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PRESENT EXPERIMENTAL DATA ON CROSS CHANNEL DEEP-INELASTIC PROCESSES AND THE GRIBOV — LIPATOV RELATION

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1 Introduction

In the previous paper [1] the importance of the measurement of inclusive proton production in e^+e^--annihilation at LEP energies was emphasized. The interest in this process was advocated by the fact that the $e^+e^- \to p + X$ process is connected by the crossing-symmetry relation with the process of deep-inelastic scattering (DIS) $e^-p \to e^- + X$ in which the proton structure functions are well measured. At the level of the parton model the connection between the structure functions defining these two deep inelastic processes is given by the Gribov-Lipatov (GL) relation [2].

In the present paper we shall study the question how well does this relation agrees in practice with the existing data in these two cross channels. Namely, we shall take DASP [3], TASSO [4,5] and ARGUS [6] data for the inclusive annihilation (IA) production of the proton in the final state and transform them with the help of the GL relation into the predictions for the DIS structure functions. The obtained values of $F_2^p(x, Q^2)$ will then be compared with the experimental data of SLAC [7], BCDMS [8], UA1 [9] and UA2 [10] collaborations.

2 Gribov-Lipatov reciprocity relation

The GL-relation looks as follows [2]:

$$-\overline{W}(\omega) = \frac{1}{\omega} W\left(\frac{1}{\omega}\right), \tag{1}$$

where for the DIS channel the following equality for structure functions is supposed to hold

$$W(\omega, q^2) \equiv 2F_1(\omega, q^2) = \omega F_2(\omega, q^2), \tag{2}$$

while for the inclusive annihilation (IA) process an analogous equality has the form:

$$-\overline{W}(\omega,q^2) \equiv 2\overline{F}_1(\omega,q^2) = \omega\overline{F}_2(\omega,q^2). \tag{3}$$

Here $pq = M\nu$, with M being the mass of the proton, and ω is a scaling variable

$$\omega = \frac{-2pq}{q^2} \tag{4}$$

unique for both channels. In (4) p is the momentum of the proton and q is the momentum transferred from the leptonic (e^+e^-) or (e^-e^-) block to the hadronic one in the corresponding Feynman diagrams. Thus, for the e^+e^-- annihilation $q^2 = (k_{e^-} + k_{e^+})^2 = Q^2 > 0$ and for DIS $q^2 = (k_{e^-} - k'_{e^-})^2 = -Q^2 < 0$.

It is convenient to rewrite the relation (1) separately for different structure functions

$$\overline{F}_1(\omega) = \frac{1}{\omega}F_1\left(\frac{1}{\omega}\right), \tag{5}$$

$$\overline{F}_2(\omega) = \frac{1}{\omega^3}F_2\left(\frac{1}{\omega}\right). \tag{6}$$

The variable ω in the annihilation channel is connected with the variable

$$z = \frac{E_h}{E_{beam}} (\equiv x_F),$$

usually used as an argument of the IA fragmentation functions $\overline{D}^h_{p_i}(z,Q^2)$ that describe the fragmentation of the parton p_i ($p_i = q_i, \overline{q}_i, g$) into the hadron h, by the following relation (see for instance [11])

$$IA: \qquad \omega = \frac{-2pq}{q^2} = \frac{-2pq}{Q^2} = -z. \tag{7}$$

At the same time, in DIS channel

$$DIS: \qquad \omega = \frac{2pq}{-q^2} = \frac{2pq}{Q^2} = \frac{1}{x}, \tag{8}$$

where x is a usual argument of the DIS structure functions $F_i^{ep}(x,Q^2)$ $(i = 1,2)$.

3 The connection of F_2^{ep} with the cross section of the $(e^+e^- \to p+X)$ process

The standard structure functions (3) for an inclusive annihilation process are connected with the fragmentation functions within the parton model by the following relation (see [12] and the discussion in [11]) $(N_C = 3)$

$$2z\overline{F}_1 \equiv z^2\overline{F}_T = N_C \sum_{i=1}^{N_f} e_{q_i}^2 \left[\overline{D}_{q_i}^h(z) + \overline{D}_{\bar{q}_i}^h(z)\right] \tag{9}$$

Now we pass to the discussion of the cross section formulae. We start with the formula [13]

$$\frac{d^2\sigma^{e^+e^-\to p+X}}{dE_p dcos\theta} = \frac{4\pi\alpha^2}{(q^2)^2}\frac{M^2\nu}{\sqrt{q^2}}\left(1-\frac{q^2}{\nu^2}\right)^{1/2} \times$$

$$\times \left[2\overline{W}_1(q^2,\nu) + \frac{2M\nu}{q^2}\left(1-\frac{q^2}{\nu^2}\right)\frac{\nu\overline{W}_2(q^2,\nu)}{2M}sin^2\theta\right] \tag{10}$$

that includes the prescaling terms like β^k $(k = 1,2,3)$, with $\beta = \sqrt{1-q^2/\nu^2} = p/E_p$ being the proton velocity v_p, usually set in the Bjorken limit approximation to be $\beta = 1$. In (10) E_p is the energy of the detected proton and θ is the angle of the proton momentum with respect to the e^+e^- colliding beam axis.

After passing to the standard structure functions $(M\nu = pq)$

$$F_1(\omega,q^2) = MW_1(q^2,\nu); \qquad \overline{F}_1(\omega,q^2) = M\overline{W}_1(q^2,\nu); \tag{11}$$

$$F_2(\omega,q^2) = \nu W_2(q^2,\nu); \qquad \overline{F}_2(\omega,q^2) = \nu\overline{W}_2(q^2,\nu); \tag{12}$$

formula (10), being rewritten in terms of the z variable (7), looks like

$$\frac{d\sigma^{e^+e^-\to p+X}}{dzdcos\theta} = \frac{3}{4}\sigma_0 z\beta[2\overline{F}_1(\omega,q^2) + \frac{z\beta^2}{2}\overline{F}_2(\omega,q^2)sin^2\theta], \tag{13}$$

where

$$\sigma_0 = \sigma_0(e^+e^- \to \mu^+\mu^-) = 4\pi\alpha^2/3s. \tag{14}$$

From (13) one gets after integrating over $cos\theta$ the following expression:

$$\frac{d\sigma^{e^+e^- \to p+X}}{dz} = 3\sigma_0 z\beta[\overline{F}_1(\omega, q^2) + \frac{z\beta^2}{6}\overline{F}_2(\omega, q^2)] \tag{15}$$

that in the limit $v_p = \beta \to 1$ coincides with the expression given, for example, in [11]. In what follows the preasymptotical value of β will be kept in the formulae for a convenience of passing from the differencial cross section $d\sigma/dz$ to the expression for the $e^+e^- \to p+X$ cross section with respect to the $x_p = p/E_{beam}$ variable, i.e. $d\sigma/dx_p$.

Now applying in (15) the Callan-Gross relation (3), supposed to be true within the parton model, we arrive to the formula

$$\frac{d\sigma^{e^+e^- \to p+X}}{dz} = -3\sigma_0 z^2\frac{\beta(3-\beta^2)}{6}\overline{F}_2(\omega, q^2) \tag{16}$$

that gives after the application of the GL-relation (6) the following connection between the DIS structure function

$$F(1/\omega, q) \equiv F(x, Q^2)$$

and the $e^+e^- \to p+X$ cross section

$$\frac{z}{3\sigma_0}\frac{d\sigma^{e^+e^- \to p+X}}{dz} = \frac{\beta(3-\beta^2)}{6}F_2^{ep}(x, q^2). \tag{17}$$

It is more convenient to express the structure function through the annihilation cross section normalized to the total hadronic cross section

$$F_2^{ep}(x, q^2) = \frac{2Rz}{3-\beta^2}\left[\frac{1}{\sigma_{tot}}\frac{d\sigma^{e^+e^- \to p+X}}{\beta dz}\right], \tag{18}$$

where, as usual,

$$R = \frac{\sigma_{tot}(e^+e^- \to hadrons)}{\sigma_0(e^+e^- \to \mu^+\mu^-)}, \tag{19}$$

$$\sigma_{tot}(e^+e^- \to hadrons) = 3\sigma_0 \sum_{i=1}^{n_f} e_{q_i}^2 (1 + \frac{\alpha_s(Q^2)}{\pi} + ...) \tag{20}$$

and

$$\beta = \sqrt{1 - \frac{4M^2}{sz^2}} = \frac{1}{\sqrt{1 + \frac{4M^2}{sx_p}}}. \tag{21}$$

Now, applying formula (18) to different data on the $e^+e^- \to p + X$ cross sections at some values of $s = Q^2$, we can obtain the values for the DIS structure function at the same values of Q^2. The accuracy of the theoretical prediction obtained in such a way will be of the same order as the accuracy of the parton model approximation itself.

4 The comparison of the theoretical predictions for deep-inelastic structure functions with the existing data.

Before passing to our aim we would like to mention that the data on the annihilation process $e^+e^- \to p+X$ are collected only for a finite number of values of the square of the momentum transfer Q^2 from the leptonic (e^+e^-) to hadronic blocks ($p + X$) (Q^2 concides in this case with the square of the total energy of e^+e^- -beams $(2E_{beam})^2 = s = Q^2$). At the same time, the data tables for the structure functions of DIS contain much more bins in Q^2. But they mainly do not coincide with the e^+e^- bins in $Q^2 = s$ in which the data on the inclusive annihilation are presented. Therefore, in order to compare data in two cross channels we would need to perform the averaging over Q^2 or to compare the data from the neighbouring intervals.

Thus, in fig.1 the data of the DIS structure functions of SLAC [7] (empty circles) presented at $Q^2 = 12\ GeV^2$ and the theoretical values are presented. The values of $F_2^p(x,Q^2)^{theor}$ are found with applying relation (18) of the data on the IA cross sections. The latter are taken from the table 4 of [3] at interval $\sqrt{s} = 3.60 \div 3.67\ GeV$, i.e., $Q^2 = s = 12.96 \div 13.47\ GeV^2$. The errors of the calculated theoretical predictions for the values of the structure functions, i.e. $F_2^p(x,Q^2)^{theor}$ are obtained also by the recalculation with the help of (18) of the errors, taken from the same table 4 of [3]. (The errors for the experimental values of the structure functions, taken from [7], are much smaller than those for IA and usually they are not larger than the size of the open circles).

In an analogous way, the data of the BCDMS collaboration on $F_2^p(x,Q^2)$, taken from table 3 of [14] at $Q^2 = 24.5\ GeV^2$, are shown in fig.2 together with the values of $F_2^p(x,Q^2)^{theor}$, predicted with the help of the GL relation based on the same DASP data [3] at $\sqrt{s} = 5.0\ GeV^2$, i.e. $Q^2 = 25.0\ GeV^2$.

Fig.3 contains the comparison of the BCDMS data [14] at $Q^2 = 99.0\ GeV^2$ with the prediction for the same interval, done on the basis of (18) and the data of the ARGUS collaboration [6], taken at $\sqrt{s} = 9.98\ GeV$ that corresponds to $Q^2 = 99.6\ GeV^2$.*)

In an analogous way, the comparison of the BCDMS collaboration [14] data $Q^2 = 175\ GeV^2$ with the predictions, based on (18) and TASSO data at $Q^2 = (14\ GeV)^2 = 196\ GeV^2$, is presented in fig.4.

The data of the UA1 [9] and UA2 [10] collaborations for the proton structure function $F_2^p(x,Q^2)$ at $< Q^2 > = 2000\ GeV^2$ are shown in fig.5 toghether with the theoretical prediction obtained from the data of the TASSO collaboration [5] for the values of $Q^2 = s = (34\ GeV)^2 = 1156\ GeV^2$. This prediction found for the value of Q^2 that is two times smaller than the value of Q^2, where the DIS structure function is measured, can serve of course only for the illustration of the tendency.

*) We are thankful to K.R. Schubert who draw our attention to the existing data of the ARGUS collaboration.

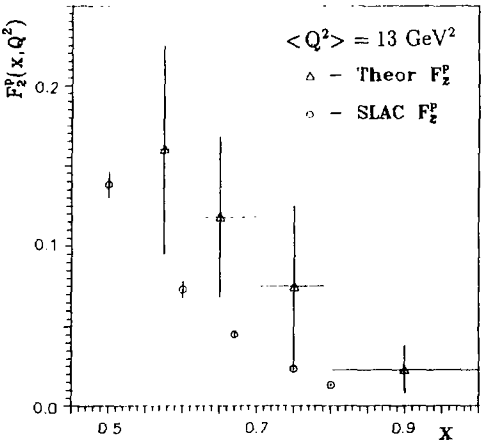

Fig.1. Open circles are SLAC data for $F_2^{ep}(x, Q^2)$ at $Q^2 = 12\ GeV^2$ taken from table 15 of [7]. The triangles show the theoretical prediction obtained by application of formula (18) to DASP data (table 4 of [3]) taken from the interval $\sqrt{s} = 3.60 \div 3.67\ GeV$ (i.e. $Q^2 = 12.96 \div 13.47\ GeV^2$).

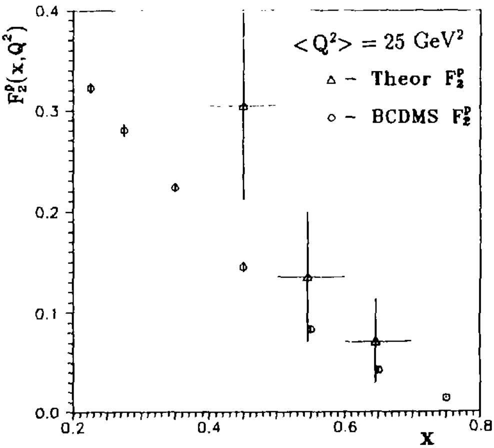

Fig.2. Open circles are BCDMS data for $F_2^{\mu p}(x, Q^2)$ at $Q^2 = 24.5\ GeV^2$ taken from table 4 of [14]. The triangles show the theoretical prediction obtained by application of formula (18) to DASP data (table 4 of [3]) at $\sqrt{s} = 5.0\ GeV$ (i.e. $Q^2 = 25.0\ GeV^2$).

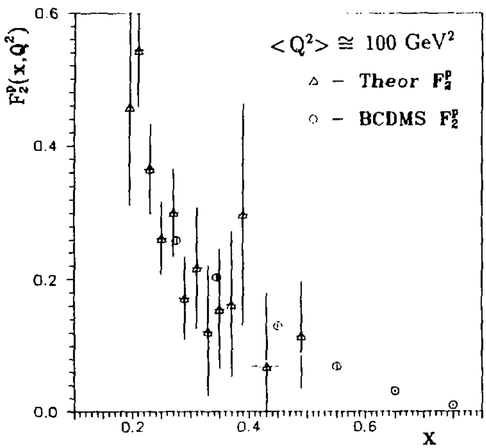

Fig.3. Open circles are BCDMS data for $F_2^{\mu p}(x,Q^2)$ at $Q^2 = 99.0\,GeV^2$ taken from table 8 of [14]. The triangles show the theoretical prediction obtained by application of formula (18) to ARGUS data (table 8 of [6]) at $\sqrt{s} = 9.98\,GeV$ (i.e. $Q^2 = 99.6\,GeV^2$).

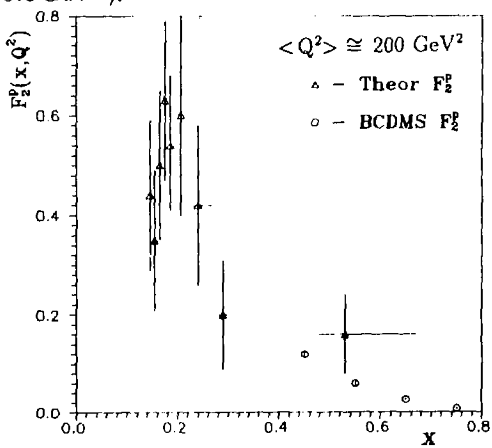

Fig.4. Open circles are BCDMS data for $F_2^{\mu p}(x,Q^2)$ at $Q^2 \cong 175\ GeV^2$ taken from table 6 of [14]. The triangles show the theoretical prediction obtained by application of formula (18) to TASSO data (table 4.b of [4]) at $s = Q^2 = (14\,GeV)^2 = 196\,GeV^2$.

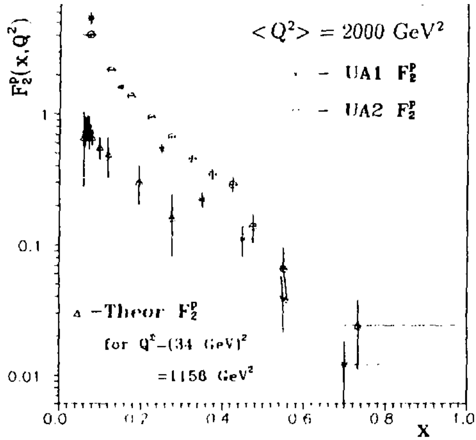

Fig.5. Crosses and open circles are UA1 (table 1 of [9]) and UA2 (table 2 of [10]) data of proton structure function $F_2^p(x,Q^2)$ at $<Q^2>= 2000\ GeV^2$ shown together with the theoretical prediction for the structure function $F_2^p(x,Q^2)$ obtained by application of formula (18) to TASSO data (table 4.b of [4]) at $s = Q^2 = (34\ GeV)^2 = 1156\ GeV^2$.

So from all thiese plots it can be concluded that although the present data do not show a complete fulfilment of the Gribov-Lipatov relation in all x region, nevertheless one can say that at $x > 0.45$ there exists a tendency for the data to follow the theoretical prediction based on this relation.

Summary

The existence of the theoretical prediction on the relation between the structure functions of two cross channels processes represents an interesting possibility of its checking. Here we have performed it at the level of the parton model. The following publications would be devoted to the problem of inclusion of QCD corrections in this checking.

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