93-25

объединенный
институт
ядерных
исследований
дубна

E2-93-25

O.Teryaev

AXIAL ANOMALY IN $e^+e^- \to \gamma\gamma$ AND ANYON SUPERCONDUCTIVITY

Submitted to «Physics Letters B»

1993

1 Introduction

The so-called EMC Spin Crisis and its resolution via the Axial Anomaly contribution (see, e.g., [1] and Ref. therein) naturally lead to the question whether this contribution manifests itself in various spin-dependent processes. Recently [2] it has been shown that such well-known spin-flip amplitudes as the anomalous magnetic moment are really closely related to the Axial (Chiral) Anomaly.

In the present Letter the concept of Axial Anomaly is slightly generalized (Section 2) to obtain the general method for "Hunting the Anomaly" (Section 3). It is applied to transverse polarized electron and positron annihilation (Section 4). As a result, an anomalous pole arises in the zero fermion mass limit. Its possible contributions to the anyon superconductivity are discussed in Section 5.

2 Axial Anomaly without the axial current

One should ask whether it is possible to consider massless singularities of diagrams different from the triangle one as manifestations of the Axial Anomaly. To verify this, a slight generalization of the concept of anomaly is required.

Usually the anomaly is understood as violation of the classical equation of motion for the axial current when the classical field is replaced by quantum operator. In the massless case it is violation of classical symmetry, because the conservation of the axial current in this case is just the consequence of the Noether theorem. From this point of view the triangle diagram, being the two-photon matrix element of the l.h.s. of the anomaly equation, is of course unique.

The classical symmetry, however, may be understood as a certain "naive" restriction for the quantum amplitudes. In the chiral symmetry case this restriction is quite obvious: the emission or absorption of the photon does not change the massless fermion helicity. This results, e.g., in the "Helicity conservation rule" in QCD [3] valid in all leading twist diagrams to all orders of perturbation theory.

Nevertheless, this restriction may be violated provided the photon is soft and collinear. The kinematical smallness in the nominator proportional to the fermion mass (it is assumed to be small but finite) is compensated by the mass singularity in the denominator. As a result, the helicity flip amplitude zero-mass limit is finite. This phenomenon was mentioned as early as 1964 by Lee and Nauenberg in their classical paper [4]. It was, in fact, the discovery of the generalized infrared approach to the axial anomaly I discuss here.

A crucial step was made by Dolgov and Zakharov [5] in 1970 immediately after the standard ultraviolet treatment of the anomaly by Adler, Bell and Jackiw [6]. The zero mass limit was shown to produce the $\delta(q^2)$ singularity for the imaginary part of the AVV triangle diagram. The corresponding massless pole leads to the nonzero axial current divergence matrix element for massless fermions.

The present Letter is devoted to the application of the Dolgov and Zakharov approach to diagrams different from the triangle one. I am following the original Lee and Nauenberg idea, not restricted to any perfect diagram. It is of course possible to call every new singularity the new anomaly. However, I prefer to use the same term "Axial Anomaly" for all singularities associated with the helicity flip in the zero fermion mass limit. I would like to remind the history of the discovery of longitudinal real photons interaction via the box diagram, made by Gorski, Ioffe and Khodjamirian [7] few years ago. It was called "new anomaly", but later was related by the authors to the well-known conformal anomaly.

3 The general method

It is just a straightforward generalization of the infrared approach [5]: the anomaly manifests itself as a massless pole in the zero fermion mass limit. Twenty years ago Dolgov and Zakharov discovered, as mentioned above, that the imaginary part of the famous triangle diagram (Fig. 1a) tends not to zero, but to the δ-function in the limit $m \to 0$

$$ImM \sim \frac{m^2}{(q^2)^2} \ln\frac{1+\beta}{1-\beta} \to_{m\to 0} const \cdot \delta(q^2). \tag{1}$$

Here q is the axial current momentum, $\beta = \sqrt{1-x}(x = 4m^2/q^2)$. The factor m^2 in the numerator is due to the helicity flip by the axial source and one photon. This kinematical smallness is compensated by the dynamical one in the denominator of the "horizontal" propagator when the scattering angle is small.

This result implies the appearance of pseudoscalar (due to photon pair quantum numbers) massless excitation. For massless quarks it is just the pion. This proof of its existence (the so-called t'Hooft consistency principle [8]) is a complement to the Goldstone theorem.

How much "anomalous" is such a behaviour (1)? It would seem not very much. The imaginary part of an arbitrary form factor $F(q^2, m^2)$ of the dimension m^{-2} (like the total cross-section), which is suppressed in the chiral limit as m^2, can be written as

$$ImF(q^2, m^2) = \frac{4m^2}{(q^2)^2} f(x),\ x \equiv \frac{4m^2}{q^2}. \tag{2}$$

Integrating it from the threshold $q^2 = 4m^2$ up to infinity one has

$$\int_{4m^2}^{\infty} dq^2 ImF(q^2, m^2) = \int_0^1 dx f(x) \equiv C. \tag{3}$$

Provided the integral converges and taking into account that the $ImF(q^2, m^2)$ becomes narrower and higher as $m \to 0$(it is clear from the expression $ImF = x^2 f(x)/4m^2$), one can conclude:

$$ImF(q^2, m^2) \to_{m\to 0} C\delta(q^2). \tag{4}$$

There is a counterpart of this unexpected behaviour: in the "opposite" limit $m \to \infty$ one should obtain zero. It is just a manifestation of the well-known phenomenon of the cancellation of normal and anomalous divergences (see, e.g., [11, 1]): F contains both the normal and anomalous contributions, the latter being its singular zero mass limit. It has to be subtracted in order to obtain the normal one. It is impossible to observe this cancellation directly in the imaginary part that is equal to zero just because the limit under consideration corresponds to an unphysical region. One should write down the dispersion relation

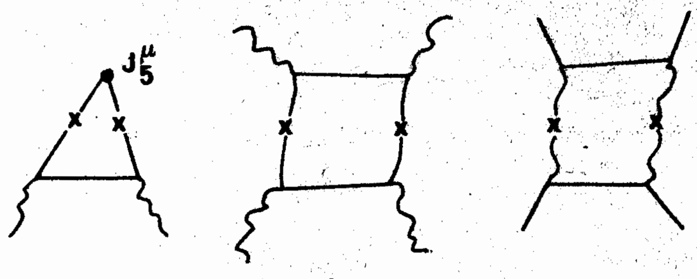
$$F(q_0^2, m^2) = \frac{1}{\pi}\int_{4m^2}^{\infty} \frac{ImF(q^2, m^2)dq^2}{q^2 - q_0^2} = \frac{1}{\pi}\int_0^1 \frac{f(x)dx}{4m^2x^{-1} - q_0^2} \to_{m\to\infty} \frac{1}{4\pi m^2}\int_0^1 f(x)x dx \to 0. \tag{5}$$

Note that the last integral converges better than (3), and no subtraction is needed. The dispersion relation for the axial current divergence, however, requires a subtraction: it is just the axial anomaly [5].

4 The anomalous contribution to $e^+ \uparrow e^- \downarrow \to \gamma\gamma$

What are candidates for the anomalous behaviour apart from the mentioned triangle diagram? Note that its upper side is only the fermion source with the required parity, and the anomalous behaviour is due to the lower one, namely, the polarized Compton amplitude. It is therefore promising to consider the processes resulting from this amplitude squared. One possibility (Fig. 1b) is just the one providing the anomalous gluon contribution [1] to the polarized deep-inelastic scattering. Although the anomalous contribution to its first moment [9, 10, 11] is reduced to the triangle (Fig. 1a), it is possible to extract a similar contribution, exploring the Adler–Bardeen theorem, in other moments, too [12]. Therefore, we obtain the Axial Anomaly without the axial current! The expression is very simple

$$E_g(x) = \frac{\alpha_s}{\pi}(x-1) \tag{6}$$



a b c

Fig. 1. The processes with axial–like anomalies.

and differs significantly from the whole box contribution [10, 11]. The phenomenological consequence of the difference [13] is, however, negligible.

The following natural candidate is the e^+e^-annihilation (Fig. 1c). It appears that the best way to extract the chiral–limit suppressed contribution, consists in taking the difference between the total cross-sections with parallel and antiparallel transverse polarized particles, calculated long ago [14]:

$$\Delta\sigma^T = \frac{\pi r_e^2 x^2}{4\beta}[6\beta^{-2} - 4 + (\beta^{-1} + 2\beta - 3\beta^{-3})\ln\frac{1+\beta}{1-\beta}]. \quad (7)$$

The straightforward computation [1] of the integral (3) gives:

$$\Delta\sigma^T_{m\to 0} \to \pi\alpha^2(\frac{\pi^2}{2} - 10)\delta(q^2). \quad (8)$$

It is just the axial (chiral) anomaly contribution (without the axial current again).

5 Anomalous contribution and anyon superconductivity

The transverse direction of polarization is of major importance for the manifestation of anomaly. Although one should naively expect a similar contribution to the cross-section with parallel helicities, only the logarithmic term is suppressed by m^2[14] (in contradiction with the statement of the canonical textbook [16]). The nonsuppressed term leads to the divergence of (3).

[1] It is an error (16 instead 10) in the early version [15] of this paper. However, it did not affect the sign of the anomalous contribution and its interpretation given below.

The most clear way to ensure the transverse polarization is to make electrons move within a 2–dimensional plane with spins aligned transverse to it by a magnetic field. It is just the system representing the remarkable phenomenon of anyon superconductivity [17]. As the relevant Chern–Simons term is the component of anomalous photonic current, it is of interest to discuss possible implications of the anomalous contribution (8) for the superconductivity problem.

Dealing with 2+1 - dimensional models of anyon superconductivity, one should relate them to the real 3+1 - dimensional world. Due to the uncertainty principle it is impossible to neglect the transverse degree of freedom in the coordinate and momentum space simultaneously. Usually one makes this reduction in the coordinate space, assuming the energy to be low enough to excite the transverse degree of freedom. Owing to dealing with the scattering amplitudes in the momentum representation, it seems natural to start with the 2+1 - dimensional momentum space. As high T_c superconductors represent the layer structure, the uncertainty principle immediately leads to the important conclusion: the electron cannot be confined to the single CuO layer and the coherent multilayer behaviour is of major importance. Therefore, one should choose the Neumann boundary conditions for the transverse Schrödinger equation and obtain the nonzero transverse electromagnetic current. As a result, it is not conserved in 2+1 dimensions: the effective 2+1 theory should be non–gauge invariant! Note that the nonconservation of the vector current in 2+1 can lead to the appearance of a zero-mass pole completely analogous to the ghost pole in QCD [18] (see also [1] and Ref. therein). As this pole is a signal of superconductivity [19], we have a new mechanism of it. The corresponding physical picture is the charge escape in the transverse direction and the return in another place, i.e. some 'wormhole' in 2+1 - dimensional space. Note that the commonly accepted mechanism of high-T_c superconductivity in the Luttinger liquid model also requires the interlayer coherent transport, if one passes below T_c (see, e.g.,[20]). As this theory (and, in particular, the transverse coherent transport below T_c[21]) is strongly supported by the experimental data, the incorporation of the transverse dynamics into the anyon superconductivity theory via the electromagnetic current nonconservation, seems to be reasonable. Recent papers [22],[23] are dealing with two-layer systems, but I would like to stress that a macroscopically large number of layers is required to obtain the 2+1 - dimensional momentum space.

From another point of view, pseudoscalar (just like a pion) massless excitation ϕ (Fig. 2a), governed by Eq. (8), leads to the long–range spin–dependent interaction between two fermions. Note that for sufficiently large interaction radius it may be cancelled by the normal contribution, as described in Section 1 (there are arguments in favour of the noncancellation in the *real* fermions case to be published elsewhere). This cancellation, however, is normally slow enough to allow an experimental observation of this effect.

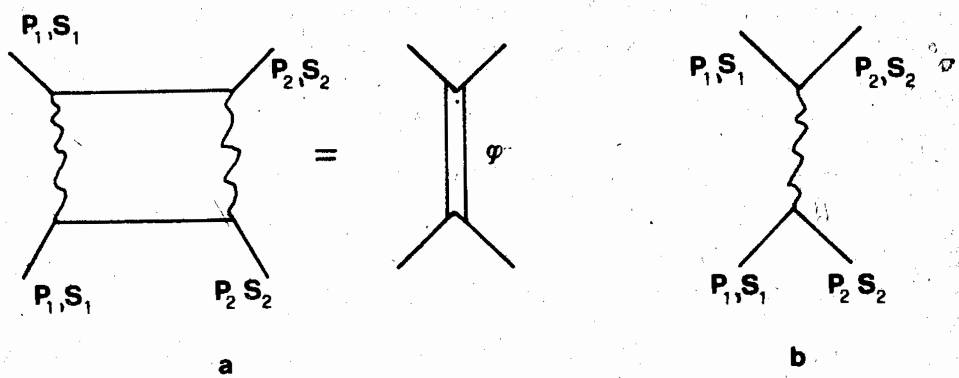


Fig.2. Anomaly correction (a) to the Coulomb potential (b).

What can we say about the sign of the anomalous interaction? It provides a term to the electron scattering amplitude at rest of type (Fig. 2a), proportional to the product of the transverse polarizations $s_1 s_2$. Note that this amplitude should be cancelled by the exchange amplitude due to the Pauli principle, and only spin singlet states of type $[p, s] - [p, (-s)]$ contribute to the long-range interaction. In the Coulomb amplitude case (Fig. 2b), the sum of the normal and exchange singlet contributions appears to be equal to the normal triplet contribution under consideration. As it is nonzero only for $s_1 = s_2$, it clearly differs in sign from the anomalous contribution (Fig. 2a). This sign difference is of a simple physical nature: the integral (3), leading to (8), is dominated by the low-energy $x \sim 1$ region. The photon pair angular momentum is then zero (just as in the familiar triangle diagram case), requiring $s_1 = -s_2$. Therefore, the anomalous interaction leads to the attraction of electrons with parallel spins. This is a new pairing mechanism necessary for the superconductivity description. Note that the anomalous attraction, as mentioned above, is confined to the plane in the momentum space. Making a Fourier transformation to the coordinate space, it is possible to obtain even a slower varying potential like lnR instead of R^{-1}. The attraction would dominate for long distances, while the repulsion for short distances. One may expect the formation of a bound state in this potential.

6 Conclusions

The Chiral Anomaly is often thought to manifest itself in the triangle diagram only, with possible rare exceptions. Nevertheless, it has been shown that it may be attributed to a wide class of chiral-limit suppressed amplitudes. The simple criterion of anomalous behaviour (3) is presented. It is applied to the polarized e^+e^- total cross-section case. As a result the existence of the anomalous pole is shown and its coupling to electrons (8) is calculated. The possible applications of this phenomenon to the anyon superconductivity problem are discussed. First, the new mechanism of anyon superconductivity arises due to the coherent multilayer behaviour and electromagnetic current nonconservation. Second, anomalous pole may lead to the long-range attraction between two electrons, and to the bound state formation. The relations between these two effects as well as to the Luttinger Liquid theory, require further investigation.

Acknowledgements

The author would like to thank Professor Abdus Salam, the International Atomic Energy Agency and UNESCO, for hospitality at the International Centre for Theoretical Physics, Trieste, where this work was started. He is indebted to A.V. Efremov for stimulating discussions and to G. Calucci, H. Fritzsch, D.I. Kazakov, E.A. Kuraev, Yu Lu and J. Strathdee for useful comments and interest in the work.

References

[1] A.V. Efremov, J. Soffer, O.V. Teryaev, Nucl.Phys. **B346** (1990) 97.

[2] O.V. Teryaev, Phys. Lett. **B265** (1991) 185.

[3] S.J. Brodsky, G.P. Lepage, Phys.Rev. **D24** (1981) 2848.

[4] T.D. Lee, M. Nauenberg, Phys.Rev. **133B** (1964) 1549.

[5] A.D. Dolgov, V.I. Zakharov, Nucl.Phys. **B27** (1971) 525.

[6] S.L. Adler, Phys.Rev **177** (1969) 2426;
J.S. Bell, R. Jackiw, Nuovo Cim. **60A** (1969) 47.

[7] A.S.Gorski, B.L.Ioffe, A.Yu.Khodjamirian, Phys.Lett. **B227** (1989) 474.

[8] G. t'Hooft in *Recent Development in Gauge Theories*, (New York, Plenum Press, 1980).

[9] J. Kodaira, Nucl. Phys. **B165** (1980) 129.

[10] G. Altarelli, G.G. Ross, Phys.Lett. **B214** (1988) 381.

[11] R.D. Carlitz, J.C. Collins, A.H. Mueller, Phys. Lett. **B214** (1988) 229.

[12] A.V. Efremov, O.V. Teryaev, in *SPIN-89*, Protvino (1990) 79.

[13] V.T. Kim, O.V. Teryaev, in *SPIN–91*, Protvino (to be published).

[14] L. Page, Phys. Rev. **106** (1957) 394.

[15] O.V. Teryaev, ICTP Preprint IC/91/390(1991).

[16] V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, *Quantum Electrodynamics* (Nauka, 1989), Sect. 88.

[17] F. Wilczek, *Fractional Statistics And Anyon Superconductivity* (World Scientific, 1990).

[18] G. Veneziano, Nucl. Phys. **B159** (1979) 213;
D.I. Dyakonov, M.I. Eides, ZhETF **81** (1981) 434.

[19] A.L. Fetter, C.B. Hanna, R.B. Laughlin, Phys. Rev. **B39** (1989) 9679;
Y.-H. Chen, F. Wilczek, E. Witten, B. Halperin, Int. J. Mod. Phys. **B3** (1989) 1001.

[20] P.W. Anderson, in *Princeton RVB Book* (to be published), Chapter 2.

[21] K. Tamasaku, Y. Nakamura, S. Uchida, Nagoya University Preprint 92-06-23(1992).

[22] Z.P. Ezawa, A. Iwazaki, Tohoku University Preprint TU-402(1992).

[23] H. Fertig, S. He, S. Das Sarma, Phys. Rev. Lett. **68** (1992) 2676.

**Received by Publishing Department
on January 26, 1993.**