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TWO-DIMENSIONAL BLACK HOLES WITH TORSION

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Recently much attention has been payed to the investigation of two-dimensional dilaton gravity. This is mainly inspired by string theory, and also by the fact that it gives the simplest model for the dynamical description of a two-dimensional gravity [1-5], the gravitational variables are dilaton and metric fields $(\phi, g_{\mu\nu})$. In empty (without matter) space the classical equations of motion are exactly integrated [1-3] and the solution discribes the two-dimensional black hole. On the quantum level it was shown [4] that this model is renormalizable. Since in two dimensions many things are simpler and models (classical and quantum) become solvable, one can consider the 2D dilaton gravity as "toy model" for the study of old problems of black hole formation and evaporation [5].

On the other hand, numerous recent attempts to formulate the theory of gravity in the framework of a consistent gauge approach resulted in constructing the gauge gravity models for the de Sitter and Poincaré groups (for a review see, e.g., [6]). The independent variables are now vielbeins $e^a = e^a_\mu dx^\mu$ and Lorentz connection one-form $\omega^a{}_b = \omega^a{}_{b,\mu} dx^\mu$. The application of these methods in two dimensions was justified by attempts to give an alternative description of two-dimensional dynamical gravity in terms of variables $(e^a, \omega^a{}_b)$. It was argued also that investigation of simple two-dimensional model leads to a better understanding of four-dimensional gravity and its quantization [7]. It was shown in [7] that the Lagrangian $L = \gamma R^2 + \beta T^2 + \lambda$ is the most general one quadratic in curvature R and torsion T, and containing a cosmological constant λ. The classical equations of motion were analized in conformal gauge [7] and in light cone gauge [8] and their exact integrability was demonstrated.

In this note we will consider the model for two dimensional de Sitter gravity. The constants γ, β, λ are fixed in this case with only one free parameter α^2 and the action is of the Yang-Mills type [6]. We will show that the exact solution of equations of motion are most easily found in coordinates given by torsion's components (the generalization of result to the case of arbitrary constants γ, β, λ is straightforward). For certain choice of integrating constant this solution is of the black hole type.

1. In two dimensions the gauge gravity is described in terms of zweibeins $e^a = e^a_\mu dz^\mu, a = 0, 1$ (the 2D metric on the surface M^2 has the form $g_{\mu\nu} = e^a_\mu e^b_\nu \eta_{ab}$) and Lorentz connection one-form $\omega^a{}_b = \omega \varepsilon^a{}_b$, $\omega =$

$\omega_\mu dz^\mu$ ($\varepsilon_{ab} = -\varepsilon_{ba}$, $\varepsilon_{01} = 1$). The de Sitter curvature two-form $\mathcal{R}$ [6] in two dimensions takes the form:

$$\mathcal{R} = \begin{pmatrix} R\epsilon^a{}_b + \alpha^2 e^a \wedge e_b & \alpha T^a \\ \alpha T_b & 0 \end{pmatrix}.$$

where α is the coupling constant, and curvature and torsion two-forms are: $R = d\omega$, $T^a = de^a + \varepsilon^a{}_b\omega \wedge e^b$.

The dynamics of gravitational variables (e^a, ω) is determined by the action of the Yang-Mills type [6]:

$$S = \int_{M^2} \frac{1}{4} Tr * \mathcal{R} \wedge \mathcal{R}$$

$$= \int_{M^2} \frac{\alpha^2}{2} * T^a \wedge T^a + \frac{1}{2} * R \wedge R - \frac{\alpha^4}{2}\varepsilon_{ab}e^a e^b + 2\alpha^2 R \qquad (1)$$

where $*$ is the Hodge dualization. The last term in (1) is the boundary one and it does not affect the equations of motion.

Let us consider variables $\rho = *R$ and $q^a = *T^a$. Then using identity $\varepsilon_{ab}\varepsilon^{cd} = (\delta_a^d\delta_b^c - \delta_a^c\delta_b^d)$ it is easy to show that the action (1) takes the form:

$$S = -\tfrac{1}{4}\int[\alpha^2 q^2 + (\rho + \alpha^2)^2]e^a\varepsilon_{ab}e^b,$$

which is positive in euclidean signature; here $q^2 = q^a q^b \eta_{ab}$ ($\eta_{ab} = diag(+1, -1)$).

Variation of action (1) with respect to zweibeins e^a and Lorentz connection ω leads to the following equations of motion:

$$d\rho = -\alpha^2 q^a \varepsilon_{ab} e^b \qquad (2)$$

$$\nabla q^a = -\frac{1}{2\alpha^2}[\rho^2 + \alpha^2 q^2 - 2\alpha^4]\varepsilon^a{}_b e^b, \qquad (3)$$

where $\nabla q^a \equiv dq^a + \omega\varepsilon^a{}_b q^b$.

2. One particular solution of (2)-(3) is evident. Assuming $q^2 = constant$ one gets from (2)-(3), provided e^a are linearly independent everywhere on M^2):

$$\rho^2 = \alpha^4,\ q^a = 0$$

in all points of the two-dimensional manifold. That is, torsion is zero and M^2 is the de Sitter space.

Let now q^2 be nonconstant, and hence non zero identically everywhere in M^2. Then from eqs.(2)-(3) we have the following equation connecting q^2 and ρ:

$$\frac{dq^2}{d\rho} = \frac{1}{\alpha^4}\Phi, \tag{4}$$

where $\Phi(\rho, q^2) = \rho^2 + \alpha^2 q^2 - \alpha^4$.

The solution of this equation has the form:

$$q^2(\rho) = -\frac{1}{\alpha^2}(\rho + 2\alpha^2)^2 + \epsilon e^{\frac{\rho}{\alpha^2}}, \tag{5}$$

where ϵ is integrating constant, we will see that it is proportional to ADM mass. Notice that due to pseudoeuclidean signature q^2 can take both positive and negative values.

One can see that for large negative ρ independently of the value of integration constant ϵ function $q^2(\rho)$ has the asymptotics $q^2 \sim -\frac{1}{\alpha^2}(\rho + \alpha^2)^2$. The form of this function for positive ρ depends on the constant ϵ.

A. $\epsilon > 0$

In this case for large positive ρ function q^2 is positive and approximately $q^2 \sim \epsilon e^{\frac{\rho}{\alpha^2}}$.

The critical points of function $q^2(\rho)$ (5) (where $\frac{dq^2}{d\rho} = 0$) are solutions of following equation:

$$\rho_c = -\alpha^2 + \frac{\epsilon}{2} e^{\frac{\rho_c}{\alpha^2}} . \tag{6}$$

One can show that there are no such points for $\epsilon > 2\alpha^2$; for $\epsilon = 2\alpha^2$ one gets one critical point $\rho_c = 0$; for $0 < \epsilon < 2\alpha^2$ the function has two critical points: the first one is positive ($\rho_{c1} > 0$) and the second is negative ($\rho_{c2} < 0$).

In general case $q^2(\rho)$ in critical point is equal to the following value:

$$q_c^2 = \frac{\epsilon}{2} e^{\frac{\rho_c}{\alpha^2}} (1 - \frac{\rho_c}{\alpha^2}). \tag{7}$$

One can see that q_c^2 is positive if $\rho_c < 0$ (since $\epsilon > 0$). The sign of q^2 in positive critical point ρ_{c1} depends on value of constant ϵ. If ϵ is slightly smaller than $2\alpha^2$ then q_{c1}^2 is still positive. The point ρ_{c1} is a minimum which goes down with decreasing constant ϵ and reaches zero value $q_{c1}^2 = 0$ if, as follows from (7), $\rho_{c1} = \alpha^2$. One can see from (6) that it corresponds to $\epsilon = \frac{4\alpha^2}{e}$ [1] . Thus we come to following conclusion about the behaviour of function $q^2(\rho)$.

For $\epsilon > \frac{4\alpha^2}{e}$ the function $q^2(\rho)$ has only one zero at a negative $\rho < -\alpha^2$. If $\epsilon = \frac{4\alpha^2}{e}$ there are two such zeros: at $\rho < -\alpha^2$ and $\rho = \alpha^2 > 0$. For $0 < \epsilon < \frac{4\alpha^2}{e}$ the function $q^2(\rho)$ vanishes at three points: one for $\rho < -\alpha^2$ and two for $\rho > -\alpha^2$ (one of which satisfys $\rho > \alpha^2$).

B. $\epsilon = 0$

In this case function (5) reduces to $q^2 = -\frac{1}{\alpha^2}(\rho + \alpha^2)^2$ which is negative everywhere except for a point $\rho = -\alpha^2$ where it vanishes.

C. $\epsilon < 0$

As one can see from (5) the function $q^2(\rho)$ has no zeros in this case and it is negative for any ρ. Evidently there is only one critical point (maximum) ρ_c which lies in the interval $-\alpha^2 - \frac{|\epsilon|}{e} < \rho_c < -\alpha^2$.

3. Thus eqs.(2)-(3) determine q $(= \sqrt{q^2})$ as a function of ρ. Further analysis of (2) easily shows that $\xi(q) = 0$, where we denoted 1-form $\xi = q_c e^c$. Let us use this and introduce a new coordinate system which is (pseudo)polar with q playing the role of a 'radial' coordinate, while the 'angular' coordinate ϕ is then clearly such that its differential is proportional to ξ. Assuming (for definiteness) that $q^2 = (q^0)^2 - (q^1)^2 > 0$, one can write the torsion components in the form: $q^0 = q\cosh\phi$, $q^1 = q\sinh\phi$.

Let us consider q, ϕ as the new local coordinates on M^2. The differentials $\{dq, d\phi\}$ form basis in the space of one-forms. Since q is a function of ρ, we can use an equivalent basis $\{d\rho,\ d\phi\}$. From the construction of q, ϕ (see above) and (2)-(3) we get

$$q^a \varepsilon_{ab} e^b = -\frac{d\rho}{\alpha^2}$$
$$q_a e^a = \xi = B d\phi \qquad (8)$$

where B is some function of variables ρ and ϕ. Equations (8) are easily

[1] $e = 2.7...$ is the Euler number

solved with respect to zweibeins e^a:

$$e^0 = \frac{B}{q}\cosh\phi d\phi - \frac{1}{\alpha^2 q}\sinh\phi d\rho$$
$$e^1 = \frac{B}{q}\sinh\phi d\phi - \frac{1}{\alpha^2 q}\cosh\phi d\rho \,. \tag{9}$$

Let us find the function B. From (9) one calculates the volume 2-form $V = \frac{1}{2}\varepsilon_{ab}e^a \wedge e^b = \frac{B}{q^2\alpha^2}d\rho \wedge d\phi$, and from (8) thus $d\xi = q^2\alpha^2 B^{-1}\frac{\partial B}{\partial\rho}V$. However, it is straightforward to see that (3) can be rewritten as

$$d\xi = (\frac{\Phi}{\alpha^2} - q^2)V =$$

$$= q^2\alpha^2(q^{-2}\frac{dq^2}{d\rho} - \frac{1}{\alpha^2})V,$$

and thus B satisfies

$$B^{-1}\frac{\partial B}{\partial\rho} = q^{-2}\frac{dq^2}{d\rho} - \frac{1}{\alpha^2}.$$

From this we find finally

$$B = q^2 B_0 \exp(-\tfrac{\rho}{\alpha^2})$$

where B_0 is an arbitrary function of ϕ. Consequently the metric has the form:

$$d^2s = (e^0)^2 - (e^1)^2 = q^2(\rho)\exp(-\frac{2\rho}{\alpha^2})(d\phi)^2 - \frac{1}{\alpha^4 q^2(\rho)}(d\rho)^2 \tag{10}$$

where $q^2(\rho)$ is known function (5), and without any loose of generality we redefined the 'angular' variable $B_0(\phi)d\phi \to d\phi$ (denoting the new coordinate by the same letter). Several remarks are in order. First of all, let us note that this result (10) is also valid in region where $q^2 < 0$ (in which case one should use other formulas describing the introduction of new coordinates, $q^0 = q\sinh\phi$, $q^1 = q\cosh\phi$). Secondly, the metric (10) is obviously stationary (angle ϕ is of course the time coordinate), however the Lorentz connection one-form ω has a non-stationary part due to the time-dependent torsion. It should be noted also that the demonstration of integrability of the model (1) in coordinates of torsion

q^a is similar to the analysis of the dilaton gravity without matter in the coordinate system where the dilaton field ϕ plays the role of one of coordinates [1]. As was shown in [7], in conformal gauge the action of the type (1) leads to the essentially nonlinear equations of motion, the exact solution of which is much more complicated. Thus the conformal gauge, though standard, is not always the best one. The model (1) gives us an example of a theory when the equations dictate the natural gauge.

4. It is straightforward to see that the above results are valid (with slight appropriate modifications) also for the theory with the most general action, $S = \int LV$,

$$L = \beta T^a{}_{\mu\nu} T_a{}^{\mu\nu} + \gamma R_{\mu\nu} R^{\mu\nu} + \lambda. \tag{11}$$

Equations (2), (3) are then replaced by the generalized equations of motion

$$d\rho = -\alpha^2 q^a \varepsilon_{ab} e^b \tag{12}$$

$$\nabla q^a = -\frac{1}{2\alpha^2}[\rho^2 + \alpha^2 q^2 - \alpha^4 - \Lambda^2]\varepsilon^a{}_b e^b, \tag{13}$$

where the following notation was introduced:

$$\alpha^2 = \frac{\beta}{\gamma}, \quad \Lambda^2 = -\frac{\lambda}{2\gamma} - \frac{\beta^2}{\gamma^2}.$$

Notice that squares were introduced in order to get formal similarities with the above considered de Sitter model, but actually neither newly defined α^2 nor Λ^2 are positive, in general. As is seen from (1), the de Sitter model is recovered when $\gamma = 1$, $\beta = 2\alpha$, $\lambda = -2\alpha^4$, so that $\Lambda = 0$ and old and new α's coincide.

Equations (12), (13) yield a modified equation for q^2 which replaces (4),

$$\frac{dq^2}{d\rho} = \frac{1}{\alpha^4}\Phi', \tag{14}$$

where $\Phi'(\rho, q^2) = \rho^2 + \alpha^2 q^2 - \alpha^4 - \Lambda^2$.

The solution of this equation has the form:

$$q^2(\rho) = -\frac{1}{\alpha^2}(\rho + \alpha^2)^2 + \frac{\Lambda^2}{\alpha^2} + \epsilon e^{\frac{\rho}{\alpha^2}}. \tag{15}$$

The behaviour of this function is very similar to (5); in fact if one confines to the case $\alpha^2 > 0$, the quantitative analysis of q^2 almost duplicates the discussion in sect.2.

It is easy to see that equations (12)-(15) are such that the introduction of the q, ϕ coordinates proceeds precisely as described in the sect.3, including the same solution for the function B.

Hence, the general theory (11) has the same structure of the line-element (10), with q^2 determined from (15).

5. The coupling with matter in general case breaks this exact integrability in coordinates q^a. The exception is the coupling with Yang-Mills fields. In this case the total action

$$S = \int_{M^2} \frac{1}{4}(Tr * \mathcal{R} \wedge \mathcal{R} + Tr * F \wedge F),$$

where $F = dA + A \wedge A$ is strength of gauge field $A = A^a_\mu \tau^a dz^\mu$, yeilds the equations of motion for the gauge field A^a_μ which reduce to the conservation law $df^2 = 0$, $f^2 = Tr[*F * F]$. This means that $f^2 = constant$; in abelian case, moreover, $f = *F = constant$ is a charge. In equations (2)-(3) the coupling of the gauge gravity with the Yang-Mills matter is manifested only in the shift of "cosmological" constant $\lambda = -2\alpha^4$ as $\lambda \to -2\alpha^4 + f^2$. Correspondingly the function q^2 changes: $q^2 \to q^2 - \frac{f^2}{2\alpha^2}$ so that the metric again has the form (10).

6. Let us remind that when varying the action in order to get an equation of motion one usually drops out the surface term which arises when integrating by parts. The correct way of doing so is to impose appropriate boundary conditions. Assuming the variations $\delta\omega$ and δe^a are arbitrary at spatial infinity (which in the polar coordinate system corresponds to the infinite value of the usual radial coordinate) one gets from the action (1) the boundary conditions:

$$\rho|_\infty = -\alpha^2; \quad q^a|_\infty = 0 . \qquad (16)$$

The constraint that torsion at space infinity is zero is too strong. It leads to the constraint $\epsilon = 0$ in (5), so most of solutions are omitted.

Let us add to the action (1) following term:

$$S_b = -a\alpha^2 \int_{\partial M^2} \gamma^{\frac{1}{2}} d\tau \qquad (17)$$

where $\gamma = det\gamma_{\mu\nu}$, $\gamma_{\mu\nu} = g_{\mu\nu} - \kappa n_\mu n_\nu$ is metric induced on the boundary ∂M^2 with normal vector n_μ ($n_\mu n_\nu g^{\mu\nu} = \kappa$), $\kappa = 1$ for spacelike boundary and $\kappa = -1$ for timelike boundary. Then variation of total action $S_{tot} = S + S_b$ (note that it is still positive in euclidean signature for $a > 0$) leads to the modified boundary conditions at space infinity, which for metric (9)-(10) take the form

$$\rho|_\infty = -\alpha^2; \quad q|_\infty = a\,. \tag{18}$$

It means that integrating constant in (5) $\epsilon = a^2 e$. Thus the found solution discribes the two-dimensional asymptotically de Sitter space with two kinds of possible singularities: where $\rho = -\infty$ and $\rho = +\infty$. Remember that ρ is curvature of the Lorentz connection one-form ω: $\rho = *(d\omega)$. One can show that relationship of ρ and "metrical" curvature $\rho_0 = *[d(*(de^a)e^a)]$ determined for metric (10) is given by the formula:

$$\rho_0 = \rho + \frac{1}{\alpha^2}\rho^2 - \alpha^2\,. \tag{19}$$

So we have $\rho_0 = \rho$ if $\rho = \pm\alpha^2$ and singularity of ρ means the singularity of ρ_0, one can see from (19) that at any kind of singularity ρ_0 is positive.

7. The most interesting solution is of the type **A** with ρ laying in the interval $-\alpha^2 \leq \rho < +\infty$. One can see that metric (10) describes the two-dimensional asymptotically de Sitter space-time with sigularity ($\rho = +\infty$) and horizons at points where the function $q^2(\rho)$ has zeros.

As was described above, for $\epsilon > \frac{4\alpha^2}{e}$ such points are absent and we have naked singularity. For $0 < \epsilon < \frac{4\alpha^2}{e}$ we obtain two horizons which coincides when $\epsilon = \frac{4\alpha^2}{e}$. Thus the metric (10) for $0 < \epsilon \leq \frac{4\alpha^2}{e}$ resembles the charged two-dimensional black hole type solution [3]. The case $\epsilon = \frac{4\alpha^2}{e}$ corresponds to the extremal black hole.

In support of this analogy we note that the equation (2) is similar to the Maxwell equation $df = *j$ where $f = *(dA)$ is strength of abelian gauge field A and j is charged matter current one-form. Then the second gravitational equation (3) is similar to the equation of motion for charged matter. It is not surprising because the local Lorentz symmetry in two dimensions is abelian and analogous to the $U(1)$-symmetry of Maxwell theory.

From eq.(3) we get that the corresponding Lorentz current one-form $*J = -\alpha^2 q^a \varepsilon_{ab} e^b$ is conserved, $d * J = 0$. Integrating $*J$ over any spacelike hypersurface Σ we get that total charge $Q = \int_\Sigma *J$ is equal to curvature ρ at infinity[2] :

$$Q = \rho|_\infty \tag{20}$$

and consequently for the boundary conditions (18) the total charge $Q = -\alpha^2$.

8. To calculate the ADM mass for the black hole solution (9), (10) let us assume [3] that only the equation for ω (2) is satisfied and consider the zweibein energy-momentum one-form $T^a = T^a_\mu dz^\mu$ which can be determined as follows: $\delta_e S = \int - * T^a \wedge \delta e^a$. For action (1) it takes the form:

$$\tilde{T}^a \equiv - * T^a = \alpha^2 \nabla q^a + \tfrac{1}{2}[\rho^2 + \alpha^2 q^2 - \alpha^4]\varepsilon^a{}_b e^b \,.$$

Multiplying this expression on $q^a exp(-\frac{\rho}{\alpha^2})$ we obtain that

$$\begin{aligned} T &= \tilde{T}^a q^a exp(-\frac{\rho}{\alpha^2}) \\ &= \alpha^2 exp(-\frac{\rho}{\alpha^2})(\frac{1}{2}dq^2 - \frac{1}{2\alpha^4}(\rho^2 + \alpha^2 q^2 - \alpha^4)d\rho) \end{aligned} \tag{21}$$

is obviously conserved: $dT = 0$. It implies that there exist such a scalar function m that

$$T = dm \,. \tag{22}$$

Straightforward calculations show that the mass function m at point ρ can be written in the following explicit form:

$$m = \frac{\alpha^2}{2} exp(-\frac{\rho}{\alpha^2})(q^2 + \frac{1}{\alpha^2}(\rho + \alpha^2)^2) \,. \tag{23}$$

In the case when the field equations $*T^a = 0$ are satisfied eq.(22) implies that $m = constant$ and for $q^2(\rho)$ in the form (5) we get that $m = \frac{\alpha^2 \epsilon}{2}$.

In order to see that defined in such way m is indeed ADM mass let us consider the metrical energy-momentum tensor $T_{\mu\nu} = \frac{1}{2}(T^a_\mu e^a_\nu + T^a_\nu e^a_\mu)$,

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[2] Note that the same formula is valid in (1+1)-electrodynamics:

$Q \equiv \int_\Sigma *j = f|_\infty$ [3]

$T^a_\mu = -\varepsilon_\mu{}^\alpha \tilde{T}^a_\alpha$. Then for component $T_{00} = T^a_0 e^a_0$ since $e^a_0 = q^a \exp(\frac{-\rho}{\alpha^2})$ we obtain that

$$
\begin{aligned}
T_{00} &= -\varepsilon_0{}^\alpha \tilde{T}^a_\alpha q^a \exp(\frac{-\rho}{\alpha^2}) \\
&= -\varepsilon_0{}^\alpha \partial_\alpha m
\end{aligned}
\tag{24}
$$

where m takes the form (23). The ADM mass is ordinary determined [2,3] as integral of T_{00} over space-like hypersurface Σ with tangent vector $v^\mu = \delta^\mu_1$ and normal $n^\mu = \varepsilon^\mu{}_\alpha v^\alpha$:

$$M = \int_\Sigma T_{00} n^0 d\rho \ .$$

One can see from (24) that it is reduced to a surface term

$$M = m(\rho)\big|_{\rho=-\frac{\alpha^2}{2}} = \frac{\alpha^2\epsilon}{2} \ .$$

Hence only the solution of the type **A** describes the positive mass configuration (solutions of the type **B** and **C** have correspondingly zero and negative mass).

9. In conclusion we considered the two-dimensional gauge gravity of de Sitter group (generalization to the Poincare gravity is straightforward) and shown that the classical equations are exactly integrated in coordinate system determined by components of 2D torsion $q^a, a = 0, 1$. The general solution is two-dimensional asymptotically de Sitter space and for some choice of integrating constant it turns out to be of the charged bkack hole type. The square of torsion $q^2 = q^a q^b \eta_{ab}$ is shown to be function of curvature ρ and zeros of $q^2(\rho)$ are points of horizons. We calculate the conserved charge corresponding to the local Lorentz symmetry and ADM mass which is positive for the black hole type solution.

As the next step it would be of interest to consider the coupling of gauge gravity with matter and analyze the Hawking radiation and back-reaction for this type of black holes. This work is in progress.

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