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REALISTIC NN POTENTIAL FOR THE LOW APPROACH

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1. Introduction

Meson-theoretical approaches to low energy NN physics remain the only quantitative methods until the present days. Notwithstanding the considerable success in this direction, an intensive work in the field is continued. The main reason of such a nonreducible interest is *simplicity*, a general principle in physics. Meson theory provides a very simple description of the low energy NN picture by the exchange of a few low-lying mesons mediating an internucleonic interaction. Then, it is a reality of nowadays that baryons and mesons are the only hadrons visible in experiment; so, in this sense, the meson-exchange model looks like an *elementary* process as well. And at last, the realistic meson-nucleon picture can serve as a serious test for the actually *elementary* quark-gluon dynamics of the generally accepted fundamental theory QCD of strong interactions in this energy region.

There already exist a few realistic OBE models [1, 2, 3] successfully describing nuclear matter as well as NN bound state. Hence, one may ask, why the construction of a new schemes like those are still of interest?

All the existing approaches, in general, differ by their energy dependence, off-shell behavior or by prescription for potential construction (number of exchanged mesons, antiparticles etc.). Whereas an energy-dependent scheme is more appropriate near thresholds (e.g. for meson production above the corresponding threshold values), an instantaneous approach is more efficient in few- and many-body bound-state problems. In various versions of the Bonn OBE model [4] or of Gross approach [2] the resulting T-matrices coincide on-shell, but have a quite different off-shell behavior. However, more often than not the different off-shell behavior of the interaction has no physical consequences; it may reflect the fact that a different calculational scheme is used to relate the meson-baryon couplings to the physical observables [5].

With time it becomes obvious that the convenience of a specific calculational scheme depends on the issue under study. A nice example is the role and amount of three-body forces or meson-exchange currents which can differ remarkably in different schemes.

The present paper deals with one of the approaches to the low energy meson-nucleon physics [6] based on the spectral decomposition of the scattering T-matrix, being a field-theoretical case of the Low equations [7]. Usually πN scattering is considered within the frame of this approach [8, 9, 10] NN scattering problem was suggested in the paper [11], where, however, several assumptions were made. The present paper concerns those assumptions and thus completes our scheme of realistic description of low energy NN-scattering. That is, at this stage we have applied the constructed potential neither to nuclear matter nor to deuteron problem, though it will be of considerable interest, because the dominant contribution to the potential is presented by the equal-time interaction, while the effects of retardation, nucleon off-shellness or of real meson exchanges are separated in other terms which, as we will see below, can be considered as correction contributions. So, our goal is just to estimate these contributions (as in [11]) which complicate the potential and usually arise when the three-dimensional approach to the problem is considered.

In our previous work [11] we have suggested a relativistic three-dimensional Lippman-Schwinger (LS) type equation for the NN scattering T-matrix

$$T(\mathbf{p}',\mathbf{p};E) = U(\mathbf{p}',\mathbf{p};E) + \int U(\mathbf{p}',\mathbf{p}'';E)\frac{d\mathbf{p}''}{E(\mathbf{p}'') - E(\mathbf{p}) + i\epsilon}T(\mathbf{p}'',\mathbf{p};E), \quad (1)$$

derived from the field-theoretical Low equation. The potential of this nonlinear integral Low equation W=Y+V uniquely determines the energy-dependent potential of the suggested LS type equation Eq. (1)

$$U(\mathbf{p}',\mathbf{p};E) = <\mathbf{p}',-\mathbf{p}' \mid Y+V \mid \mathbf{p},-\mathbf{p}> = A(\mathbf{p}',\mathbf{p}) + EB(\mathbf{p}',\mathbf{p}), \quad (2)$$

where $E = 2E(\mathbf{p}) = 2\sqrt{M^2+\mathbf{p}^2}$ for half-energy-shell amplitude, and A and B are hermitian matrices [11]. So, the potential U Eq. (2) consists of two parts: the first part

$$Y = \langle p' \mid \{J(0), b^+(0)\} \mid p \rangle, \qquad (3)$$

equal-time anticommutator (often called as seagull term), when calculated with the use of the simplest phenomenological meson-nucleon Lagrangians, reproduces the One Boson Exchange (OBE) NN potential, while the second part -

$$\langle p_1'p_2' \mid V \mid p_1p_2 \rangle = \qquad (4)$$

$$\sum_{m=\pi,\rho..} \langle p_1' \mid \bar{J}_{p_1}(0) \mid m \rangle \frac{\delta^{(3)}(p_1' - p_1 - P_m)}{E_{p_1'} - E_{p_1} - E_m} \langle m \mid J_{p_2'}(0) \mid p_2 \rangle$$

$$- \sum_{m=\pi,\rho..} \langle 0 \mid J_{p_2'}(0) \mid p_2, m \rangle \frac{\delta^{(3)}(p_1 - p_1' - P_m)}{E_{p_1} - E_{p_1'} - E_m} \langle m, p_1' \mid \bar{J}_{p_1}(0) \mid 0 \rangle.$$

is defined by the on-mass-shell meson-exchange terms and contains the meson-nucleon vertex functions with one nucleon off-mass-shell.

In that work [11] we have done the following assumptions:

a) we did not include some terms in the NN potential arose from evaluation of the equal-time anticommutator Y,

b) neglected the retardation in the V-term, i.e. took a transferred three-momentum $-t = -(p' - p)^2$, and also

c) neglected the second term of the vector meson propagators

$$-g^{\mu\nu} + (p' - p)^\mu (p' - p)^\nu / m_v^2 \qquad (5)$$

not vanishing in both Y and V -terms of the potential. Hereafter, for convenience, we'll refer to this term as a vector meson "gauge" term. In the case of Y it remains because a nucleon current is not conserved, while in V it does't vanish even the retardation is presented: in the Low-type NN potentials the above mentioned term contributes due to the off-shell nucleon in the meson-nucleon vertex functions. In our later paper [12] we have evaluated this gauge term for the V potential and estimated its contribution to the NN phase shifts and polarization characteristics.

So, in the present work we shall evaluate the rest Y_2 term of the Y anticommutator and take into account the retardation effect in V.

2. Equal-time anticommutator

Let us begin with the equal-time anticommutator Eq. (3) and present it in the form $Y = Y_1 + Y_2$, where Y_1 was evaluated in ref. [11] (Y_{ps}^2 of ref. [11] is already included in Y_1), Y_2 is just a term we had dropped in ref. [11], and as it will be shown below, leads to a four-fermion contact interaction.

To exhibit the origin of these terms let us recall the Lagrangian exploited in our approach. For this purpose it is enough to present it's part corresponding to vector mesons

$$\mathcal{L}_{NN v} = \frac{1}{2} m_v^2 \varphi_\mu^2 - \frac{1}{4}(\varphi_{\mu\nu})^2 + g_v \bar{\Psi}\gamma^\mu \Psi \varphi_\mu + \frac{f_v}{4M} \bar{\Psi}\sigma^{\mu\nu}\Psi\varphi_{\mu\nu} \qquad (6)$$

where

$$\varphi_{\mu\nu} = (\partial_\mu \varphi_\nu - \partial_\nu \varphi_\mu).$$

Then, the nucleon current in Eq. (3) one can present in the form

$$J_v = A_v \Psi \qquad (7)$$

with

$$A_v = g_v \gamma^\mu \varphi_\mu + \frac{f_v}{4M} \sigma^{\mu\nu} \varphi_{\mu\nu}$$

The Lagrangian Eq. (6) is a singular one: there are only three independent components of the vector meson field φ_μ and therefore a constraint relation exists. The E-L equation

$$(\Box + m_v^2)\varphi_\mu + \partial_\mu \partial^\nu \varphi_\nu = g_v \bar{\Psi}\gamma_\mu \Psi + \frac{f_v}{4M} \partial_\nu (\bar{\Psi}\sigma_{\mu\nu}\Psi) \qquad (8)$$

then yields the explicit constraint expressions

$$\varphi_0 = \frac{1}{m_v^2}(-\partial^k \pi_k - \frac{g_v}{2} \bar{\Psi}\gamma_\mu \Psi)$$

$$\varphi_{0k} = \pi_k + \frac{f_v}{4M} \bar{\Psi}\sigma_{0k}\Psi \qquad (k = 1, 2, 3) \qquad (9)$$

where

$$\pi_k = \frac{\partial \mathcal{L}_v}{\partial \dot{\varphi}_k}.$$

Hence, $A = A(\Psi)$ in Eq. (7) and the anticommutator Eq. (3) splits into two parts

$$\{J, b^+\} = \{A\Psi, b^+\} = A\{\Psi, b^+\} + [b^+, A]\Psi \qquad (10)$$

corresponding to Y_1 and Y_2 respectively.

Considering the rest part of the Lagrangian one can immediately conclude that scalar interaction doesn't contribute to the Y_2, while for the pseudovector interaction we just have the following constraint

$$\partial_0 \varphi = \pi_\varphi - g_{ps} \bar{\Psi} \gamma^5 \gamma^0 \Psi$$

So, evaluating Y_2 commutator for the vector and pseudoscalar mesons one can reduce it to the form

$$Y_2 = \left(c_{ps} \left(\frac{g_{ps}}{2M}\right)^2 + c_v \left(\frac{g_v}{2m_v}\right)^2 + c_{vt} \left(\frac{f_v}{4M}\right)^2 \right) \bar{u}(-\mathrm{p}\prime) \Gamma^\alpha u(-\mathrm{p}) < \mathrm{p}\prime \mid j_\alpha \mid \mathrm{p} > \qquad (\alpha = s, ps, v); \qquad (11)$$

where c_α are the combinations of istopic and Firtz coefficients,

$$< \mathrm{p}_{1\prime} s_{1\prime} \mid j_\alpha \mid \mathrm{p}_2 s_2 >= g_\alpha \bar{u}_{\mathrm{p}_{1\prime}, s_{1\prime}} u_{\mathrm{p}_2 s_2} \Lambda_\alpha(t), \qquad (12)$$

with

$$\Lambda_\alpha(t) = \left[\frac{\Lambda_\alpha^2 - m_\alpha^2}{\Lambda_\alpha^2 - t} \right]^{n_\alpha}, \qquad (13)$$

which was used for the evaluation of the Y_1 term (eq. (36) of [11]).

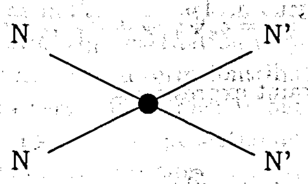

Fig. 1. Four-nucleon contact interaction.

As it is seen Y_2 doesn't lead to the boson-exchange interaction. In the other words Y_2 matrix elements have no dependence on $t = -(\mathrm{p}\prime - \mathrm{p})^2$, except of the form factors $\Lambda_\alpha(t)$. So, one can consider it as a contact interaction of four nucleons Fig. 1. It is not surprising: though the OBE Lagrangian doesn't contain the explicit four-fermion interaction, it effectively exists (for example in Eq. (6)) via the constraint relations Eq. (9).

There is one more contribution to the Y part of the potential. The contribution from the vector meson gauge term Eq. (5) corresponding in the configuration space to the second term in the l.h.s. of Eq. (8) doesn't vanish when 3-dimensional transferred momentum $t = -(\mathrm{p}\prime - \mathrm{p})^2$ is considered. This contribution has been calculated in ref. [13].

3. Off-shell nucleon contribution

It was stated in ref. [11], that the second term V of the potential Eq. (2) corresponds to the meson-exchange contributions too, but it is built by means of the off-shell nucleon vertex functions. The difference between the Y and the relativistic V-parts causes in the meson-nucleon vertex functions and in retarded meson propagators and matrix elements of V (Notice, that there is an opportunity to include the antinucleon degrees of freedom in V when cutting the full set of intermediate states [11]). On the energy shell when both, initial and final, nucleons are restricted on their mass-shell the contribution of V potential does vanish. Therefore, this part of the potential stands for the pure off-mass-shell contribution of the nucleons and thus affects the on-shell scattering amplitude and NN observables via the unitarazing equation.

Retardation is the most characteristic feature of this part of the potential differing it from the equal-time seagul term – Y. Except of the retardation effect one have to take into account the contribution arising from the vector meson gauge term Eq. (5). In the case of V potential it does not vanish due to the off-shell nucleon at the meson-nucleon vertex. As it was pointed out, this contribution had been estimated in ref. [12]. Now, we'll exermine this term taking into account the retardation.

4. Numerical results and summary

Considering the numerical results, first of all it is to be noted that in the case when retardation is included, stronger cut-off form factors are necessary. This is due to the well-known fact that the retarded sum converges more slowly to the physical result than the equal-time sum. Hence, we have replaced the monopole form factors with dipole ones and fitted NN phase shifts to experimental data. The resulting NN phases are depicted in fig 2 and the parameters of the exchanged mesons are given in table 1.

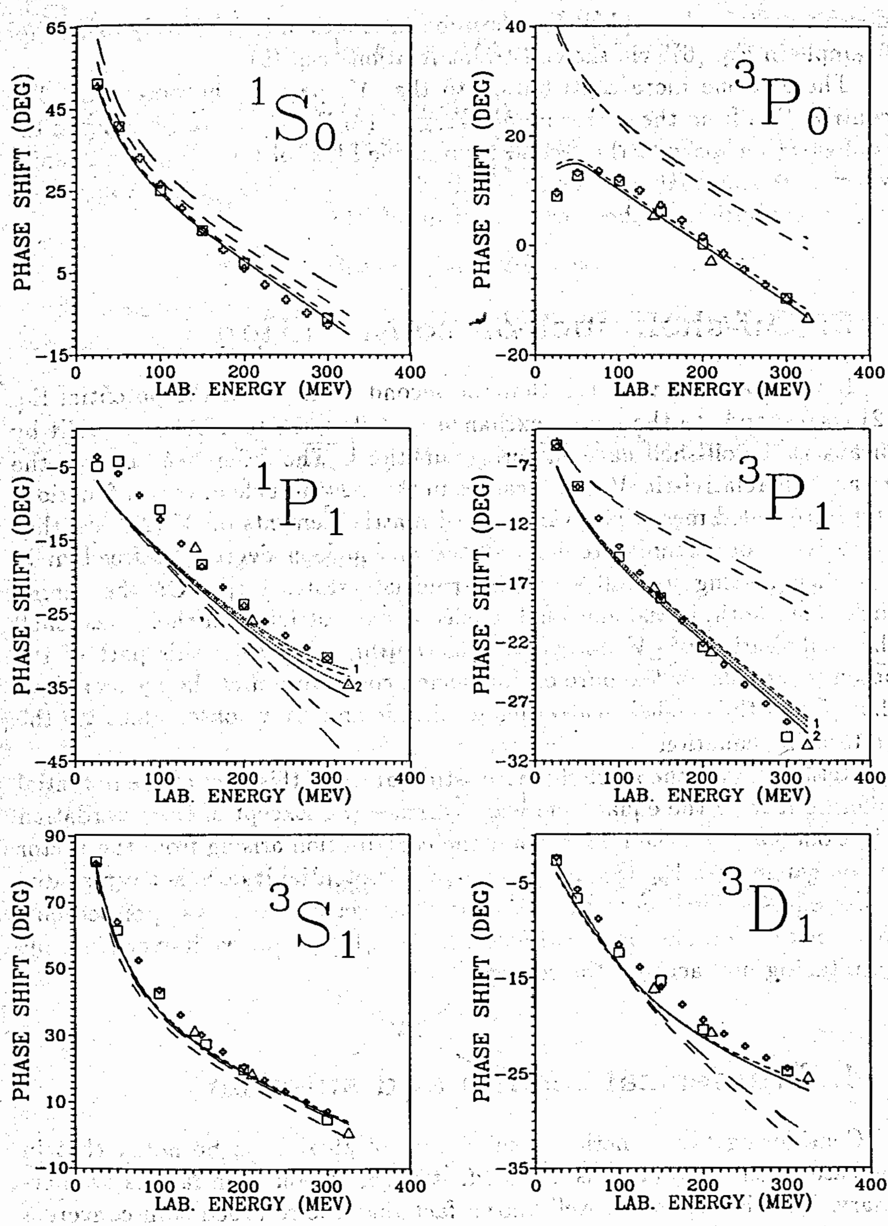

Fig. 2. NN phase shifts. Crists and boxes refers to the energy-dependent and energy-independent phase-shift analysis of Arndt et al.[14] respectively, and triangle to the energy-independent analysis of Bugg et al.[15]. Curves are explained in the text.

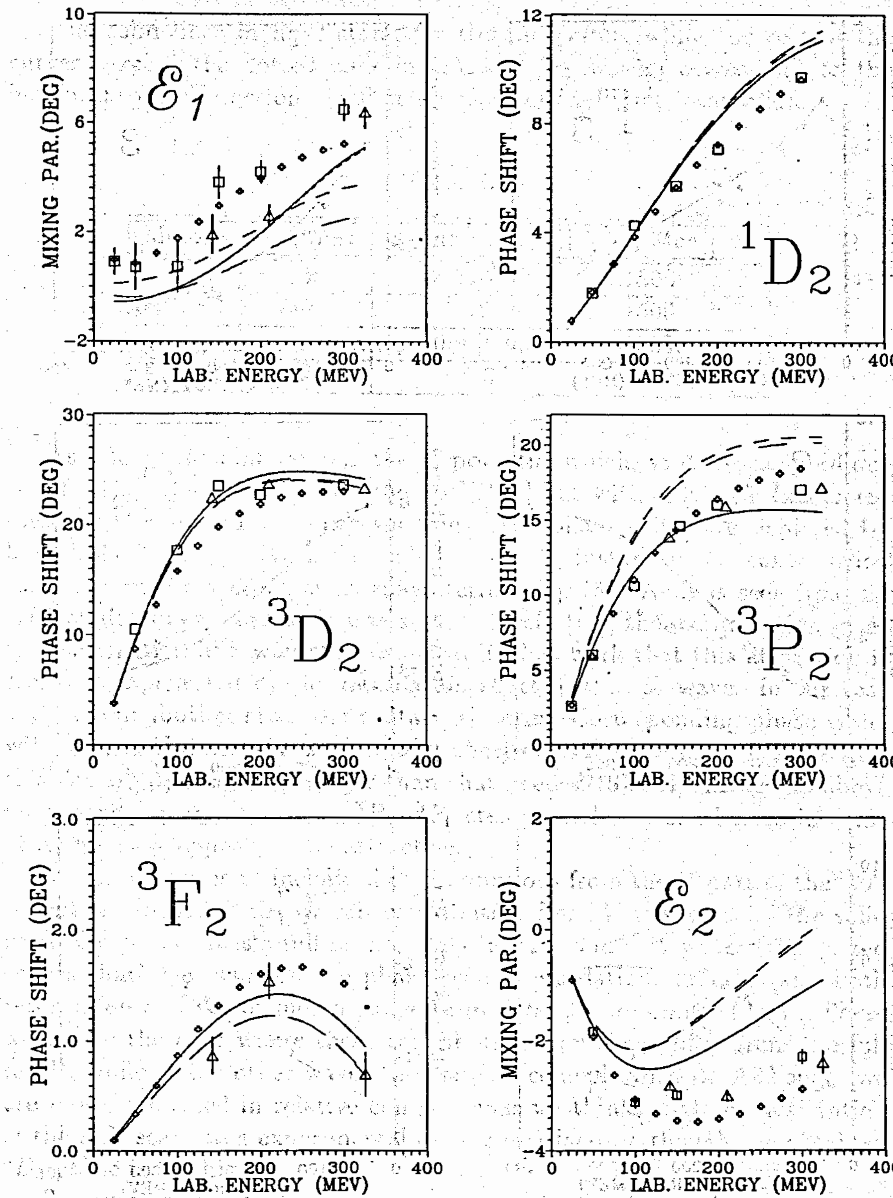

Fig. 2. (Continued)

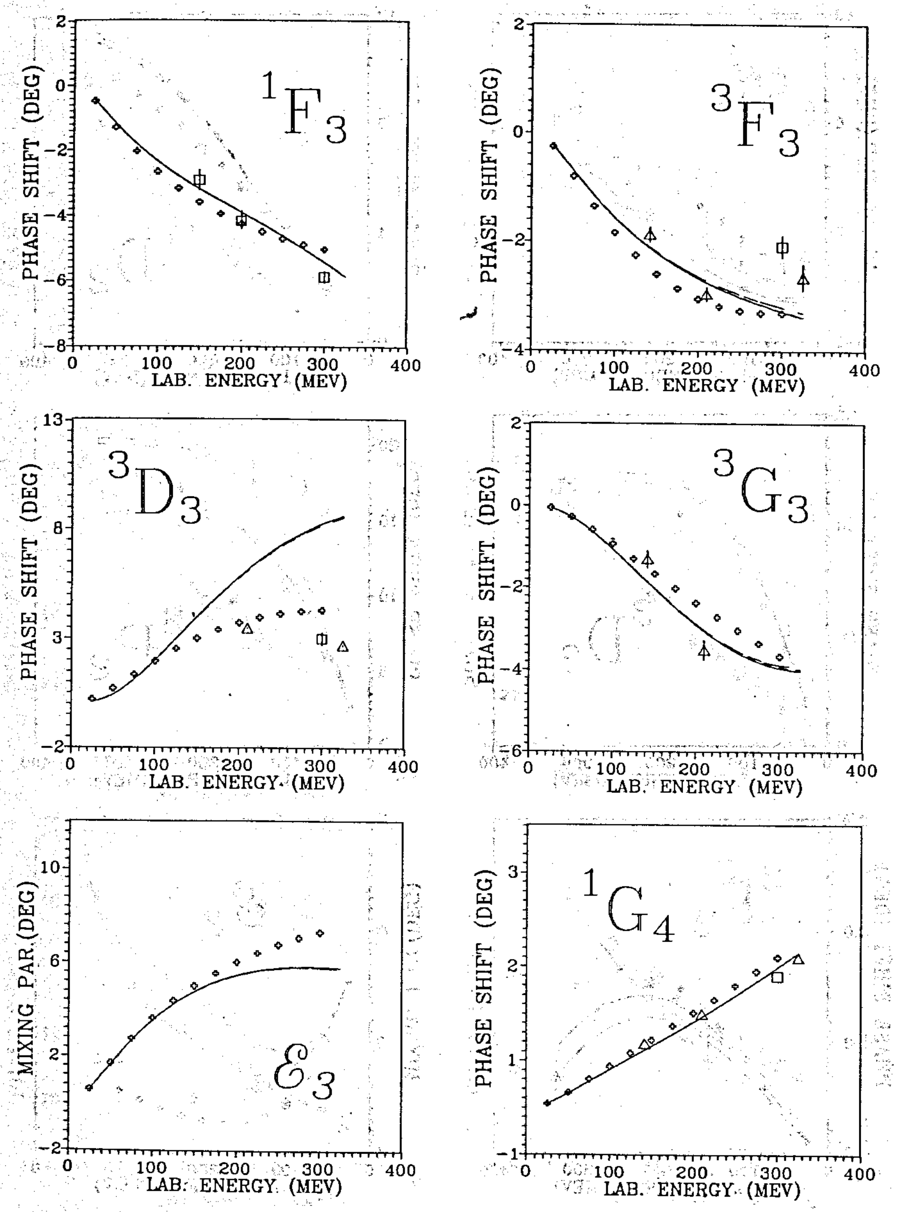

Fig. 2. (Continued)

The solid lines in fig 2 stand for the full result, while the rest of the curves (except the dotted ones in $^1P_1$ and $^3P_1$ waves) correspond to the instantaneous interaction. At first we consider the latter interaction.

Table 1.

Mesons	m_α, Mev	$g_\alpha^2/(4\pi)^2$, $[f_v/g_v]$	Λ_α, Mev	n_α
π	138	14	1500	2
ω	782.6	12	1500	2
ρ	769	0.9 [6.0]	1650	4
σ	520	6.6	1500	2

As a starting point we took the Y_1 potential which, as it was pointed out in ref. [11], almost reproduces the OBEP [1] but with only four exchanged mesons π, σ, ρ, and ω. Corresponding NN phase shifts are depicted by long-dashed curves. Then, we added to Y_1 the instantaneous contribution arising from the vector meson gauge term [2] Eq. (5). As it is seen from fig 2 (dash-dot lines), and as it was stated in ref. [13], this contribution gives some attraction in S-waves. It is shown in that work that this attraction is almost compensated by the retardation effect in the $^1S_0$-wave. In our case such a contribution gives the contact Y_2 term. Corresponding phase shifts with added Y_2 term are depicted by short-dashed curves. It is seen that this contribution is much larger than that needed to compensate the above mentioned attraction in $^1S_0$, $^3P_0$, $^3P_1$ etc. partial waves, whereas in some other waves it appears to be attractive.

The next step is to include the contributions from the V part of the NN potential. In fig. 2 the overall contribution from V is shown by the solid lines. Off course, it is a full result. Only in $^1P_1$ and $^3P_1$ waves (the dotted curves) have we depicted the phases when retardation (label 1) and both retardation and vector meson gauge term (label 2) are omitted in V. These waves are the ones where these contributions somehow differ from the full result, while in the other waves they almost coincide with it. Although, we are more interested in relative contributions we think, that the description of the NN scattering experimental data is satisfactory, though the obtained $^3D_3$ phase is too high.

So, even with a first-glance view, one immediately concludes – almost the whole interaction occurs by the seagull term. Contributions from the V potential even in the low-lying partial waves are very small. The meson parameters (Table 1) are like those in OBEP, though here we have only one

[2] Note, that this term exists due to the particular lagrangian exploited in OBE models. If one takes, for example, $1/2\, \partial_\mu\varphi_\nu\partial^\mu\varphi^\nu$ instead of $\varphi_{\mu\nu}^2 = (\partial_\mu\varphi_\nu - \partial_\nu\varphi_\mu)^2$ then this term vanishes.

set of scalar meson adjustable parameters, as in ref [2]. One should also note the relatively low value of the omega coupling $g_\omega^2/(4\pi)^2$=12. It is mainly due to the repulsion caused by the contact interaction. There is one more free parameter in the above mentioned work [2]. It is the ratio of tensor to vector coupling f_v/g_v for ω mesons (usually it is set to zero in other OBEP). Then, we have examined our result taking Gross's value f_ω/g_ω=0.14 for this parameter. The result is in fig 3.

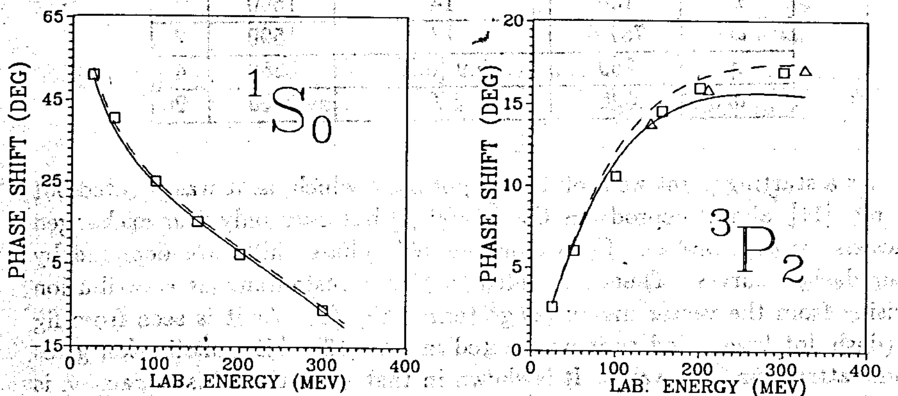

Fig. 3. With the solid lines full result is shown as in fig. 2, while the dash curves correspond to the phases when the f_ω/g_ω=0.14 contribution is added.

It leads to the only significant contribution in the $^3P_2$ wave, while in other waves this contribution is of the same order as in the $^1S_0$ wave, or less. That is, the introduction of the f_ω/g_ω contributes further to the lowering of the ω coupling.

It is to be noted that influence of the contact term contribution on the ^3P waves and on the behavior of ε_1 mixing parameter indicates a significant change of a total tensor force. Therefore, the application of this potential to the deuteron problem will be of great interest.

The main shortcoming of the constructed potential is that we have omitted retardation in the Y_1 term. Though, it is a part of the *equal-time anticommutator*, the fourth component of the four-momentum transfer is not zero. Retardation in Y_1 term leads to a great amount of attraction especially in the $^3S_1$ partial wave and it seems to us that one have to include the antinucleon degrees of freedom when working with the retarded Y_1 potential, as it was done in [2].

So, with the usual Lagrangian exploited in OBE models, we have evaluated the (previously neglected) second Y_2 term (for vector and pseudovector mesons, only) of the equal-time anticommutator Y using the same meson-nucleon vertex functions as that used in the construction of the Y_1 potential. Namely, for Y_1 we have

$$Y_1 \sim \bar{u}(-p\prime)\Gamma^\alpha u(-p) < p\prime \mid \varphi_\alpha \mid p >,$$

while for Y_2,

$$Y_2 \sim \bar{u}(-p\prime)\Gamma^\alpha u(-p) < p\prime \mid j_\alpha \mid p >,$$

$$(\alpha = s, ps, v).$$

Therefore, there is no particle propagation in Y_2 and we conclude it to be an instantaneous contact interaction of four nucleons. The four nucleon contact interaction is one characteristic feature of the Low approach NN potential and, as it is seen from the fitting results, leads to considerable repulsion thus lowering the ω coupling. Another feature of this potential is a real-meson-exchange term - V, which turned out to give a small contribution even in the low-lying waves. The contributions to the V, such as retardation and/or vector meson gauge term, may be safely neglected. We are making such a statement because we have included all the possible contributions to the NN potential except the retardation in the Y_1 term. We think, that it is necessary to introduce the antinucleon degrees of freedom when one calculates the Y_1 term with retardation.

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