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SOME REMARKS TO THE PROBLEM OF NEUTRINO OSCILLATION

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Within the standard approach [1] to the neutrino oscillation it is assumed that the mass matrix M for physical neutrino states ν_e, ν_μ, ν_τ is non-diagonal:

$$M = \begin{vmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{vmatrix}. \qquad (1)$$

Diagonalizing this mass matrix M we go to new states ν_1, ν_2, ν_3 (we shall call them asymptotic states), which are eigenstates of a Hamiltonian. In this case it is assumed that the physical states ν_e, ν_μ, ν_τ do not have a certain mass and transitions (oscillation) between these neutrinos proceed in accordance with the corresponding formulae [1]. If a heavier neutrino decays in flight in vacuum, one can observe products of this decay. Below we shall apply the term «real oscillation» to that type of oscillation when there occurs a real transition of particles of one type to particles of another type in vacuum. It should be mentioned, however, that the above oscillation scheme is rather idealized. If the mass matrix M was non-diagonal during production of particles, then ν_1, ν_2, ν_3 should be produced instead of ν_e, ν_μ, ν_τ. The asymptotic eigenstates ν_1, ν_2, ν_3 cannot arise as real states in all cases because in real processes particles are produced within finite time periods and with certain masses (i.e. the four-momentum is invariant). Besides, the requirement to conserve proper numbers in production of particles in interactions of the given type prevents realization of asymptotic states. Then, while the particles produced are moving in vacuum, the transition to asymptotic states will occur. Whether this transition is real or virtual depends on the eigenmass value of the asymptotic state. With $K^0, \overline{K}^0$, for example, the asymptotic states K_1^0, K_2^0 will be real because $m_{K^0K^0} = m_{\overline{K}^0\overline{K}^0} = (m_{K_1^0} + m_{K_2^0})/2$. If $m_{K^0K^0} \neq m_{\overline{K}^0\overline{K}^0} \neq (m_{K_1^0} + m_{K_2^0})/2$, these asymptotic states would only be realized as virtual states.

Let us consider a system of oscillating particles as a bound system. Phenomenologically, the degree of binding of these particles will be determined by non-diagonal mass terms of the mass matrix (we shall consider a system of three particles (1)), or, more precisely, by ratios of non-diagonal and diagonal terms of this matrix:

$$A_{ik} = m_{ik}/m_{ii} \quad i \neq k; \quad i, k = 1 \div 3 \qquad (2)$$

and

$$B_{ijk} = m_{ik}/|m_{ii} - m_{jj}| \quad i \neq j; \quad i \neq k; \quad i, j, k = 1 \div 3. \qquad (3)$$

If $A_{ik} << 1$, the given system can be regarded as a weakly bound system, and the physical states ν_e, ν_μ, ν_τ will be realized as real states, while at $A_{ik} \lesssim 1$ this system will be strongly bound and the asymptotic states ν_1, ν_2, ν_3 will be realized as real states (and not virtual ones). The asymptotic states can also be realized as real states in a special case where $A_{ik} << 1$ and $m_{11} = m_{22} = m_{33}$.

Let us proceed to the phenomenological analysis of two examples of vacuum oscillations $K^0 \leftrightarrow \overline{K}^0$, and $\gamma \leftrightarrow \rho^0$. Then we shall apply the results to analysis of neutrino oscillation.

a) $K^0, \overline{K}^0$ oscillation

The mass matrix of $K^0, \overline{K}^0$-mesons has the form

$$\begin{vmatrix} m_{K^0K^0} & 0 \\ 0 & m_{\overline{K}^0\overline{K}^0} \end{vmatrix} = \frac{m_{K^0K^0}}{m_{\overline{K}^0\overline{K}^0}} = m_{K^0}. \qquad (4)$$

When $K^0, \overline{K}^0$-mesons pass through vacuum, their mass matrix becomes nondiagonal. Diagonalizing it, we go to the mass matrix of K_1^0, K_2^0-mesons

$$\begin{vmatrix} m_{K^0K^0} & m_{K^0\overline{K}^0} \\ m_{K^0\overline{K}^0} & m_{\overline{K}^0\overline{K}^0} \end{vmatrix} \to \begin{vmatrix} m_{K_1^0K_1^0} & 0 \\ 0 & m_{\overline{K}_2^0\overline{K}_2^0} \end{vmatrix}$$

$$K^0 = (K_1^0 + K_2^0)/\sqrt{2}, \quad \overline{K}^0 = (K_1^0 - K_2^0)/\sqrt{2}$$

$$m_{K^0K^0} = (m_{K_1^0} + m_{K_2^0})/2 \qquad (5)$$

and the mixing angle θ is equal to $\pi/4$ [2]. To estimate the degree to binding of the $K^0, \overline{K}^0$-meson system due to the nondiagonal mass term appearing in (4), we shall use expression (2)

$$m_{K^0\overline{K}^0}/m_{K^0K^0} \approx 3.47 \cdot 10^{-15}.$$

So, $K^0, \overline{K}^0$-mesons are quite a weakly bound system.

$K^0, \overline{K}^0$-mesons produced in strong interactions are eigenstates of this interaction and have certain masses (strangeness is preserved in strong interactions). Then oscillation or mixing of quarks arising from violation of strangeness must be virtual. How will the oscillation between K^0- and $\overline{K}^0$-mesons occur? Since K^0- and $\overline{K}^0$-mesons incorporate $\overline{s}$, d and $\overline{d}$, s quarks, their masses are the same and there must be real transitions between K^0- and $\overline{K}^0$-mesons (these transitions can be described by the appropriate formulae [1]). Since $m_{K^0 \overline{K}^0} = (m_{K_1^0} + m_{K_2^0})/2$, the asymptotic states K_1^0, K_2^0 will be realized as real states (particles). Possible types of $K^0 \leftrightarrow \overline{K}^0$ oscillation with allowance for decay widths and difference in masses, if $m_{K^0} \neq m_{\overline{K}^0}$, are considered in ref. [3].

b) $\gamma \leftrightarrow \rho^0$ oscillation

Let us consider mixing (oscillation) that arises in the vector dominance model [4]. This model considers mixing of vector fields in strong interaction $V_\mu(\rho^0)$ and electromagnetic interaction A_μ. The initial fields $\begin{pmatrix} V_\mu \\ A_\mu \end{pmatrix}$ are mixed when the strong and electromagnetic interactions are included [5].

$$V'_\mu = \cos\varphi\, V_\mu - \sin\varphi\, A_\mu, \quad \cos\varphi = G/\sqrt{G^2 + e^2}$$

$$A'_\mu = \sin\varphi\, V_\mu + \cos\varphi\, A_\mu, \tag{6}$$

G and e are the strong and the electromagnetic interaction constants. Owing to gauge invariance in electromagnetic interaction (and isospin conservation in strong interaction) the mass matrix of the fields V_μ, A_μ must be diagonal:

$$\begin{vmatrix} m^2_{A_\mu} & \mu^2 \\ \mu^2 & m^2_{V_\mu} \end{vmatrix} = \begin{vmatrix} 0 & 0 \\ 0 & m^2_\rho \end{vmatrix}. \tag{7}$$

How the $\gamma \leftrightarrow \rho^0$ oscillation will occur? Since the fields A_μ and V_μ are noticeably different in mass, there must be no real oscillation between them as it is between K^0 and $\overline{K}^0$. There must be virutal oscillation between these fields (particles). It means that the transition to the asymptotic states A'_μ, V'_μ must be virtual (so, there must be no real transition of a γ quantum to a ρ^0-meson followed by its decay into products that can be observed). To make this transition (oscillation) real (for $\gamma \to \rho^0$), the γ quantum must participate in the interaction in order to come to the ρ^0-meson mass shell.

c) ν oscillation

Knowing that the lepton numbers l_e, l_μ, l_τ are well conserved in the standard theory of weak interaction [6], which is highly accurate, one can draw a conclusion that a neutrino system (see mass matrix (1)) is a weakly bound system, i.e. the interaction that violates the above numbers must be weak and, consequently, the ratios $m_{ik}/m_{ii} \ll 1$ $(i \neq k;\ i, k = 1-3)$ must be small. The physical neutrino states ν_e, ν_μ, ν_τ arising from weak interactions will have certain masses. The asymptotic states ν_1, ν_2, ν_3 violating the lepton numbers l_e, l_μ, l_τ will be realized in motion of neutrinos in vacuum, and the type of their realization will depend on the masses m_{ν_e}, m_{ν_μ}, m_{ν_τ}.

Let us classify possible types of neutrino oscillations by analogy with K^0, $\overline{K}^0$ and γ, ρ^0 oscillations.

1) If masses of ν_e, ν_μ, ν_τ are equal, there must be real oscillation between these neutrinos in full analogy with K^0, $\overline{K}^0$ oscillation. In this case the asymptotic states ν_1, ν_2, ν_3 will be realized in vacuum as real states.

2) If masses of ν_e, ν_μ, ν_τ are different, there must be virtual oscillation by analogy with $\gamma \leftrightarrow \rho^0$ oscillation (asymptotic states ν_1, ν_2, ν_3 will be realized in vacuum as virtual states as well). To find real neutrino oscillation in this case one must have interaction of this neutrino for its transition to the mass shell of another neutrino. For example, if a ν_e neutrino oscillates to ν_μ, ν_τ neutrinos and these neutrinos have decay modes, then these oscillations and decay modes can be seen only if ν_e participates in an interaction and there is a transition to the corresponding mass shells of ν_μ, ν_τ.

In case 2) for experimental observation of neutrino oscillation neutrinos must be passed through matter so that a real transition of a neutrino of one type to a neutrino of another type could occur. When neutrinos born inside the Sun pass through the Sun, there is a natural possibility of this transition. The bulk of the Earth can be used for the same purpose. The facilities Super-Kamiokande [7] and SNO [8], which are under construction now, may allow one to observe neutrino oscillation of the indicated type.

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