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E2-93-257

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TO THE LAGRANGIAN FORMULATION OF NJL-MODEL WITH SEPARABLE INTERACTION

Submitted to «ЯФ»

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1993

1. INTRODUCTION

The Nambu-Jona-Lasinio (NJL) model [1], and other models motivated by it [2]-[10] are powerful instruments for the study of the composite structure of hadrons. Actually, the first success of this model has been related to the explanation of the spontaneous breaking of chiral symmetry and the small pion mass [2].

To give a sense to various loop integrals arising in this approach, a momentum cutoff is usually introduced and the detailed momentum dependence of the hadronic vertices which characterize the composite structure of hadrons is neglected. In this rough approximation it was shown that the NJL-model reproduces the standard formulation of the σ-model [2].

More realistic generalizations of the NJL-model use the nonlocal four-quark interactions, usually in a separable form. In this way hadron wave functions and global hadronic characteristics can be connected [3]-[5],[15, 16].

There exist more fundamental approaches [6] which realize the NJL mechanism starting from the QCD bosonization, but this requires to introduce bilocal hadronic fields producing equations that are difficult to solve. Any simplifications of this approach yield a kind of NJL-model with nonlocal interactions and/or modified quark propagators.

Using QCD bosonization a special formulation of quark confinement has been introduced in [9, 10, 15]. It was assumed that the hadron-quark vertices are local but the quark propagators inside the quark loop are described by entire analytical functions providing both a quark confinement and ultraviolet convergence of all diagrams.

The main goal of this paper is to give a Lagrangian formulation of the NJL-model with separable interaction both for mesons and for the first time for baryons. We check the Goldstone theorem in this approach which means that a zero-mass pion appears in the chiral limit.

In fact, we do not pay much attention to the Schwinger-Dyson (SD) equation for constituent quark masses and the Bethe-Salpeter(BS) equation for hadron masses because they have too many free parameters to be predictive. Actually, these equations may be considered only as the self-consistent constraints which connect the quark and hadron masses with the NJL coupling strength.

All important information about the composite structure of hadrons is concentrated in the matrix elements of the physical processes, in particular in the electromagnetic form factors characterizing the response of a bound state to the interaction with a photon. Here, we introduce the electromagnetic interactions by means of the time-ordering P-exponent in the nonlocal quark currents. This reproduces automatically the Ward-Takahashi identities and electromagnetic gauge invariance in each step of calculation.

There are two adjustable parameters, a range parameter Λ appearing in the separable interaction and a constituent quark mass m_q. As in the papers [4, 5],the weak decay constant f_π, the two-photon decay width $\Gamma_{\pi^0\to\gamma\gamma}$, as well as the charge form factor $F_\pi(q^2)$ and the $\gamma^*\pi^0 \to \gamma$ transition form factor $F_{\gamma\pi}(q^2)$ are calculated. Here we consider monopole form factors. We do not take into account ρ-meson contributions because they are small (see, e. g. [4]).

2. THE NJL-MODEL WITH SEPARABLE INTERACTION

For the convenience of the reader we give the Lagrangian of the NJL-model with separable interaction (SI) [3]

$$L^{SI}_{NJL} = \bar{q}i\,\partial\!\!\!/ q + \frac{G}{2}\{J_S^2 + J_P^2\} \tag{1}$$

with J given by

$$\begin{aligned} J_S &= \int dy\bar{q}(x+y/2)f(y^2)q(x-y/2) \\ J_P^i &= \int dy\bar{q}(x+y/2)f(y^2)i\gamma^5\tau^i q(x-y/2). \end{aligned} \tag{2}$$

Here the form factor $f(y)$ characterizes a region of a quark-antiquark interaction. In the original NJL-model the form factor was chosen to be a δ-function (or unity in momentum space). The Lagrangian (1) is invariant under the global axial ($q \to e^{i\gamma^5 \vec{\tau}\vec{\theta}}q$) and vector ($q \to e^{i\vec{\tau}\vec{\theta}}q$) transformations.

The standard way of the bosonization of the NJL-model may be found in many papers (see for instance [2, 3]) so that we just give a short sketch of some points which will be needed further. Let us consider the vacuum generating functional

$$Z = \int \delta q \int \delta \bar{q} \exp\{i \int dx L^{SI}_{NJL}\} \tag{3}$$

(an infinite renormalization constant is omitted).

Using the Gaussian transformation for the quadratic interaction of quark currents and then integrating over quark fields one obtains

$$Z = \int \delta\sigma \int \delta\vec{\pi} \exp\{iW_{\rm eff}[\sigma, \vec{\pi}]\} \tag{4}$$

with the effective action $W_{\rm eff}$ given by

$$W_{\rm eff}[\sigma, \vec{\pi}] = -\frac{m_0^2}{2} \int dx[\sigma^2(x) + \pi^2(x)] - iN_c \mathrm{tr} \ln[i \not{\partial} - \tilde{\sigma} - i\gamma^5 \tilde{\pi}], \tag{5}$$

where N_c is a number of colors, $m_0^2 = 1/G$ is a bare meson mass, and the fields $\tilde{\sigma}$ and $\tilde{\pi}$ are given by

$$\tilde{\sigma}(x_1, x_2) = \sigma(\frac{x_1 + x_2}{2}) f((x_1 - x_2)^2) \qquad \tilde{\pi} = \vec{\tau}\vec{\pi}(\frac{x_1 + x_2}{2}) f((x_1 - x_2)^2). \tag{6}$$

Assuming that the field σ has a nonvanishing vacuum expectation σ_0

$$\sigma(x) = s(x) + \sigma_0 \tag{7}$$

and varying the action (5) $\delta W_{\rm eff}[\sigma_0, 0]/\delta\sigma_0 = 0$, one obtains a gap equation

$$1 = 4GN_cN_f i \int \frac{d^4k}{(2\pi)^4} \frac{f^2(k^2)}{k^2 - \Sigma^2(k^2)} \tag{8}$$

with the quark mass operator defined by

$$\Sigma(k^2) = \sigma_0 f(k^2) \tag{9}$$

where $f(k^2)$ is the Fourier-transform of the vertex form factor.

In the calculation of physical values [4], the momentum-dependent mass operator is approximated by an effective mass $<\Sigma(k^2)>= m_q$ (we neglect here the bare quark masses). The integral (8) is calculated by transition to the Euclidean region $k^0 \to ik_4$, so that $k^2 \to -k_E^2$. This procedure is well-defined for a wide class of form factors $f(k^2)$ decreasing rapidly in the Euclidean region (see, for details [9]).

Further we would like to show how to extract the kinetic terms from Eq. (5). To do this, consider the leading order in the series of Eq. (5)

$$W^{(2)}_{\rm eff} = -\frac{m_0^2}{2}\int dx(s^2+\vec{\pi}^2)+\frac{i}{2}N_c {\rm tr}[S(\tilde{s}+i\gamma^5\tilde{\pi})]^2, \tag{10}$$

where we have introduced the notation for the quark propagator

$$S(x)=[i\not{\partial}-\Sigma(-\partial^2)]^{-1}\delta(x). \tag{11}$$

After simple transformations, one obtains

$$W^{(2)}_{\rm eff}=\frac{1}{2}\sum_{\phi=s,\pi}\int dx_1\int dx_2\phi(x_1)\{-m_0^2\delta(x_1-x_2)+\Pi_\phi(x_1-x_2)\}\phi(x_2) \tag{12}$$

with $\Pi_\phi(x)$ given by

$$\Pi_\phi(x)=iN_cN_f\int dy_1\int dy_2 f(y_1^2)f(y_2^2){\rm tr}[S(x-\frac{y_1+y_2}{2})\Gamma_\phi S(-x-\frac{y_1+y_2}{2})\Gamma_\phi], \tag{13}$$

where

$$\Gamma_\phi = I(\phi=s),\ \text{or}\ \ i\gamma^5(\phi=\pi).$$

Further we represent the Fourier-transform of the two-point function of Eq. (13) in the form

$$\Pi_\phi(p^2)=\int dx e^{ipx}\Pi_\phi(x)=\Pi_\phi(m_\phi^2)+\Pi'_\phi(m_\phi^2)(p^2-m_\phi^2)+\Pi^{\rm ren}_\phi(p^2),$$

where m_ϕ^2 is the physical meson mass. Using this expansion one obtains

$$\begin{aligned} W^{(2)}_{\rm eff} &= \frac{1}{2}\sum_{\phi=s,\pi}\{\int dx\phi(x)\left[(-m_0^2+\Pi_\phi(m_\phi^2))+(\Box-m_\phi^2)\Pi'_\phi(m_\phi^2)\right]\phi(x)+ \\ &+\int dx_1\int dx_2\phi(x_1)\Pi^{\rm ren}_\phi(x_1-x_2)\phi(x_2)\}. \end{aligned} \tag{14}$$

It is readily seen that if we require fulfilment of the condition

$$
\begin{aligned}
1 &= G\Pi_\phi(m_\phi^2) \qquad (15)\\
&= iGN_cN_f\int\frac{d^4k}{(2\pi)^4}f^2(k^2)\\
&\cdot\mathrm{tr}\left\{\Gamma_\phi\left[\frac{1}{\not k+\not p/2-\Sigma((k+p/2)^2)}\right]\Gamma_\phi\left[\frac{1}{\not k-\not p/2-\Sigma((k-p/2)^2)}\right]\right\}_{p^2=m_\phi^2}
\end{aligned}
$$

the physical pole appears in the meson Green function.

Putting the pion mass in Eq.(16) to zero one has the gap equation (8), thereby reproducing the Goldstone theorem.

Scaling the fields ϕ (s or π) in Eq. (5) by the factor $1/\sqrt{\Pi'_\phi(m_\phi^2)}$ one obtains $W_{\rm eff}$

$$
W_{\rm eff}[s,\vec{\pi}]=\frac{1}{2}\sum_{\phi=s,\pi}\{\int dx\phi(x)(\Box-m_\phi^2)\phi(x)+\sum_{n=2}^{\infty}\frac{1}{n}\mathrm{tr}\left[S\frac{\hat{\phi}}{\sqrt{\Pi'_\phi(m_\phi^2)}}\right]^n\}. \qquad (16)
$$

One can see that the only connection of this expression with the original NJL-Lagrangian is via the quark mass operator $\Sigma(k^2)$ in the gap equation (8). Hereafter we shall use the approximation $<\Sigma(k^2)>=m_q$ (see, [4, 5] for details) for the calculation of physical observables.

We would like to remark that the effective action (17) can be obtained from the quantum field theory defined by the following Lagrangian

$$
L=L_0+L_{int}\,. \qquad (17)
$$

where

$$
\begin{aligned}
L_0 &= \bar{q}(i\not\partial-m_q)q+\frac{1}{2}s(\Box-m_s^2)s+\frac{1}{2}\vec{\pi}(\Box-m_\pi^2)\vec{\pi} \qquad (18)\\
L_{int} &= \frac{g_s}{\sqrt{2}}s(x)J_S(x)+\frac{g_\pi}{\sqrt{2}}\vec{\pi}(x)\vec{J}_P(x) \qquad (19)
\end{aligned}
$$

if the renormalization constants of the meson fields are set equal to zero

$$Z_\phi = 1 - \frac{g^2}{2}\Pi'_\phi(m_\phi^2) = 0. \tag{20}$$

This condition reflects the composite nature of the hadrons (dressed states in quantum field theory). It is the so-called *compositeness condition* discussed in many papers (see, for instance [11, 12] and the applications in [9, 10]).

Our formulation of the NJL-model with separable interaction may be extended to describe the interactions of any physical states. For instance, we give here the Lagrangians describing octets of vector (axial), pseudoscalar (scalar) mesons, and baryons.

1. Mesons $M = \frac{1}{\sqrt{2}}\sum\limits_0^8 \lambda^i\phi^i$.

$$L_M^0(x) = \pm\frac{1}{2}\mathrm{tr}M(x)(\Box - m_M^2)M(x) \qquad (+ \text{ for S, P} \; - \text{ for V, A}) \tag{21}$$

$$L_M^{int}(x) = g_M\int dy f(y^2)\bar{q}(x+y/2)\Gamma_M M(x)q(x-y/2). \tag{22}$$

2. Baryons $B = \frac{1}{\sqrt{2}}\sum\limits_1^8 \lambda^i\psi^i$.

$$L_B^0(x) = \mathrm{tr}\bar{B}(x)(i\not\partial - m_B)B(x), \tag{23}$$

$$\begin{aligned} L_B^{int}(x) = {} & \bar{B}^{km}(x)\int dy_1\int dy_2\int dy_3\delta\left(x - \frac{y_1+y_2+y_3}{3}\right) \\ & \cdot f\left((\frac{y_3-y_2}{2\sqrt{3}})^2\right) f\left((\frac{y_2+y_3-2y_1}{6})^2\right) \\ & \cdot\left\{ig_V J_V^{mk}(y_1,y_2,y_3) + g_T J_T^{mk}(y_1,y_2,y_3) + \mathrm{circle}(1,2,3)\right\} + \mathrm{h.c.} \end{aligned} \tag{24}$$

where

$$J_V^{mk}(y_1,y_2,y_3) = \lambda_i^{mm_1}\gamma^\mu\gamma^5 q_{a_1}^{m_1}(y_1)\left(q_{a_2}^{m_2}(y_2)\varepsilon^{km_2n}\lambda_i^{nm_3}C\gamma^\mu q_{a_3}^{m_3}(y_3)\right)\varepsilon^{a_1a_2a_3}, \tag{25}$$

$$J_T^{mk}(y_1,y_2,y_3) = \lambda_i^{mm_1}\sigma^{\mu\nu}\gamma^5 q_{a_1}^{m_1}(y_1)\left(q_{a_2}^{m_2}(y_2)\varepsilon^{km_2n}\lambda_i^{nm_3}C\sigma^{\mu\nu}q_{a_3}^{m_3}(y_3)\right)\varepsilon^{a_1a_2a_3} \quad (26)$$

The notation implied is as follows: k,m,n and a are flavor and color indices, $\sigma^{\mu\nu} = \frac{i}{2}[\gamma^\mu,\gamma^\nu]$, and C is the charge conjugation matrix, respectively. The choice of variables in the form factor of the separable interaction implies the use of the center of mass frame

$$y_1 = x - 2\xi_1 \qquad y_2 = x + \xi_1 - \sqrt{3}\xi_2 \qquad y_3 = x + \xi_1 + \sqrt{3}\xi_2$$

so that

$$\xi_1 = \frac{y_2 + y_3 - 2y_1}{6} \qquad \xi_2 = \frac{y_3 - y_2}{2\sqrt{3}}.$$

Now we introduce the electromagnetic interaction into this scheme. Note that this was done in Ref.[4] by using the minimal substitution $\partial^\mu \to \partial^\mu - ie_q A^\mu$ both in the free quark Lagrangian and in the interaction part which has a form factor. Restoring gauge invariance in this case requires a complicated procedure which is fairly arbitrary.

Here we would like to suggest to introduce the electromagnetic fields to the interaction Lagrangian using the time-ordering P-exponent. In this case the gauge invariant meson-quark vertex has the form

$$\begin{aligned} L_M^{\rm int}(x) &= g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1+y_2}{2}\right) f\left((y_1-y_2)^2\right) \qquad (27) \\ &\cdot \bar q(y_1) P \exp\left\{ieQ\int_{y_1}^{x} dz^\mu A^\mu(z)\right\}\Gamma_M M(x) P\exp\left\{ieQ\int_x^{y_2} dz^\mu A^\mu(z)\right\} q(y_2) \end{aligned}$$

where $Q = \frac{1}{2}(\lambda^3 + \frac{1}{\sqrt{3}}\lambda^8) = \mathrm{diag}(2/3, -1/3, -1/3)$.

For neutral mesons one obtains

$$L_{M^0}^{\rm int}(x) = g_M \int dy_1 \int dy_2 \delta\left(x - \frac{y_1+y_2}{2}\right) f\left((y_1-y_2)^2\right) \qquad (28)$$

$$\cdot\bar{q}(y_1)P\exp\left\{ieQ\int\limits_{y_1}^{y_2}dz^\mu A^\mu(z)\right\}\Gamma_M M^{(0)}(x)q(y_2).$$

For baryons this interaction is introduced in a similar way.

We shall use the S-matrix defined by

$$S = T\exp\{i\int dx L^{\rm int}(x)\} \tag{29}$$

to derive one-loop quark diagrams describing the physical processes. The T-product is defined in a standard manner

$$<0|T(q(x)\bar{q}(y))|0> = \int\frac{d^4k}{(2\pi)^4i}e^{-ik(x-y)}\frac{1}{m_q - \not{k}}. \tag{30}$$

The hadron-quark coupling constants g_M in Eq. (23) and g_B in Eq.(25) are defined from the compositeness condition (21).

3. MODEL PARAMETERS AND PION DECAY CONSTANTS

First we would like to discuss the model parameters. Of course, the form factor $f(k^2)$ characterizing the composite structure of hadron is an unknown function. Detailed analysis of form factors is presented in [17]. Here, we consider one of kinds of widely used form factors :

- monopole $f(k^2) = \frac{\Lambda^2}{\Lambda^2 - k^2}$

All Feynman diagrams are calculated in the Euclidean region ($k^2 = -k_E^2$) where the form factors decrease rapidly so that no ultraviolet divergences arise. For convenience the form factors are chosen to be dimensionless.

The three-dimensional Fourier-transforms of the form factors can be considered as nonrelativistic potentials (in Born approximation). Putting $k^0 = 0$ one can get

$$V(r) = \int\frac{d^3\vec{k}}{(2\pi)^3}e^{i\vec{k}\vec{r}}f(-\vec{k}^2) = \Lambda^3 v(r\Lambda). \tag{31}$$

We obtain

• monopole $v(r) = \frac{1}{4\pi r}e^{-r}$

Thus there are two adjustable parameters, Λ characterizing the region of quark-antiquark interaction, and the constituent quark mass m_q. We shall define these parameters by fitting the experimental pion decay constant f_π ($f_\pi^{\rm expt}$= 132 MeV) and $g_{\pi\gamma\gamma}$ ($g_{\pi\gamma\gamma}^{\rm expt}$= 0.276 GeV^{-1}).

We shall imply that all masses and momenta inside the Feynman integrals are expressed in the unit Λ. Also we shall neglect the pion mass when calculating the physical pion decay constants.

1. Pion-quark coupling constants.

As mentioned above the pion-quark coupling constants are defined from the compositeness condition (21) with the pion mass operator given by

$$\Pi_\pi(p^2) = 6\int\frac{d^4k}{(2\pi)^4 i}f^2(k^2)\mathrm{tr}\left\{\gamma^5\left[\frac{1}{m_q - \not k - \not p/2}\right]\gamma^5\left[\frac{1}{m_q - \not k - \not p/2}\right]\right\}. \qquad (32)$$

Neglecting the pion mass one has

$$\left(\frac{3g_\pi^2}{4\pi^2}\right)^{-1} = \frac{1}{4}\int\limits_0^\infty du\,u f^2(-u)\frac{(3m_q^2+2u)}{(m_q^2+u)^3}. \qquad (33)$$

2. Pion weak decay

The weak decay of the pion is defined by the diagram of Fig.1. After simple transformations of the Feynman integral, we have

$$\begin{aligned} f_\pi &= \frac{3g\pi}{4\pi^2}m_q\int\frac{d^4k}{\pi^2 i}\frac{f(k^2)}{[m_q^2-(k+p/2)^2][m_q^2-(k-p/2)^2]} \\ &\simeq \frac{3g\pi}{4\pi^2}m_q\int\limits_0^\infty du\,u f(-u)\frac{1}{(m_q^2+u)^2}. \end{aligned} \qquad (34)$$

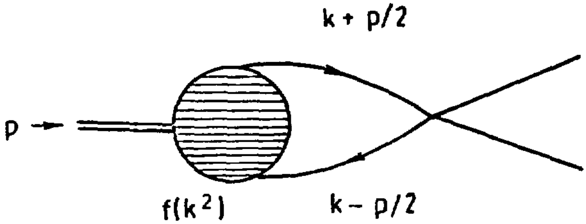

Fig.1. The diagrams describing the weak pion decay.

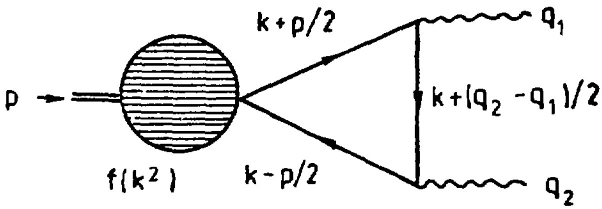

Fig.2. The diagrams describing the pion two-photon decay.

Table 1

Form Factors	Λ (MeV)	m_q (MeV)	f_π (MeV)		$g_{\pi^0\gamma\gamma}$ (GeV^{-1})	
			NJL SI	EXP [14]	NJL SI	EXP [14]
monopole	400	267	132	132	0.251	0.276

3. Pion two-photon decay

The two-photon decay of the pion is defined by the diagram of Fig.2. After similar transformations we have

$$G_{\pi\gamma\gamma}(p^2,q_1^2,q_2^2) = \frac{g_\pi}{2\sqrt{2}\pi^2}\frac{m_q}{\Lambda^2}\cdot\int\frac{d^4k}{\pi^2 i}\frac{f(k^2)}{[m_q^2-(k+p/2)^2][m_q^2-(k-p/2)^2]} \tag{35}$$

$$\cdot\frac{1}{[m_q^2-(k+(q_1-q_2)/2)^2]}.$$

The two-photon decay coupling constant is obtained from Eq. (36) where both photons are on the mass shell:

$$g_{\pi\gamma\gamma} = G_{\pi\gamma\gamma}(m_\pi^2,0,0) \simeq \frac{g_\pi}{2\sqrt{2}\pi^2}\frac{m_q}{\Lambda^2}\int\limits_0^\infty du u f(-u)\frac{1}{(m_q^2+u)^3}. \tag{36}$$

The numerical results for the physical observables for the best fit are shown in Table 1. Inserting the best values for Λ and m_q into the gap equation (8), gives $G = 3.039\pi^2\Lambda^2$ for the monopole form factor. One can check also that the low-energy relation $f_\pi g_{\pi\gamma\gamma} = 1/(2\sqrt{2}\pi^2)$ is reproduced with good accuracy ($\leq 7\%$).

4. The $\gamma^*\pi^0 \to \gamma$ form factor

The form factor for the $\gamma^*\pi^0 \to \gamma$ transition was measured for space-like momentum transfer $Q^2 > 0$ of the virtual photon [13] by making use of the two-photon process $\gamma\gamma \to \pi^0$, where the two photons are radiated virtually by colliding e^+e^- beams.

In the extended NJL model this form factor is expressed as

$$F_{\gamma\pi}(Q^2) = e^2 G_{\pi\gamma\gamma}(m_\pi^2,-Q^2,0) \simeq e^2\frac{g_\pi}{2\sqrt{2}\pi^2}\frac{m_q}{\Lambda^2}R_{\pi\gamma}(Q^2/\Lambda^2) \tag{37}$$

with the structure function $R_{\pi\gamma}$ given in the Appendix.

Our results for monopole form factor are shown in Fig.3. The experimental data are described by the monopole fit with

$$F(Q^2) = \frac{e^2 g_{\pi\gamma\gamma}}{1+Q^2/\Lambda_\pi^2} \qquad \Lambda_\pi = 0.77\text{GeV}. \tag{38}$$

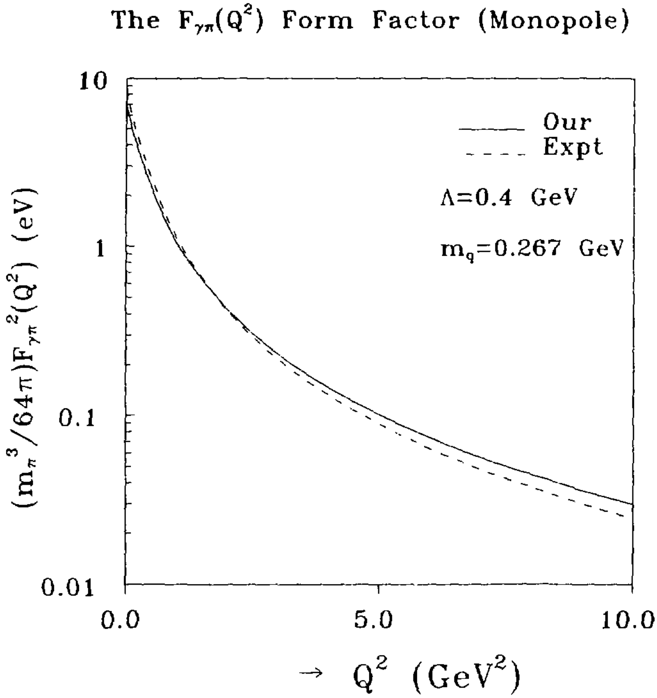

Fig.3. The form factor of the $\gamma^*\pi^0 \to \gamma$ transition for spacelike photons $0 \le Q^2 \le 5\mathrm{GeV}^2$. The dashed line is the result of the monopole fit with $\Lambda = 770$ MeV. The solid lines are our predictions for monopole form factors. The experimental data are from [13].

Table 2

Vertex Function	$r_{\pi\gamma}$ (fm) NJL	$r_{\pi\gamma}$ (fm) EXP [14]
monopole	0.655	0.65 ± 0.03

The radius for $\gamma^*\pi^0 \to \gamma$ transition is defined by

$$< r^2_{\pi\gamma} >= -6\frac{F'_{\pi\gamma}(0)}{F_{\pi\gamma}(0)} \cdot \tag{39}$$

where

$$F_{\pi\gamma}(0) = \int\limits_0^\infty duu\frac{f(-u)}{(m_q^2+u)^3} \qquad F'_{\pi\gamma}(0) = -\frac{1}{2}\frac{m_q^2}{\Lambda^4}\int\limits_0^\infty duu\frac{f(-u)}{(m_q^2+u)^5} \cdot \tag{40}$$

The numerical results for the radius $r_{\pi\gamma}$ are given in Table 2. Excellent agreement with the available experimental data is reached.

5. The pion electromagnetic form factor

The pion charge form factor is defined by the diagrams of Fig.4. These diagrams are not gauge invariant separately. The sum of the diagrams can be written as

$$\begin{aligned}
\Lambda^\mu(p,p') \;=\; & \frac{q^\mu}{q^2}[\Pi_\pi(p^2)-\Pi_\pi(p'^2)] + \\
& +\frac{3g_\pi^2}{4\pi^2}\int\frac{d^4k}{4\pi^2 i} f\left(\left[k+\frac{p}{2}\right]^2\right) f\left(\left[k+\frac{p'}{2}\right]^2\right) \\
& \cdot \mathrm{tr}\left[\gamma^5 S(k+p')\left(\gamma^\mu - \frac{q^\mu \not q}{q^2}\right) S(k+p)\gamma^5 S(k)\right] + \\
& +\frac{\eta^\mu}{\eta^2}\frac{3g_\pi^2}{4\pi^2}\int\frac{d^4k}{4\pi^2 i}\int\limits_0^1 dt f(k^2) f'\left((k+qt/2)^2\right) \\
& \cdot k\eta \cdot \mathrm{tr}\left[\gamma^5 S\left(k+\frac{p'}{2}\right)\gamma^5 S\left(k-\frac{p'}{2}\right)\right] \\
& -\mathrm{tr}\left[\gamma^5 S\left(k+\frac{p}{2}\right)\gamma^5 S\left(k-\frac{p}{2}\right)\right] \cdot
\end{aligned} \tag{41}$$

where we use the following notation

$$\eta^\mu = P^\mu - q^\mu\frac{P\cdot q}{q^2} \qquad P = p + p'.$$

The Ward-Takahashi identity directly follows from Eq.(42)

$$q_\mu \Lambda^\mu(p,p') \equiv \Pi_\pi(p^2) - \Pi_\pi(p'^2). \tag{42}$$

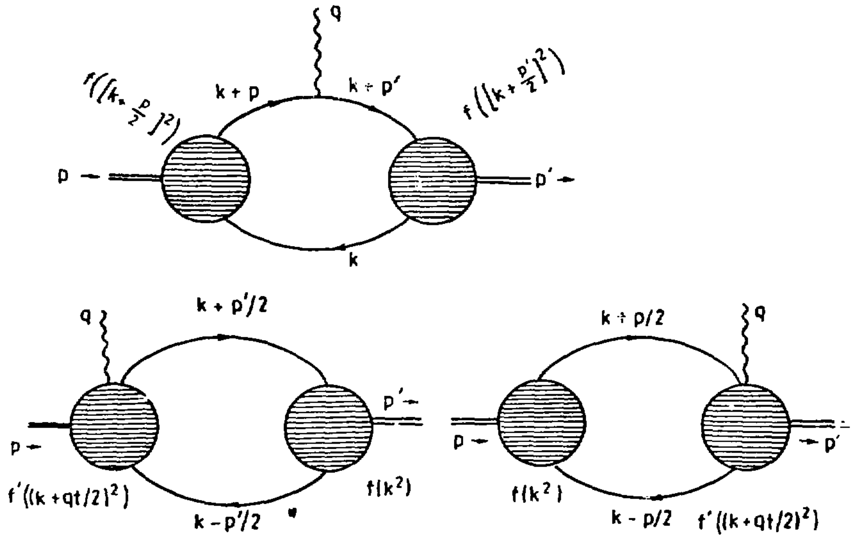

Fig.4. The diagrams describing the pion charge form factor.

Taking $q = 0$, one obtains on the one hand

$$\Lambda^\mu(p,p) = \frac{\partial\Sigma_\pi(p^2)}{\partial p^\mu} = 2p^\mu \frac{\partial\Sigma_\pi(p^2)}{\partial p^2}\,, \tag{43}$$

where $\Sigma_\pi(p^2) \equiv (3g_\pi^2/4\pi^2)\Pi_\pi(p^2)$, and, on the other hand,

$$\Lambda^\mu(p,p) = 2p^\mu F_\pi(0), \tag{44}$$

where $F_\pi(0)$ is the pion charge form factor at the origin. It follows from the comparison of Eq.(44) and Eq. (45) on the pion mass shell that the compositeness condition $\Sigma'(m_\pi^2) = 1$ is equivalent to the normalization of the pion form factor at the origin $F_\pi(0) = 1$.

Note that the implementation of gauge invariance in the context of the minimal substitution [4] leads to

$$\int_0^1 dt f'\left((k+qt/2)^2 + q^2t(1-t)/4\right) = \frac{f((k+q/2)^2) - f(k^2)}{kq + q^2/4}$$

in Eq.(42) while the gauge invariant vertex (28) leads to

$$\int_0^1 dt f'\left((k+qt/2)^2\right).$$

For practical purposes this difference is not important in our case.

The numerical computation of the pion charge form factor is performed in the Breit frame

$$q=(0,\vec{q}), \qquad p=(E,\vec{q}/2) \qquad p'=(E,-\vec{q}/2) \qquad E=\sqrt{m_\pi^2+\vec{q}^2}. \qquad (45)$$

The analytical expressions for the form factor are given in the Appendix.

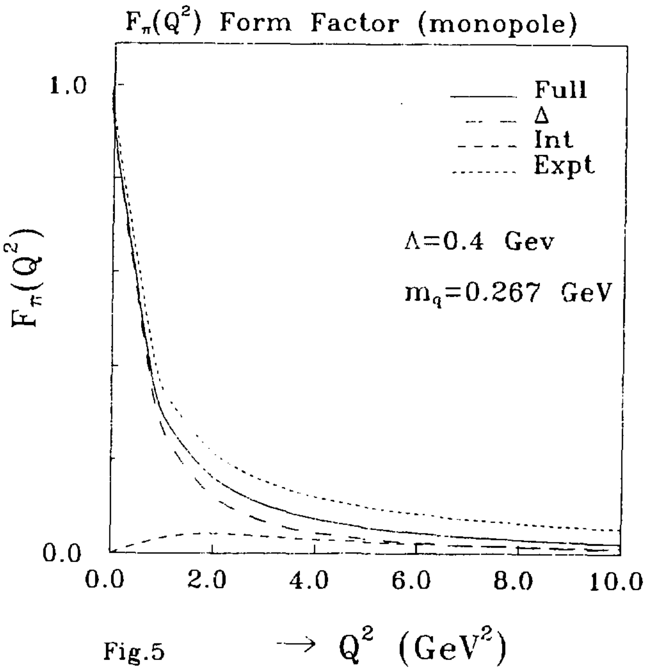

Fig.5. The pion charge form factor $F_\pi(Q^2)$ for spacelike photons $0 \le Q^2 \le 10\mathrm{GeV}^2$. Separate contributions from the triangle and bubble diagrams are shown as dashed and dotted lines, respectively. The solid line is the total result. The experimental data are from [14].

Our results for monopole form factors are shown in Fig.5 to 6. The contributions to the pion charge radius coming from the triangle (Δ) and bubble (o) diagrams are written down

$$< r_\pi^2 >^\Delta = -6\frac{1}{\Lambda^2}\frac{\Phi_1(0)}{\Phi_0(0)} \qquad < r_\pi^2 >^\circ = -6\frac{1}{\Lambda^2}\frac{\Phi_2(0)}{\Phi_0(0)} \cdot \tag{46}$$

where

$$\Phi_1 = -\int_0^\infty duu\{\frac{f^2(-u)}{(m_q^2+u)^5}\,[\frac{1}{12}u^2+\frac{3}{8}m_q^2u+\frac{3}{8}m_q^4\,]+u\frac{[f'(-u)\,]^2}{(m_q^2+u)^3}\,[\frac{7}{96}u+\frac{3}{32}m_q^2\,]\}$$

$$\Phi_2 = \int_0^\infty duu\{\frac{1}{16}\frac{f^2(-u)}{(m_q^2+u)^5}m_q^2\,[m_q^2-u\,]-\frac{1}{48}\frac{[f'(-u)\,]^2}{(m_q^2+u)^3}u^2\}$$

$$\Phi_0 = \int_0^\infty duu\frac{f^2(-u)}{(m_q^2+u)^3}\,[\frac{3}{4}m_q^2+\frac{u}{2}\,]\cdot$$

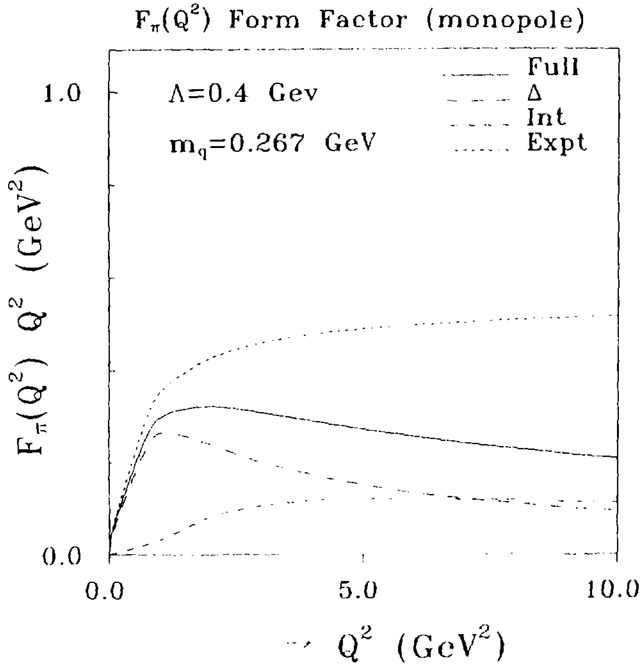

Fig.6.The pion charge form factor multiplied by Q^2 for spacelike photons $0 \le Q^2 \le 10\text{GeV}^2$. The notation is the same as in Fig.5.

The numerical results for the radius are given in Table. 3. One can see that our results are in good agreement with the available experimental data till 1 GeV^2.

Table 3

Form	NJL SI			EXP [14]
Factor	$< r_\pi^2 >^\Delta$ (fm^2)	$< r_\pi^2 >^\circ$ (fm^2)	total (fm^2)	(fm^2)
monopole	0.545	-0.012	0.533	0.430

6. SUMMARY

We have formulated the Nambu-Jona-Lasinio model with separable interaction using the Lagrangian with the compositeness condition and non-minimal inclusion of the electromagnetic interaction. This allows to calculate any low-energy physical processes on one-loop level maintaining the relativistic and electromagnetic gauge invariance in each step of calculation. On one hand the form factors in the hadron-quark vertices take into account the composite structure of hadrons thereby being related to a quark-antiquark potential, on the other hand, they make the Feynman integrals convergent.

We have calculated the pion weak decay constant, the two-photon decay width, as well as the form factor of the $\gamma^*\pi^0 \to \gamma$-transition, and the pion charge form factor. The two adjustable parameters, the range parameter Λ appearing in the separable interaction and the constituent quark mass m_q, have been fixed by fitting the experimental data for the pion decay constants.

ACKNOWLEDGMENTS

We thank G. Efimov, H. Ito, A. Korchin, A. Shebeko and M. Locher for useful discussions.

APPENDIX

(1) Explicit expression for the form factor of the $\gamma^*\pi^0 \to \gamma$ transition:

$$
\begin{aligned}
R_{\pi\gamma}(Q^2) &= \int_0^\infty du u \frac{f(-u)}{m_q^2+u} \int_0^1 d\alpha \\
&\left\{ \frac{1}{\sqrt{\left(m_q^2+u-\frac{Q^2\alpha}{2}\right)^2 + 2Q^2 m_q^2 \alpha}} \cdot \right. \\
&\cdot \frac{1}{\sqrt{\left(m_q^2+u-\frac{Q^2\alpha}{2}\right)^2 + 2Q^2 m_q^2 \alpha} + m_q^2 + u + \frac{Q^2\alpha}{2}} + \\
&+ \frac{1}{\sqrt{\left(m_q^2+u-\frac{Q^2\alpha}{2}\right)^2 + 2Q^2 \alpha (m_q^2 + 2u(1-\alpha))}} \cdot \\
&\left. \cdot \frac{1}{\sqrt{\left(m_q^2+u-\frac{Q^2\alpha}{2}\right)^2 + 2Q^2 \alpha (m_q^2 + 2u(1-\alpha))} + m_q^2 + u + \frac{Q^2\alpha}{2}} \right\} .
\end{aligned}
$$

(2) Expression for the pion charge form factor. The contributions coming from the triangle ($\triangle$) and bubble ($\circ$) diagrams are denoted by

$$
\begin{aligned}
F_\pi^\triangle(Q^2) &= \frac{\Phi_1(Q^2)}{\Phi_1(0)}, \qquad F_\pi^\circ(Q^2) = \frac{\Phi_2(Q^2)}{\Phi_1(0)} \qquad (47) \\
\Phi_1(Q^2) &= \frac{3}{\pi} \int_0^\infty dk k^3 \int_0^1 dx \frac{x^2}{\sqrt{1-x^2}} \int_{-1}^1 dy \\
&\frac{f(-k^2) f\left(-k^2 - \frac{Q^2}{4} - kxy\sqrt{Q^2}\right)}{\left[m_q^2 + k^2 + \frac{3}{2}kxy\sqrt{Q^2} + \frac{Q^2}{2}\right]^2 + \frac{k^2Q^2}{4}(1-x^2)} \\
&\cdot \left\{ 2 \frac{\left[m_q^2 + \frac{(1+2x^2)}{3}k^2 - \frac{1}{2}kxy\sqrt{Q^2}\right]\left[m_q^2 + k^2 + \frac{1}{2}kxy\sqrt{Q^2} + \frac{Q^2}{4}\right]}{\left[m_q^2 + k^2 - \frac{1}{2}kxy\sqrt{Q^2}\right]^2 + \frac{k^2Q^2}{4}(1-x^2)} - 1 \right\} .
\end{aligned}
$$

$$\Phi_2(Q^2) = \frac{4}{\pi}\sqrt{Q^2}\int_0^\infty dk k^6 \int_0^1 dx\, x^3\sqrt{1-x^2}\int_{-1}^1 dy y \int_0^1 dt$$

$$\cdot f(-k^2) f'(-k^2 - \frac{Q^2 t^2}{4} - kxyt\sqrt{Q^2})$$

$$\cdot \frac{m_q^2 + k^2}{\left[\left(m_q^2 + k^2 + \frac{1}{2}kxy\sqrt{Q^2}\right)^2 + \frac{k^2Q^2}{4}(1-x^2)\right]}$$

$$\cdot \frac{1}{\left[\left(m_q^2 + k^2 - \frac{1}{2}kxy\sqrt{Q^2}\right)^2 + \frac{k^2Q^2}{4}(1-x^2)\right]}.$$

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Received by Publishing Department
on July 7, 1993.