# объединенный ИНСтитут ядерных исөледований 

E2-93-286
V.M.Korotkiyan, O.V.Teryaev*

## ON THE COMPTON NON-ABELIAN TWIST-3 ASYMMETRY

Submitted to «Physics Letters B»

[^0]It is known that large asymnetries appear in processes with a single polarized particle. For instance, asymmetries observed in pion production with high transverse momentum are of the order of ten or more percent[1]. Because of this fact single asymmetrics can be very convenient for the study of polarization effects in QCD. However, for their appearance a mass parameter and an additional imaginary phase are necessary. As a result, these asymmetries are absent in the leading approach of QCD and for their calculation one has to take into account high twist contributions. In this letter we calculate single asymmetries for processes $\gamma N \uparrow \rightarrow g X$ and essentially non-Abelian $g N \uparrow \rightarrow g X$ and give some qualitative predictions for processes which can be studied in existing and future accelerators.

Some years ago transverse polarization in quantum chromodynamics was investigated in the framework of the Ellis, Furmansky, Petronsio factorization scheme [2]. The hadron density matrix in QCD was analized to establish that the mass of a polarized hadron is the true mass parameter of transverse polarization [3]. It was shown that contributions to single asymmetries proportional to two-argument distribution functions (parton correlation densities) can receive :maginary parts even from Born subprocess [4]. Recently similar results were obtained by G. Sterman and J. Qiu [6] whe used the so-called special propagators technique. The correspondence and discrepancy between these approaches will be discussed below.

The term in the cross section proportional to correlators can be expressed in the form [4]

$$
\begin{equation*}
d \sigma_{s}=\int d x_{1} d x_{2} \frac{1}{4} S_{p}\left[S_{\mu}\left(x_{1}, x_{2}\right) T_{\mu}\left(x_{1}, x_{2}\right)\right] \tag{1}
\end{equation*}
$$

where $S_{\mu}\left(x_{1}, x_{2}\right)$ is the codficient function of parton subprocesses with two quark and one gluon legs (Fig. 1) and $T_{\mu}\left(x_{1}, x_{2}\right)$ depend on parton correlation densities:

$$
\begin{equation*}
T_{\mu}\left(x_{1}, x_{2}\right)=\frac{M}{2 \pi}\left(\hat{p}_{1} \gamma^{5} s_{\mu} b_{A}\left(x_{1}, x_{2}\right)+i \gamma_{\rho} t^{\rho_{\mu} s p_{1}} b_{V}\left(x_{1}, x_{2}\right)\right) \tag{2}
\end{equation*}
$$

where $\epsilon^{\rho \mu s p_{1}}=\epsilon^{\rho \mu \alpha \beta} s_{\alpha} p_{1 \beta}, s_{\mu}$ is the covariant hadron polarization vector and $M$ is the hadron mass. Two-argument distributions $b_{A}$ and $b_{V}$ are real and dimensionless. They possess symmetry properties which follow from $T$-invariance

$$
\begin{equation*}
b_{A}\left(x_{1}, x_{2}\right)=b_{A}\left(x_{2}, x_{1}\right), \quad b_{V}\left(x_{1}, x_{2}\right)=-b_{V}\left(x_{2}, x_{1}\right) . \tag{3}
\end{equation*}
$$

In the case of the standard parton picture the imaginary part can appear only in one-loop approximation, and consequently asymmetry is proportional to $\alpha_{\mu}$. The situation is different in the case of correlators. Besides the usual imaginary part of the propagator (Fig. 1a)

$$
\frac{1}{x_{1}(s+u)+t+i \epsilon}=P \frac{1}{x_{1}(s+u)+t}-i \pi \delta\left(x_{1}(s+u)+t\right)
$$

$\left(\left(p_{1} x_{2}+p_{2}\right)^{2}=x_{2} s, \hat{u}=\left(p_{1} x_{2}-p_{3}\right)^{2}=x_{2} u, t=\left(p^{2}-p_{3}\right)^{2}\right)$ which corresponds to cut on $M_{N}^{2}$, it is mecessary to take jnto accomat the additional inaginary part which appears for instance in the diagram of the lig. la where the ghon is attached to a Born subprocess. The imaginary part of the popagator

$$
\begin{equation*}
\frac{1}{x_{2} s+i c}=\rho \frac{1}{x_{2} s}-i \pi \delta\left(r_{2} s\right) \tag{1}
\end{equation*}
$$

leads to the appearance of an additionat jmaginaty part in the hatron-phark amplitude. We consider diagrams which have cuts in so 1 to the kft of the cat in $M_{X}^{2}$. The ucut corresponds to an antiquark comribution, as nsmal [1].

The asymmetry in this case does not dopend on $a_{s}$. since $a_{s}$ is inchoded in the distri-
 $\alpha_{s}$ but proportional to the hadron mass) single asymuntrics. The twist-3 asymmetrics of the direct photons produced in the Compton subprocess were calculated a few years ago. The Abelian asymmetry for the process $\gamma N^{\top} \uparrow \rightarrow \gamma \times[1]$ is

$$
\begin{equation*}
A_{\gamma \gamma}=\frac{b_{A}(0, x)-b_{V}(0, x)}{f(x)} \frac{x_{F}\left(1-x_{F}\right)}{\left(1+x_{F}^{2}\right)} \frac{2 M M_{p}}{m_{T}^{2}}=\frac{b_{A}(0, x)-b_{V}(0, x)}{x f(x)}\left(1+x_{F}^{2}\right)^{-1} \frac{2 M p_{T}}{s} . \tag{5}
\end{equation*}
$$

The non- $A$ belian asymmetry for $g N \uparrow \rightarrow \gamma X[5]$ is

$$
\begin{align*}
& A_{3 \gamma}=\frac{b_{A}(0, x)-b_{V}(0, x)}{f(x)} \frac{x_{F}\left(1-x_{F}\right)\left(C_{V}-\left(x_{1}+1\right)\left(C_{A} / 2\right)\right.}{\left(1+x_{j}^{2}\right)} \frac{2 M_{T}}{m_{T}^{2}}= \\
& \frac{G_{A}(0, x)-G_{V}(0, x)\left[C_{r}-(x+1)(x / 2]\right.}{x f(x)} \frac{2 A / T_{r} r}{s} . \tag{6}
\end{align*}
$$

$x_{F}=-u / s, x=-u(s+u), m_{T}^{2}=u t / s, C_{A}=N,\left(C_{F}=\left(N^{2}-1\right) / 2 N\right.$. (In the artiches $[4,5]$ the sign of $x_{F}$ is wrong in some places). $b_{s}(0, x), b_{v}(0, x)$ and $f(x)$ are quarkghon correlators and "ordinary" quark distribution. respectively. Here and below the expressions for "raw" asymmetries are presented: all parts mated to mpolarized hadrons are omittel. To pass to the hadron case one should change $t, s \rightarrow t y, s y(y$ being the gluon momentum fraction) and integrate over $y$ (sepanately!) the properly nomalized mumerator and denominator, If we have hadroms in the foral state it is neeessary to take into account fragmentation processes. Ilowever, it is pussible to measure the asymmetry of the gluon jet. This allows one to avoid complications comected with hadronization of the final ghon. Note that asymmetry (5) is the natural "partonometer" for the cortelators (the additional $y$ integration is absent). The coeflicients of 1 and $x p$ in the expression (6) are proportional to the culor liaturs of $s$ and $u$ - channel diagrans $C_{F}-C_{A} / \geq$ and $C_{F}$, respectively. This is a consouplone of the symunetry of the Abelian result moker interchange of $u$ athe $s$.

Calculations of $h_{\text {g }}$ do wot differ from calculations for $A_{y}$, atul after all transformations we lave

$$
\begin{array}{r}
A_{\gamma 3}=\frac{b_{A}(0 . x)-b_{V}(0, x)}{f(x)} \frac{\left(1-x_{l}\right)\left(C_{F} x_{F}-\left(x_{F}+1\right) C_{A} / 2\right)}{\left(1+x_{F}^{2}\right)} \frac{2 I_{P T}}{m_{T}^{2}}= \\
\frac{b_{A}(0, x)-b_{1} \cdot(0, x)}{x f(x)} \frac{\left[C_{F}-\left(x_{F}+1\right) / x_{F} C_{A} / 2\right]}{\left(1+x_{F}^{2}\right)} \frac{2 M_{p_{T}}}{s} \tag{i}
\end{array}
$$

Note that the expressions in square brackets in the second equalities in (6) and (7) differ be interchange of $x_{F}$ and $1 / x F$. It is natural since processes $g N \uparrow \rightarrow \gamma X$ and $\gamma \mathrm{N} \uparrow \rightarrow \mathrm{g} \boldsymbol{f}$ diller by interchange of s amb $a$.

The calculations of $A_{y g}$ are more complicated becanse of the essemtially non- Ab belian nature of the ghon Compton shbpocess. New diagrans of the 1ype in Fig. lb appear

 get

$$
\begin{align*}
& A_{a,}=\frac{b_{1}(0, x)-b_{i} \cdot(0 . x)}{f(x)} \frac{\left(C_{r}-(a / 2)(1-x)\right.}{\left(1+x_{j}^{2}\right)} x \\
& \times \frac{\left(\left(r_{F}^{2}+1\right) C_{A} / 2-C_{F} x_{F}\left(1-x_{F}\right)^{2}\right)}{\left(x_{F}\left(C_{F}-C_{A} / 2\right)-\left(1+x_{F}^{2}\right) C_{F} / 2\right)} \frac{2 M_{P}}{m_{T}^{2}}= \\
& \frac{b_{A}(0, x)-b_{\cdot} \cdot(0, x)}{x f(x)} \frac{\left(C_{F}-C_{A} / 2\right)}{\left(1+x_{F}^{2}\right)} \times \\
& \times \frac{\left(\left(2_{F}^{3}+1 / x_{F}\right) C_{A} / 2-C_{F}\left(1-x_{F}\right)^{2}\right)}{\left(x_{F}\left(C_{F}-C_{A} / 2\right)-\left(1+r_{F}^{2}\right) C_{F} / 2\right)} \frac{2}{s} . \tag{8}
\end{align*}
$$

Note that the expressions (6), (7) and (S) reproluce the inclusion Compton asymmetry (5) in the "Abelian" limit $C_{i}=0 . C_{F}=1$.

The calculations are perfomed using an axial-type gange. for this gange the ghon density matrix and the mumerator ol ghom propagator are

$$
\begin{equation*}
\rho_{\mu \nu}\left(k^{\prime}\right)=-g_{1, \prime}+\pi \frac{k_{u} n_{\nu}+k_{\nu} n_{\mu}}{\left(k_{n}\right)} \tag{9}
\end{equation*}
$$

where $n$ is the normal vector $\left((n \cdot A)=0, n^{2}=0\right)$. The parameter a is introdited to control the result. In fact, we performed the calculations keeping it free. The batem is gauge invariant, because the leymman rules for it gencrate on-shell amplitudes with almost standard external fermion lines. Therefore, it should not depend on $a$. This is not the case for the coefferent of $b_{r}$, which is gange-dependent and equal, up to a sign, to that of $b_{A}$, provided the axial gange is adopted. Setting $a=1$, we get the axial gauge and structure $b_{A}-b_{V}$. The apprarance of the same combination $b_{A}-b_{1}$. in diferent processes is related to the positive $t^{\prime}$ - dianmel patity [5, i] ( $t^{\prime}$ corresponds to the forward scattering amplitude amb, of conrse, is (qual to zero). The substitution





 of correlators and kinematical variables, with the "iemonice poles" contribations of Qiu and Sterman \{6]. In their work "glamic pules" were intruduced (diagrams of Fig. 1e type ). Thes are related to the phase space region in which hadron momentmen fraction carsied $\mathrm{l}_{\mathrm{g}}$ gluon tends to zero $\left(\left(x_{1}-x_{2}\right) \rightarrow 0\right)$. These authors believe that "gluonic poles" give the main contribution to asymmetry. Itowerer the "gluonic poles" contribution appears to be equal to zero in onr apmonch.

Onr calculations give for the "ghonic poles" comtrilmions results ( lar all prucesses) which are proportional to be (x, 人). In the paper [6] they are proportional be the semedators $T_{i}^{F}(x, x)$ which have no connterart in on axial-gange lasod appoad. Honever, alhongh the Feymman gange is adopted in [6], all these cometators are gange invari-
 does not contribute wo the color singlet matrix chement ratated to 7 . The use of the equations of motion lead to $\eta_{1}^{\prime \prime}\left(x_{1} . x_{2}\right) \sim\left(r_{1}-r_{2}\right) T_{1}^{\prime \prime}\left(x_{1}, x_{2}\right) \sim\left(x_{1}-x_{2}\right) b_{5}\left(x_{1}, x_{2}\right)$. Conseguently, the coeficients attached to he nust be equal to zero for coincidence with results of [ 6 ] (and we can get nonzero result if $b_{b}\left(x_{1}, r_{2}\right)$ has a pole at $x_{1}=x_{2}$ ). However, these coeflicjents are nomzeru. Therefore, we believe that by has no pole and $b_{V}(x, x)=0$ (otherwise, we get a meaningless inlinite result). We conchade that cither the Ellis, Furmanski, Petronzio approach is inadequate for the ghonic poles contribution, or the latter is absent altogether.

Our results allow us to give some qualitative predictions for the obtained asymmetries. The single asymmetries for pion (jets) production predominate over the direct photon asymmetries in the region of target fagmentation ( $x^{2} F \sim 0$ ). It can be seen from (5), (6), (7) and (8) that $A_{\gamma,}$ and $A_{y 7}$ dilfer by a sign from $A_{\gamma g}$ and $A_{3 \gamma}$. The dependencies on $x_{f}$ of $A_{g y}$ and $A_{y^{\prime}}$, (the kinematical factor $M_{H} / \mu_{2}^{2}$ is onitted) are shown in Fig. 2. These predictions catn protably be seen experimentally.

The plamed polarization program of the STMR collahoration at RFilC is most adapted [or measuring the direet photon and jets asymmetric's [ 8 ]. As a result one can check new important predictions of (2C1) and resolve its controversies.

The authors wond like to thank A.V. Efremov for the stimulating discussions and kind support and E.A. Kurace for useful comments. ().T. is indebted to G. Sterman for explanations of the details of paper [6].

(1a)

( 1 b )

(1c)

Figure 1. The typical diagrams with "fermionic" (a,b) atd "gluonic" (c) poles.


Figure 2. The parton asymmetries for the processes of direct $\gamma$ and jets inclusive production.

## References


 Le:41.64. 9!5(1990).




 $\therefore$



## Received by Publishing Department on July 22, 1993.


[^0]:    *E-mail: teryaev@theor.jinrc.dubna.su

