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THE $U(3)\times U(3)$ NAMBU — JONA-LASINIO MODEL WITH THE GLUON CONDENSATE AND AXIAL ANOMALY AT FINITE TEMPERATURE

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1. Introduction

The Nambu- Jona- Lasinio (NJL) model [1] is a very convenient instrument for investigating the behaviour of mesons in the hot and dense medium [2]- [6].

In the work [7] we have studied a simple $U(2)_f$ symmetric variant of the NJL model with a gluon condensate (GC) at finite temperature. It was shown that the GC plays a stabilizing role for the behaviour of physical quantities when the temperature changes. Now we shall investigate the $U(3) \times U(3)$ chiral symmetric variant of the NJL model with the GC and axial anomaly. The latter plays an important role for solving the $U(1)_A$ problem ($\eta - \eta'$ mass difference). We shall show that the stabilizing role of the GC at finite temperature for heavy mesons is smaller than for light mesons. We also shall show that the absolute value of the singlet- octet mixing angle increases with the temperature. The axial anomaly decreases when the temperature increases.

The paper is organized as follows. In the next section we give the *U(3) × U(3) NJL Lagrangian with the* gluon condensate fields [7, 8] and gluon anomaly term. In Section 3, the effective Lagrangian for the scalar and pseudoscalar mesons is derived and the meson mass formulae are described. Section 4 is devoted to the description of the input parameters of our model, the meson masses and the quark condensates. Then, in Section 5 we introduce the temperature and density dependence of our physical quantities. Section 6 is the discussion and conclusion. We comment on the behaviour of the constituent quark masses, quark condensates, meson masses, F_π, F_K and the mixing angle θ at changing temperature in the region $0 < T < 150$Mev. We show that the gluon condensate plays a stabilizing role for the behaviour of these quantities at changing T. In the conclusion, we give some perspectives for the subsequent development of these investigations.

2. The $U(3) \times U(3)$ NJL model

Let us consider the effective quark Lagrangian [4], [5],[7]-[11]

$$\mathcal{L} = \mathcal{L} + \mathcal{L}_4 + \mathcal{L}_6 \ . \tag{1}$$

Here

$$\mathcal{L} = \bar{q}(i\hat{\mathcal{D}} - m^0)q \ ,$$

where q are the quark fields in the fundamental representation of $SU(3)_{\mathrm{colour}} \times U(3)_{\mathrm{flavour}}$ with

$$q = \begin{pmatrix} u \\ d \\ s \end{pmatrix}$$

and each of quarks having three colours; m^0 is the current quark mass matrix $m^0 = \mathrm{diag}(m_u^0, m_d^0, m_s^0)$ (in what follows we suppose that $m_u^0 = m_d^0$); $\hat{\mathcal{D}}$ is the covariant derivative

$$\mathcal{D}_\mu q = (\partial_\mu + ig\frac{\lambda_a}{2}G_\mu^a)q \ , \tag{2}$$

where g is the QCD coupling constant; $\lambda_a/2$ are the generators of the colour group $SU(3)_c$ and G_μ^a is the gluon field yielding a nonvanishing gluon condensate

$$G^2 =< \mathrm{vac}|\frac{g^2}{4\pi}G_{\mu\nu}^a(0)G_a^{\mu\nu}(0)|\mathrm{vac} >= [330\mathrm{MeV}]^4 \ . \tag{3}$$

The value of the gluon condensate is taken from the [12].

Term $\mathcal{L}_4$ is the effective chiral four- quark Lagrangian of the NJL type

$$\begin{aligned} \mathcal{L}_4 &= \frac{\kappa_1}{2}\left[(\bar{q}\lambda_i q)^2 + (\bar{q}i\gamma_5\lambda_i q)^2\right] - \\ &- \frac{\kappa_2}{2}\left[(\bar{q}\gamma_\mu\lambda_i q)^2 + (\bar{q}i\gamma_5\gamma_\mu\lambda_i q)^2\right] \ . \end{aligned} \tag{4}$$

Here λ_i are the $U(3)$ generators with $\lambda_0 = \sqrt{2/3}\ \mathbf{1}$ ($\mathbf{1}$ is the 3×3 unit matrix) and λ_i with $1 \leq i \leq 8$ are the standard Gell- Mann matrices.

We also take into account the $U(1)_A$ breaking term $\mathcal{L}_6$ based on 't Hooft's instanton- inspired determinant interaction [5, 13]

$$\mathcal{L}_6 = d_D\left[\mathrm{det}\bar{q}(1+\gamma_5)q + \mathrm{det}\bar{q}(1-\gamma_5)q\right] \ . \tag{5}$$

This interaction conserves chiral $SU(3)_L \times SU(3)_R$ but is flavour mixing.

3. The NJL model for the scalar and pseudoscalar mesons

Let us consider only the scalar and pseudoscalar parts of the Lagrangian (4). Upon introducing meson fields the Lagrangian (1) turns into the form

$$\mathcal{L}'(q,G,\tilde{\sigma},\phi) = -\frac{(\tilde{\sigma}_i^2+\phi_i^2)}{2\kappa_1} + \bar{q}(i\hat{\mathcal{D}} - m^0 + \tilde{\sigma} + i\gamma_5\phi)q + \mathcal{L}_6'(q,\tilde{\sigma},\phi) \tag{6}$$

with $\tilde{\sigma} = \tilde{\sigma}^i\lambda_i$, $\phi = \phi^i\lambda_i$. The vacuum expectation values of the scalar fields $\tilde{\sigma}_0, \tilde{\sigma}_3, \tilde{\sigma}_8$ are not equal to zero $< \tilde{\sigma}_{0,3,8} >_0 \neq 0$ [10] (If $m_u^0 = m_d^0$, then $< \tilde{\sigma}_3 >_0 = 0$).

In order to pass to physical fields σ_i with $< \sigma_i >_0 = 0$, it is needed to introduce new quark mass matrix $m = \mathrm{diag}(m_u, m_d, m_s)$. These new quark masses are to be identified with the constituent quark masses

$$-m^0 + \tilde{\sigma} = -m + \sigma \;, \; < \sigma_i >_0 = 0 \;. \tag{7}$$

Considering the vacuum expectation values of both parts of these equations, we get the gap equations (see Fig.1):

$$\begin{aligned} m_u &= m_u^0 - < \tilde{\sigma}_u >_0 = \\ &= m_u^0 - 2\kappa_1 < \bar{u}u >_0 - 2g_D < \bar{d}d >_0 < \bar{s}s >_0 \;, \\ m_s &= m_s^0 - < \tilde{\sigma}_s >_0 = \\ &= m_s^0 - 2\kappa_1 < \bar{s}s >_0 - 2g_D < \bar{u}u >_0 < \bar{d}d >_0 \;. \end{aligned} \tag{8}$$

Here

$$\sigma_u = \frac{\sqrt{2}\sigma_0 + \sigma_8}{\sqrt{3}} \;, \; \sigma_s = \frac{\sigma_0 - \sqrt{2}\sigma_8}{\sqrt{3}} \;. \tag{9}$$

If we suppose $m_u \approx m_d$, then $< \bar{u}u >_0 \approx < \bar{d}d >_0$.

The condensates $< \bar{u}u >_0$, $< \bar{d}d >_0$, $< \bar{s}s >_0$ are given by

$$< \bar{q}q >_0^G = -4m_q I_1^G(m_q) \;, \tag{10}$$

where

$$I_1^G(m_q) = \frac{N_c}{(2\pi)^2}\int_0^{\Lambda_3} d\mathbf{p}\frac{\mathbf{p}^2}{E_q} + \frac{G^2}{48m_q^2} \ , \ E_q = \sqrt{\mathbf{p}^2 + m_q^2} \ . \qquad (11)$$

Here $I_1^{G=0}(m_q)$ is the quadratic divergent integral regularized with a cut-off parameter Λ_3 characterizing the scale of the chiral- symmetry breaking. [1]

Now let us consider the divergent and convergent quark loops described on Fig.2.

For evaluating the quark loops (quark determinants) with the external fields, we shall use the "heat kernel" technique which has been employed in [11]. We shall take into account also the corrections connected with the Lagrangian $\mathcal{L}_6'$.

After regularization of the meson fields we get the Lagrangian $\mathcal{L}(\sigma,\phi,G)$ [7, 8, 10]

$$\begin{aligned}
\mathcal{L}(\sigma,\phi,G) &= \frac{1}{4}\mathrm{Tr}\Big\{(\partial_\mu\phi)^2 + (\partial_\mu\sigma)^2 - (c_{ij}^\sigma\sigma^2 + c_{ij}^\phi Z\phi^2) - \\
&- g_{ij}^2\Big[(\sigma^2 - 2\frac{m}{g_{ij}}\sigma + Z\phi^2)^2 - [(\sigma - \frac{m}{g_{ij}}), Z^{1/2}\phi]_-^2\Big]\Big\} + \\
&+ \mathcal{L}_6''(\sigma,\phi) \ . \qquad (12)
\end{aligned}$$

Here

$$\begin{aligned}
g_{ij} &= \Big\{4I_2^G(m_i,m_j)\Big\}^{-1/2} \ , \\
I_2^G(m_i,m_j) &= \frac{I_1^{G=0}(m_i) - I_1^{G=0}(m_j)}{m_j^2 - m_i^2} + \frac{G^2}{8\cdot 48}\Big(\frac{1}{m_i^2} + \frac{1}{m_j^2}\Big)^2 \ . \qquad (13)
\end{aligned}$$

where g_{ij} is the renormalization coupling constant. $I_2^{G=0}(m_i,m_j)$ is the logarithmic divergent integral regularized with the cut- off parameter Λ_3.

[1] Here we used the three- dimensional cut- off Λ_3 because it is more convenient for introducing the temperature dependence of physical quantities. As a rule, $\Lambda_3 < \Lambda_4$ [5, 6].

$$c_{ij}^{\phi} = \frac{g_{ij}^2}{\kappa_1}\Big\{1 - 4\kappa_1^{\phi}\left[I_1^G(m_i) + I_1^G(m_j)\right]\Big\} = \frac{g_{ij}^2}{2\kappa_1}\Big(\frac{m_i^0}{m_i} + \frac{m_j^0}{m_j}\Big) \quad . \qquad (14)$$

$Z = (1 - 6m_u^2/M_{a_1}^2)^{-1}$ is the additional renormalization of pseudoscalar fields resulting from the nondiagonal transitions of the type $\pi \longrightarrow a_1$ (π is the pion field and a_1 is the axial- vector field, $g_\pi = Z^{1/2}g$).

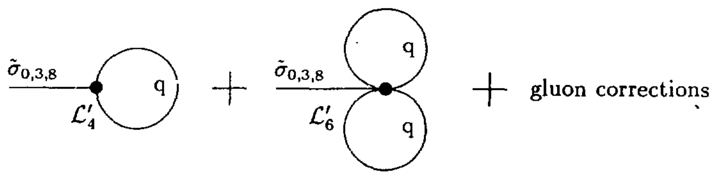

Figure 1: The gap equation for current quarks

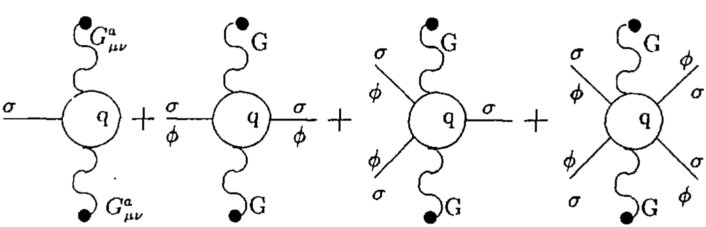

Figure 2: The quark loops with the external fields. $G_{\mu\nu}^a$ is the gluon field, φ is the pseudoscalar field and σ is the scalar field

From (12) and (14) we can see that the pseudoscalar meson masses are equal to zero if the current quark masses are equal to zero.

Taking into account $\mathcal{L}_6(\sigma, \phi)$, we get (see [5]) [2]

$$\kappa_1^{\phi} = \kappa_1 + g_D < \bar{q}q >_0 \tag{15}$$

and instead of $\mathcal{L}_6''$ in (12) there appears an additional term (in the approach quadratic in the meson fields)

$$-\frac{1}{2} d_{\phi} (Z \phi_0^2 + \sigma_0^2) \ , \tag{16}$$

where

$$d_{\phi} = -6g^2 \frac{g_D}{\kappa_1} \frac{< \bar{q}q >_0^2}{m_q} \ . \tag{17}$$

The similar term has been considered in [10] for describing the gluon anomaly.

Now we can get the following mass formulae for the pseudoscalar meson nonets.

$$M_{\pi}^2 = Z c_{uu}^{\phi} = \frac{m_u^0 m_u}{\kappa_1 F_{\pi}^2} \approx -2 \frac{m_u^0}{F_{\pi}^2} < \bar{u}u >_0 \ . \tag{18}$$

It is the Gell- Mann- Oakes- Renner formula. Here, we have used the Goldberger- Treiman identity

$$Z g_{uu}^2 = g_{\pi}^2 = \left(\frac{m_u}{F_{\pi}}\right)^2 \ , \tag{19}$$

where $F_{\pi} = 93$MeV is the pion decay constant.

$$M_K^2 = Z[c_{us}^{\phi} + (m_s - m_u)^2] \ , \tag{20}$$

[2] For the scalar- isoscalar mesons $\kappa_1^{f_0} = \kappa_1^{\phi}$ but for the scalar- isovector mesons $\kappa_1^{a_0} = \kappa_1 - g_D < \bar{q}q >_0$ (see [4]).

$$M^2_{\binom{\eta}{\eta'}} = \frac{Z}{2}\Big\{c^\phi_{uu} + c^\phi_{ss} + d_\phi \mp$$

$$\mp \left[(d_\phi - \frac{c^\phi_{ss} - c^\phi_{uu}}{3})^2 + \frac{8}{9}(c^\phi_{ss} - c^\phi_{uu})^2\right]^{1/2}\Big\} , \tag{21}$$

$$\mathrm{tg}2\theta_\phi = \frac{2\sqrt{2}(c^\phi_{ss} - c^\phi_{uu})}{(c^\phi_{ss} - c^\phi_{uu} - 3d_\phi)} .$$

θ_ϕ is the mixing angle of the singlet– octet components of the η–mesons.

The gluon condensate also gives the singlet– octet mixing for the scalar mesons $f_0 - f'_0$. As a result, we can obtain the masses f_0 and f'_0 from (21), where $Z = 1$, $c^{\sigma_0}_{uu} = c^\phi_{uu} + 4m^2_u$, $c^{\sigma_0}_{ss} = c^\phi_{ss} + 4m^2_s$.

4. Determination of the model parameters

In our model, we have eight *input* parameters:

1. The pion decay constant $F_\pi = 93$MeV.
2. The ρ meson decay constant g_ρ $(g^2_\rho/4\pi \approx 3)$.
3. The gluon condensate $G^2 = (330\text{MeV})^4$.
4. The a_1 meson decay mass $M_{a_1} \approx 1.2$MeV.
5. The pion mass $M_\pi = 140$MeV.
6. The kaon mass $M_K = 495$MeV.
7. The ρ meson mass $M_\rho = 770$MeV.
8. The mixing angle $\theta_\phi = -22^0$.

Using these input parameters we define the following *output* parameters:

1. The current and constituent quark masses $m_u = 300$MeV, $m^0_u = 4.5$MeV, $m_s = 430$MeV, $m^0_u = 92$MeV.

2. The cut- off parameter $\Lambda_3 = 652$MeV.

3. The renormalization constant $Z = 1.6$.

4. The four- quark coupling constants $\kappa_1 = 8.03\mathrm{GeV}^2$, $\kappa_2 = (g_\rho/2M_\rho)^2 = 16\mathrm{GeV}^2$ [10].

5. The 't Hooft coupling constant $g_D = -85\mathrm{GeV}^{-5}$ ($d_\phi = 0.46\mathrm{GeV}^2$).

Now we can describe the quark condensates, the masses of the scalar, pseudoscalar, vector and axial- vector meson nonets, all coupling constants of meson interactions and the behaviour of these quantities as functions of the temperature and chemical potential (see [6, 10, 11]).

Let us show how we define our *output* parameters. Consider three equations:

1. The kinetic relation between the coupling constants g_ρ and g which takes place in the NJL model

$$g_\rho = \sqrt{6}g = \sqrt{\frac{3}{2}}\left[I_2^G(m_u, m_u)\right]^{-1/2} . \tag{22}$$

2. The Goldberger- Treiman relation (19)

$$\frac{m_u}{F_\pi} = g_\pi = \frac{g_\rho}{\sqrt{6}} Z^{1/2} . \tag{23}$$

3. The expression for the renormalization constant Z

$$Z = \left(1 - \frac{6m_u^2}{M_{a_1}^2}\right) . \tag{24}$$

From (23) and (24) we derive the equation for m_u^2 with the solution

$$m_u^2 = \frac{M_{a_1}^2}{12}\left[1 - \sqrt{1 - (\frac{2g_\rho F_\pi}{M_{a_1}})^2}\right] , \quad m_u \approx 300\mathrm{MeV} \tag{25}$$

and then from (22) and (24) we get $\Lambda_3 = 652$MeV and

$$Z = 2\left[1 + \sqrt{1 - (\frac{2g_\rho F_\pi}{M_{a_1}})^2}\right]^{-1} = 1.6 . \tag{26}$$

In order to define the coupling constant κ_1 and the current quark mass m_u^0 we have to introduce the input parameter M_π and to use equations (8) and (18) [3].

[3] At the first step, we can neglect the 't Hooft term in (8).

Then, we get

$$\kappa_1^{-1} = \left(\frac{M_\pi F_\pi}{m}\right)^2 + 8I_1^G(m_u) = 0.125\mathrm{GeV}^2 \ , \ \kappa_1 = 8.03\mathrm{GeV}^{-2} \ , \tag{27}$$

$$m_u^0 = \frac{M_\pi^2 F_\pi^2 \kappa_1}{m_u} = 4.5\mathrm{MeV} \ . \tag{28}$$

Using the mass formula (20) for M_K and the gap equation (3) we obtain

$$m_s = 430\mathrm{MeV} \ , \ m_s^0 = 92\mathrm{MeV} \ . \tag{29}$$

If we use the mixing angle $\theta_\phi = -22^0$ ($\theta_\phi^{\mathrm{exp}} = -17.3 \pm 3.6$ [14]), we obtain from (21) and (17)

$$d_\phi = 0.46\mathrm{GeV}^2 \ , \ d_D = -89\mathrm{GeV}^{-5} \ , \tag{30}$$

$$M_\eta = 544\mathrm{MeV} \ , \ M_{\eta'} = 1039\mathrm{MeV} \ ,$$

$$(M_\eta^{\mathrm{exp}} = 549\mathrm{MeV} \ , \ M_{\eta'}^{\mathrm{exp}} = 958\mathrm{MeV}) \ .$$

Then, using eq.(21) with $Z = 1, c_{uu}^{\sigma_0}, c_{ss}^{\sigma_0}$, we obtain the following masses of the scalar- isoscalar mesons f_0 and f_0': $M_{f_0} = 780\mathrm{MeV}$, $M_{f_0'} = 1160\mathrm{MeV}$, and $\theta_{f_0 f_0'} = -42^0$ [4].

From formula (15) we can see that the 't Hooft correction to κ_1 is equal to 20% .

Now let us consider the quark condensates. The total quark condensate $<\bar{q}q>_0^G$ consists of two parts (see eqs. (10), (11)). The first part does not explicitly contain the gluon condensate G^2

$$<\bar{u}u>_0^{G=0} = \mathrm{Tr}\left(\frac{-i}{i\hat{\partial} - m_u}\right) = -4m_u I_1^{G=0}(m_u) = (-245\mathrm{MeV})^3. \tag{31}$$

The second part includes the gluon condensate corrections

$$\Delta <\bar{u}u>_0^G = -\frac{1}{12}\frac{g^2}{4\pi^2}\frac{<G_{\mu\nu}^a G_a^{\mu\nu}>}{m_u} \ . \tag{32}$$

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[4] Recently, the existence of the scalar state $I = 0$, $0^{++}(750)$ has been supported again [15].

In order to exclude the effect of explicit chiral symmetry breaking by nonzero current quark masses we consider instead of (31) the reduced condensates ${}^R< \bar{u}u >_0^{G=0}$ where we subtract from (31) the values obtained in the limit $\kappa_1 = 0$ (when the constituent quark mass m_q is replaced by m_q^0 see [5, 16]). This effect plays a more important role for $< \bar{s}s >$. Then, we get

$$
\begin{aligned}
{}^R< \bar{q}q >_0^G &= -4m_q I_1^G(m_q) + 4m_q^0 I_1^{G=0}(m_q^0) \\
{}^R< \bar{u}u >_0^G = {}^R< \bar{d}d >_0^G &= (-263\mathrm{MeV})^3 \ , \\
{}^R< \bar{s}s >_0^G &= (-248\mathrm{MeV})^3 \ .
\end{aligned}
\tag{33}
$$

5. The NJL model at finite temperature and chemical potential

Now let us show how these physical quantities change at the finite temperature and finite baryon number density.

The free quark propagator at the finite temperature and finite baryon number density takes the form [6, 7, 17]

$$
\begin{aligned}
S_F(p,T,\mu) &= (\hat{p}+m)\Big[\frac{1}{p^2-m^2+i\epsilon} + \\
&+ 2\pi i\delta(p^2-m^2)\big[\theta(p_0)n(p,\mu)+\theta(-p_0)\bar{n}(p,\mu)\big]\Big] \ .
\end{aligned}
\tag{34}
$$

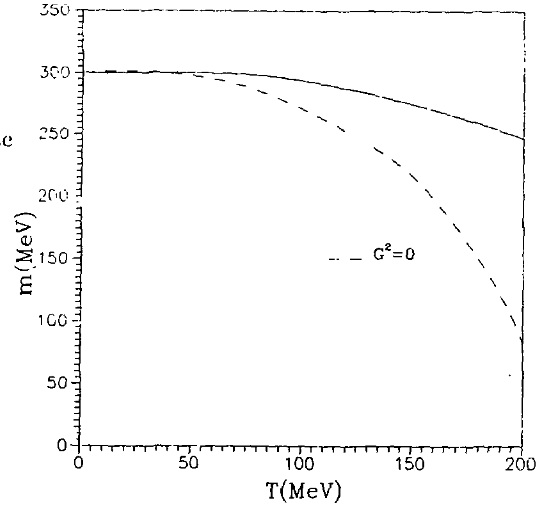
Here $n(p,\mu)$ and $\bar{n}(p,\mu)$ are the Fermi functions for the quark and antiquark.

$$
\begin{aligned}
n(p,\mu) &= \Big[1+\exp\big(\beta(E-\mu)\big)\Big]^{-1} \ , \\
\bar{n}(p,\mu) &= \Big[1+\exp\big(\beta(E+\mu)\big)\Big]^{-1} \ ,
\end{aligned}
\tag{35}
$$

where $\beta = 1/T$, μ is the chemical potential. Then, instead of the integrals $I_1^{G=0}$ and $I_2^{G=0}$ of eqs. (11), (13) we get the following T- and μ- dependent quantities:

$$
I_1^{G=0}(m,T,\mu) = \frac{N_c}{4\pi^2}\int_0^{\Lambda_3} d\mathrm{p}\, \frac{\mathrm{p}^2}{E}(1-n-\bar{n}) \ ,
$$

Figure 3:
The T- dependence of the constituent quark mass m_u with gluon condensate $G^2 \neq 0$ and without gluon condensate $G^2 = 0$



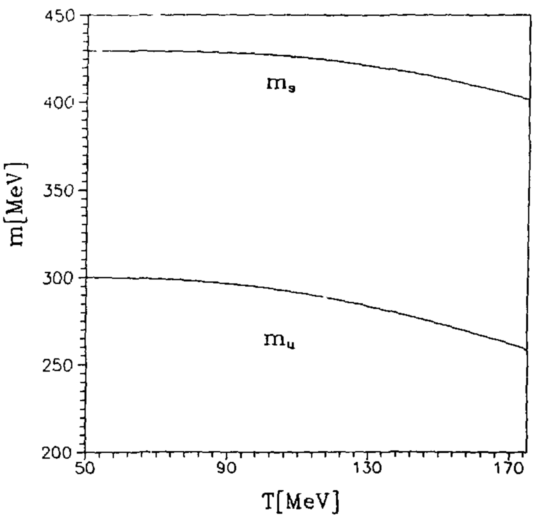


Figure 4: The T- dependence of m_u and m_s with gluon condensate

$$I_2^{G=0}(m_i,m_j,T,\mu) = \frac{N_c}{8\pi^2}\int_0^{\Lambda_3} dp\, \frac{\mathbf{p}^2}{E^3}(1-n-\bar{n}) \;, (m_i = m_j) \;. \tag{36}$$

(for $m_i \neq m_j$ see eq.(13)). The temperature behavior of the gluon condensate has the form [18] [5]

$$G^2(T) = G^2(0) - \frac{\pi^2}{135}\frac{N_f^2(N_f^2-1)}{(11N_c-2N_f)}\frac{T^8}{F_\pi^4}\left\{\ln\frac{\Lambda_P}{T}-\frac{1}{4}\right\} + \dots \;, \tag{37}$$

where $\Lambda_P = 275\text{MeV}$ is the logarithmic scale occurring in the temperature expansion of the pressure P.

Now using eqs. (36) and (37) for I_1, I_2 and G^2 we can obtain the temperature dependence of all physical quantities – the constituent quark masses m_u and m_s, quark condensates $<\bar{u}u>$, $<\bar{s}s>$, decay constants F_π and F_K, meson masses and the mixing θ_ϕ (see Figs.1-8). In our approach we neglect a possible temperature dependence of κ_1, g_D and Λ_3 [6].

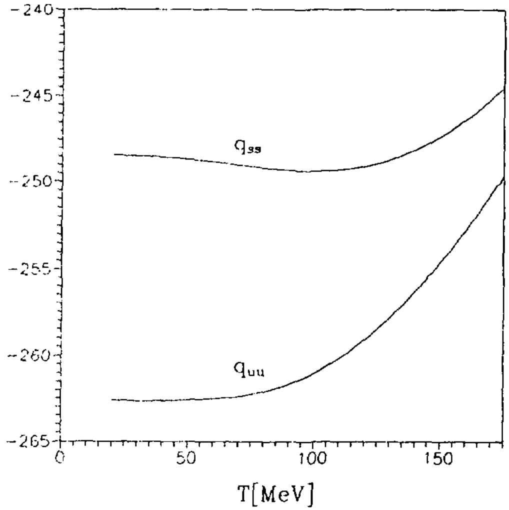
6. Discussion

Figs. 1–8 show the T– dependence of the main physical quantities in our model. The behaviour of $m_u(T)$ is shown in Figure 1. It has been found by a self- consistent solution of the thermal gap equation (8) with I_1 and G^2 given by (36), (37). For comparison we give the behaviour of this value also for the case where the gluon condensate is equal to zero. We can see that the gluon condensate plays a stabilizing role for the behaviour of the constituent quark mass (and other physical quantities) at changing temperature in the region $0 < T < 150\text{MeV}$. This takes place because the term with the gluon condensate plays a more and more important role when the temperature increases (in the region $0 < T < 150\text{MeV}$). Indeed, this term has the weakly changing

[5] Eq. (37) is legitimate in the region $0 < T < 150\text{MeV}$. The dependence on μ is neglected. The weak dependence of the gluon condensate on the temperature was found also in the $SU(2)$ lattice gauge theory in the papers [19, 20].

[6] The temperature dependence of g_D was investigated in [21], where it was shown that g_D rapidly decreases when temperature increases.

Figure 5:
The T- dependence of the quark condensates;
$q_{uu} \equiv < \bar{u}u >_0$,
$q_{ss} \equiv < \bar{s}s >_0$



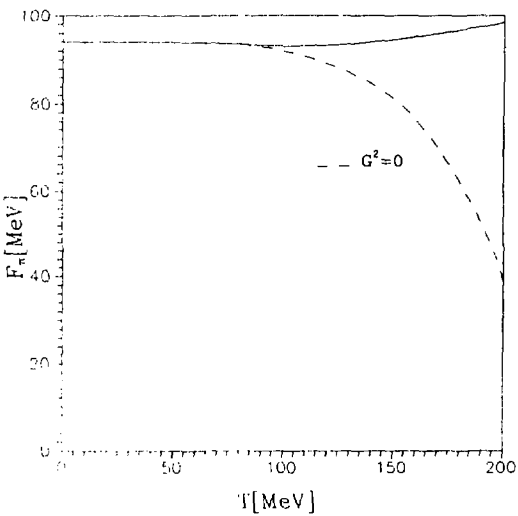
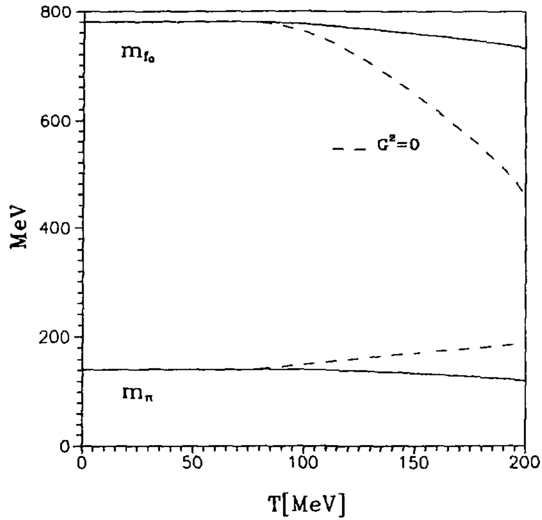


Figure 6: The T- dependence of the pion decay constant F_π with gluon condensate $G^2 \neq 0$ and without qluon condensate $G^2 = 0$

Figure 7:
The behaviour
of the meson
masses m_{f_0}, m_π
as functions
of temperature T



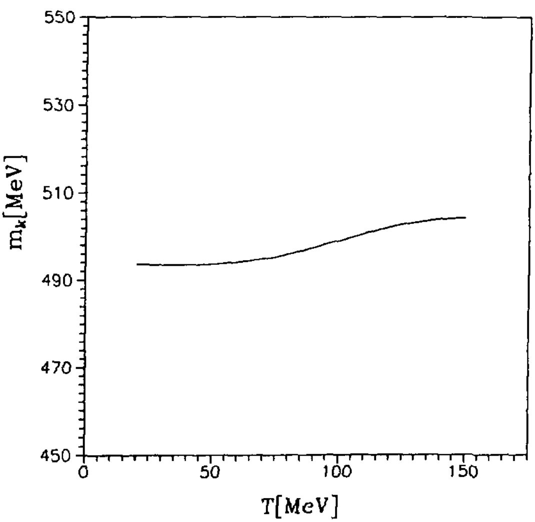
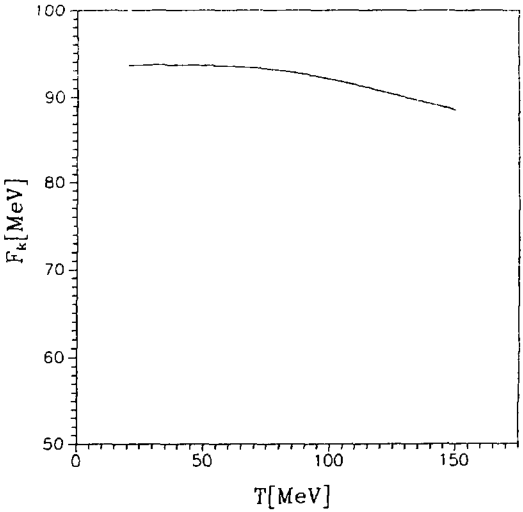


Figure 8: The behaviour of the kaon mass m_K as functions of temperature T

Figure 9:
The T- dependence
of the kaon decay
constant F_K



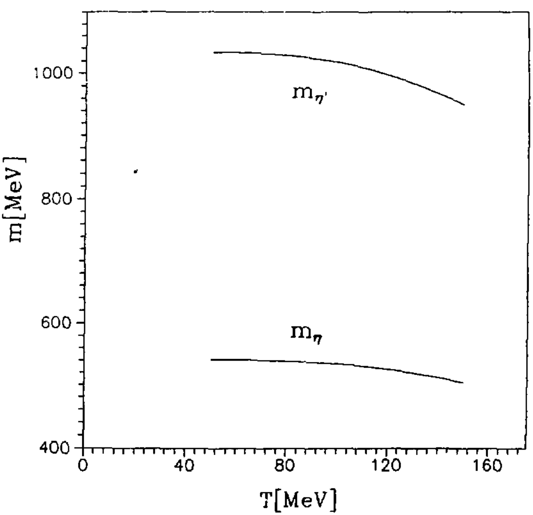


Figure 10: The behaviour of the masses m_η and $m_{\eta'}$ as functions of temperature T

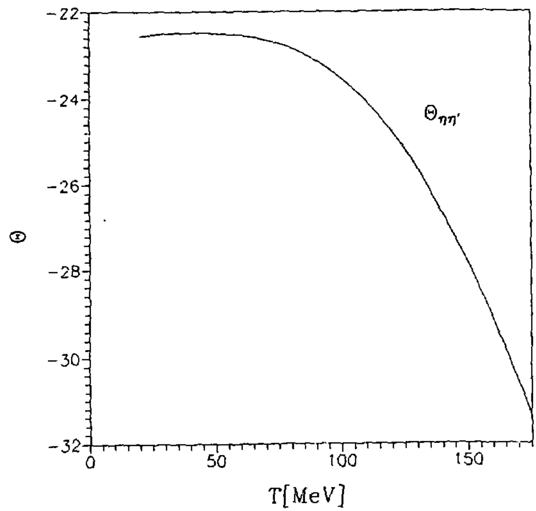


Figure 11: The T- dependence of the mixing angle $\theta_{\eta\eta'}$

with T function $G^2(T)$ in the numerator and a more rapidly decreasing function m_q^2 in the denominator (see eqs. (8), (10), (11)).

Figs. 2,3 show the T– dependence of the m_u, m_s and quark condensates $< \bar{u}u >_0$, $< \bar{s}s >_0$. Figs. 4,5 show the behaviour of F_π, M_π and M_{f_0} with G^2 and without G^2. In Fig. 6 we can see the behaviour of the kaon mass M_K and F_K. It is interesting that the last behaviour is closer to the behaviour of M_π and F_π without G^2. Indeed, in this case we have heavier quark masses in the denominator of the term with G^2 and thus the influence of this term is weaker.

Figs. 7,8 show the T– dependence of the masses of the η, η' mesons and the mixing angle θ. The absolute value of θ increases in the interval $0 < T < 150$MeV. At the critical temperature T_c, d_ϕ is equal to zero and θ coinsides with the ideal mixing angle $\theta^0 = 35.3^0$ ($\sin\theta^0 = 1/\sqrt{3}$).

In conclusion, we would like to note that here we have obtained the results which can be used only at low temperatures. Indeed, we consider the G^2– approximation following the work [22]. However, the G^2 terms rapidly increase with the temperature. So at a high temperature it is necessary to consider the whole function $f(G^2)$ but not only the first term of its expansion. This function can be found using, for instance, the scale invariance (see works [23]- [26]). In the future we want to investigate this problem.

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