)}\frac{d}{d\delta}\right)\tilde{\delta}^{-1} \quad \text{and} \quad \overline{\delta}^{\,-1} \to \left(1 + \frac{1}{ln(1/x)}\frac{d}{d\delta}\right)\overline{\delta}^{\,-1},$$

and for $\delta = 0$

$$\tilde{\delta}^{-1} = \frac{1}{2}\left(\ln\frac{1}{x} - \left[(\Psi(1+\nu)+\gamma)^2 + (\Psi^{(1)}(1+\nu) - \zeta(2))\right]\frac{1}{ln(1/x)}\right)$$
$$\overline{\delta}^{\,-1} = \frac{1}{2}\left(\ln\frac{1}{x} - \left[(\Psi(\nu)+\gamma-1)^2 + (\Psi^{(1)}(\nu) - \zeta(2)-1)\right]\frac{1}{ln(1/x)}\right).$$

Hence, the addition of the term $\ln(1/x)$ to PD leads to the change

$$\ln\frac{1}{x} \to \frac{1}{2}\ln\frac{1}{x} + 1.88 - \frac{2.51}{\ln(1/x)}$$

in eq.(21). It will have the form

$$g(x) = \frac{(1.14, 1.10, 1.05, 0.97)}{e}\frac{1}{\alpha_{LO}(Q^2)}\varphi_L\, x^{-\delta_1} \text{ for } \delta = \delta_1$$

and leads to numerical agreement of our results with the ones from [24].

# 4 Conclusions

In this paper we presented the method to extract GD from SF at small $x$. The results are true in any order of PT (all coefficients are expressed through Wilson coefficients of operator expansion) and coincide in the LO with the ones from [13].

We give also a rather full analysis in the LO and NLO where Wilson coefficients are well known. Its results agree very well with the ones of numerical analysis from [24] and demonstrate the importance of the including of $\alpha_s^2$ corrections to longitudinal SF for the extraction of GD from future **HERA** and **LEP*LHC** data.

Note that the connection between GD and DIS SF becomes strongly $\delta$ -dependent, when we add NLO corrections. This is in contrast with LO analysis. Hence, before the extraction of GD from experimental data of SF at small $x$, the fit of the parameters like in eq.(1) parametrizations for $F_L(x)$ and $F_2(x)$ will be necessary.


## Acknowledgements

The author is grateful to Gonzalo Parente for the possibility to use results of the paper [24] before its publication and the useful discussions.


# 5 Appendix

Here we present the method to replace the convolution (2) by simple product at small $x$.

**1.** Consider the basic integral

$$J_\delta(a,x) = x^a * \varphi(x) \equiv \int_x^1 \frac{dy}{y}\, y^a\, \varphi(\frac{x}{y}),$$

, where $\varphi(x) = Ax^{-\delta}(1-x)^\nu \equiv x^{-\delta}\tilde{\varphi}(x)$. Expanding $\tilde{\varphi}(x)$ near $\tilde{\varphi}(0)$ , we have

$$\begin{aligned} J_\delta(a,x) &= x^{-\delta}\int_x^1 dy\, y^{a+\delta-1}\left[\tilde{\varphi}(0) + \frac{x}{y}\,\tilde{\varphi}^{(1)}(0) + \dots + \frac{1}{k!}\left(\frac{x}{y}\right)^k \tilde{\varphi}^{(k)}(0) + \dots\right] \\ &= x^{-\delta}\left[\frac{1}{a+\delta}\,\tilde{\varphi}(0) + \frac{x}{a+\delta-1}\,\tilde{\varphi}^{(1)}(0) + \dots\right] \qquad \text{(A1)} \\ &- x^a\left[\frac{1}{a+\delta}\,\tilde{\varphi}(0) + \frac{1}{a+\delta}\,\tilde{\varphi}^{(1)}(0) + \dots + \frac{1}{k!}\,\frac{1}{a+\delta-k}\,\tilde{\varphi}^{(k)}(0) + \dots\right] \end{aligned}$$

The second term on the r.h.s. of eq.(A1) can be summed, and $J_\delta(a,x)$ has the following form

$$J_\delta(a,x) = x^{-\delta}\left[\frac{1}{a+\delta}\,\tilde{\varphi}(0) + \frac{x}{a+\delta-1}\,\tilde{\varphi}^{(1)}(0) + \dots\right] + x^a\,\frac{\Gamma(-(a+\delta))\Gamma(1+\nu)}{\Gamma(1+\nu-a-\delta)}\,\tilde{\varphi}(0)$$

Consider three important cases:



a) $a \geq 2$

$$J_\delta(a,x) = x^{-\delta}\frac{1}{a+\delta}\,\tilde{\varphi}(\frac{a+\delta}{a+\delta-1}x) + \dots$$

b) $a = 0$

$$J_\delta(0,x) = x^{-\delta}\left[\frac{1}{\delta}\,\tilde{\varphi}(0) + \frac{x}{\delta-1}\,\tilde{\varphi}^{(1)}(0) + \dots\right] + \frac{\Gamma(-\delta)\Gamma(1+\nu)}{\Gamma(1+\nu-\delta)}\,\tilde{\varphi}(0) =$$

$$x^{-\delta}\frac{1}{\delta}\tilde{\varphi}(\frac{\tilde{\delta}}{\delta-1}x) + \dots$$

with $\tilde{\delta}$ from eq.(14)

c) $a = 1$

$$J_\delta(1,x) = x^{-\delta}\left[\frac{1}{1+\delta}\,\tilde{\varphi}(0) + \frac{x}{\overline{\delta}}\,\tilde{\varphi}^{(1)}(0) + \dots\right] - x\frac{\Gamma(-(1+\delta))\Gamma(\nu)}{\Gamma(\nu-\delta)}\,\tilde{\varphi}^{(1)}(0) =$$

$$x^{-\delta}\frac{1}{1+\delta}\tilde{\varphi}(\frac{1+\delta}{\overline{\delta}}x) + \dots$$

with $\overline{\delta}$ from eq.(15)

**2.** Consider the integral

$$I_\delta(x) = K(x) * \varphi(x) \equiv \int_x^1 \frac{dy}{y}\, K(y)\, \varphi(\frac{x}{y})$$

and define the moments of the kernel $K(y)$ in the following form

$$C_n = \int_0^1 dy\, y^{n-2} K(y)$$

By analogy with case **1**, we have

$$I_\delta(x) = x^{-\delta}\,\tilde{C}_{1+\delta}\,\tilde{\varphi}(\frac{\overline{C}_\delta}{\tilde{C}_{1+\delta}}x) + \dots$$

where $\tilde{C}_{1+\delta}$ and $\overline{C}_\delta$[8] coincide with $C_{1+\delta}$ and $C_\delta$ after the replacement $1/\delta \to 1/\tilde{\delta}$ and $1/\delta \to 1/\overline{\delta}$, respectively.


# References

[1] J.Feltesse, *Proc. of the HERA Workshop* (1988) 33.

[2] J.Feltesse, *Proc. of the ECFA Workshop on LHC* Aachen (1990) 29.

[3] A.V.Kotikov, Dubna preprint; in preparation

[4] A.J.Buras, *Rev.Mod.Phys.* **52** (1980) 194.


[8] In Section 2 we droped these overhand subscrips

[5] R.G.Roberts and M.R.Whalley, *J. of Phys.* **G17** (1991) D1.

[6] J.Kwecinski, A.D.Martin, W.S.Stirling and R.G.Roberts, *Phys.Rev.* **D42** (1990) 3645.

[7] E.A.Kuraev, L.N.Lipatov and V.S.Fadin, *ZHETF* **53** (1976) 2018, **54** (1977) 128; Ya.Ya.Balitzki and L.N.Lipatov, *Yad.Fiz.* **28** (1978) 822; L.N.Lipatov, *ZHETF* **63** (1986) 904.

[8] P. Amandruz et al, *Phys.Lett.* **295B** (1992) 159.

[9] A.N.Muller, *Zeuten Workshop on "Deep Inelastic Scattering"* Teupitz (1992); Preprint CU-TP-570, New York.

[10] A.Donnachie and P.V.Landshoff, *Nucl.Phys.* **244B** (1984) 322; **267B** (1986) 690.

[11] E.Leader, *Comments Nucl.Part.Phys.* **20** (1992) 269.

[12] P.V.Landshoff and O.Nachmann, *Z.Phys.* **C35** (1987) 405.

[13] A.M.Cooper-Sarkar et al, *Z.Phys.* **C39** (1988) 281.

[14] A.V.Kotikov, Preprint P2-88-139 (1988) Dubna.

[15] A.V.Kotikov, Preprint P2-88-422 (1988) Dubna.

[16] V.I.Vovk, A.V.Kotikov and S.I.Maximov, *Teor.Mat.Fiz.* **84** (1990) 101.

[17] L.L.Enkovszky, A.V.Kotikov and F.Paccanoni, *Yad.Fiz.* **55** (1992) 2205.

[18] D.I.Kazakov and A.V.Kotikov, *Nucl.Phys.* **307B** (1988) 721; **345B** (1990) 299(E).

[19] D.I.Kazakov et al, *Phys.Rev.Lett.* **65** (1990) 1535, 2921(E).

[20] E.B.Zijlstra and W.L. van Neervan, *Phys.Lett.* **272B** (1991) 127, **273B** (1991) 476.

[21] D.I.Kazakov and A.V.Kotikov, *Phys.Lett.* **291B** (1992) 171.

[22] J.Sanchez Guillen et al, *Nucl.Phys.* **353B** (1991) 357;

[23] N.Magnussen and G.A.Schuler. *Proc. of the ECFA Workshop on LHC* Aachen (1990) 858.

[24] G.Parente and J.Sanchez Guillen. *Proc. of Multiparticle dynamics* Santiago (1992).

[25] E.B.Zijistra and W.L. van Neervan, *Nucl.Phys.* **383B** (1992) 525.

[26] A.D.Morfin and Wu-Ki Tung, *Z.Phys.* **C52** (1991) 13.

[27] S.Keller et al, *Phys.Lett.* **270B** (1990) 61.

**Received by Publishing Department on November 1, 1993.**