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INCLUSIVE (p, n) AND ($^3He, t$) REACTIONS ON NUCLEI IN THE QUASIELASTIC AND Δ-ISOBAR EXCITATION REGION

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1. INTRODUCTION

Starting from a formalism of effective numbers for the inclusive integral cross sections of (p,n) with formation of quasielastic and Δ-isobar nuclear excitation we obtain, without free parameters, a satisfactory quantitative description of the existing experimental data. The theory gives a factorized form which allows for a useful separation of the reaction and the structure of the light and heavy reaction partners. We have now been successful in using this formalism with only small modifications for $(^3He,t)$ reactions at intermediate energies. We argue from the similarity of the cross sections for the (p,n) and $(^3He,t)$ reactions obtained by removing the form factors of $(^3He,t)$ from the cross sections and bringing the experimental data for different energies to the same energy. It is shown that the reaction mechanism for the processes (p,n) and $(^3He,t)$ is in principle essentially the same.

2. PRECIS

Inclusive reactions of the type (e,e'), (p,p'), (π,π'), (p,n), $(^3He,t)$, $(^6Li,^6He)$,... provide a basic source of information on effective NN and NΔ interactions, reaction mechanisms, and on the nuclear structure at intermediate energies. The knowledge of spin and isospin components of NN and NΔ interactions is still somewhat poor in spite of a substantial amount of works (see for example the review talks in Ref.[1]).

Thus, the theoretical understanding of these reactions is far from satisfactory [2]. Specifically, since large energy and momentum transfer is involved, the traditional methods have to be re-examined to see if approximations that were made at low q and ω are still valid. New attention has to be given to how one may correctly separate the nuclear structure effects from the mechanisms of the reactions at intermediate energies. The analysis of the processes is significantly complicated by collective excitations of the type (ΔN^{-1}), i.e. Δ-isobar plus a nucleon hole [3]. The renormalization of the interaction $NN - N\Delta$ in nuclei requires further investigations [4]. Furthermore, proper calculations of the distortions in the entrance and exit channels including the "exchange" term (the Pauli principle between the projectile nucleons and the bound target nucleons) are not clear cut [5, 6, 7]. And finally, a main subject of investigation and a major part of this paper is the connection between the mechanisms of charge-exchange on a nuclear target compared to a free proton target [8].

It is necessary to keep in mind that a characteristic feature of the inclusive treatment of cross sections is the lack of sensitivity to details of the structure of the target nuclei. Following the terminology of Refs. [5, 7] we may refer to this as the "universality" of inclusive cross sections. The "universality" in the theoretical calculations displays itself as an essential independence of the results to the choice of the basic shell model functions. For example, wave functions of the Saxon-Woods potential, oscillator wave functions, quasiclassical or even the Fermi-gas wave functions give the same cross sections within an accuracy of about 5-10%. This "universality" manifests itself in the cross sections of the charge-exchange reactions, which contain all information about the $NN - N\Delta$-interactions in nuclei and of the reaction mechanism, being smooth functions of arguments such as the mass number A and the energy of the projectile.

In [8] (see also [9]) we showed that under definite conditions one can establish a link between the cross sections of the (p,n) charge exchange reactions on nuclei and on the free proton (referred to as an "elementary" process)

$$\frac{d\sigma[A(p,n)_\Delta B]}{d\Omega_n} = \overline{N}\frac{d\sigma[p+p\to n+\Delta^{++}]}{d\Omega_n}. \qquad (1)$$

We will recall our arguments in Section 3. Here $\overline{N} = (Z + \frac{1}{3}N) < f^2 >$ is the effective number of nucleons participating in the process $(p,n)_\Delta$. The quantity $< f^2 >$ is a folding of the target density with the Glauber absorption factor and leads to an effective (reduced) number of active nucleons, comparison with data and an extension of the theory are given in Sections 4 and 5.

A main aim of his contribution is also to discuss common properties of integral cross sections of inclusive $(^3He,t)$ and (p,n) reaction. for $T > 0.6$ GeV/A in the Δ-isobar excitation region, on the basis of the effective-number approximation. Aside from the form factor we will, in Section 6, argue that these reactions have the same underlying reaction mechanism and that the cross section can be given a factorized form in terms like(1) of the nuclear structure of the participants in the reaction.

3. RELATION BETWEEN THE INCLUSIVE $A(p,n)_\Delta B$ REACTION AND THE ELEMENTARY PROCESS

In Refs.[8, 10] it has been argued that the $A(p,n)_\Delta B$ reaction can be described in the framework of distorted waves. In this approach the invariant cross section is given by ($c=\hbar=1$)

$$d\sigma = \frac{2E_i E_A}{\lambda^{1/2}(s_{NA}, m_N^2, M_A^2)}\frac{1}{2}\frac{1}{2J_i+1}\sum(2\pi)^4\delta^{(4)}(P_i+P_A-P_n-P_f)\,|\,T^{pA}_{B+\Delta A}\,|^2\,d\vec{P}_n. \qquad (2)$$

where sums are over the angular momentum projection quantum numbers and the final channels f. In first order Born approximation the matrix element of the T operator can be written as (spin coordinates are suppressed)

$$T^{np}_{B+\Delta,A} = < \hat{A}\{\chi_n^{(-)}(\vec{k}_n,\vec{r})\Psi_{B+\Delta}(\vec{r}_1,\ldots,\vec{r}_A)\} \mid \sum_{j=1}^{A} V_{N\Delta}(\vec{r}_j,\vec{r}) \mid$$

$$\hat{A}\{\chi_p^{(+)}(\vec{k}_p,\vec{r})\Psi_{\alpha_i J_i M_i}(\vec{r}_1,\ldots,\vec{r}_A)\} > . \quad (3)$$

In (2) and in the following we use a notations where: $E_i = (\vec{P}_1^2 + m_N^2)^{1/2}$ is the energy of the incoming proton with impuls $\vec{P}_i$ and mass m_N, $E_A = (\vec{P}_A^2 + M_A^2)^{1/2}$ is the energy of the target-nucleus A, s_{NA} is the square of the invariant mass of the system p+A, $\lambda(x,y,z) = (x-y-z)^2 - 4yz$ is the kinematic or triangular function [11]. The quantities P_i, P_A, P_n and P_f are the four momenta of the projectile-proton, target-nucleus, registered neutron and nor-registered final fragments (for example, f=B+Δ), respectively. Formula (3) is given in the CM of p+A. It contains the wave function $\Psi_{\alpha_i J_i M_i}$ of the target-nucleus A with the spin J_i and projection M_i and other quantum numbers α_i in overlap with the wave function $\Psi_{B+\Delta}$ of the unobserved system. Furthermore, $\chi_p^{(+)}(\chi_n^{(-)})$ are the distorted waves of the incident and exit channels, $V_{N\Delta}$ is the transition operator for $NN \to N\Delta$ and $\hat{A}$ the antisymmetrizer. The factor $[2(2J_i+1)]^{-1}$ in formula (2) is due to averaging/summing over the projections of projectile and target spins.

In Refs. [5, 6, 7] it was shown that the usual method of evaluation of distorted waves $\chi_p^{(+)}$ $(\chi_n^{(-)})$ in the framework of the optical model underestimates the cross sections because this method ignores the contributions of processes due to incoherent rescattering of protons (neutrons) on nucleons in the target nucleus (residual nucleus $B+\Delta$). This effect can be taken into account using the distorted waves in Glauber approximation [5]

$$\chi_p^{(+)}(\vec{k}_p,\vec{r}) = (2\pi)^{-3/2} exp(i\vec{k}_p\cdot\vec{r}) \prod_{j=1}^{A}[1-\Gamma(\vec{b}-\vec{b}_j)\theta(z_j-z)]\chi_m(\vec{\sigma}), \quad (4)$$

$$\chi_n^{(-)}(\vec{k}_n,\vec{r}) = (2\pi)^{-3/2} exp(i\vec{k}_n\cdot\vec{r}) \prod_{j=1}^{A-1}[1-\Gamma(\vec{b}-\vec{b}_j)\theta(z-z_j)]\chi_m(\vec{\sigma}), \quad (5)$$

where $\Gamma(\vec{b})$ is the profile function.

$$\Gamma(\vec{b}) = \frac{1}{2\pi ik}\int e^{i(\vec{q}\vec{b})} A_{NN}(\vec{q})d^2q. \quad (6)$$

Here $\vec{b}$ is the impact parameter, $\vec{q}$ is the transferred momentum, $A_{NN}(\vec{q})$ the nucleon-nucleon scattering amplitude, $\chi_m(\vec{\sigma})$ the spin function of a nucleon with the spin $\vec{\sigma}$ and projection m, while $\theta(z)$ is the step function

$$\theta(z) = \begin{cases} 1 & \text{if } z \geq 0 \\ 0 & \text{otherwise}. \end{cases}$$

Using the completeness of the states of the nonregistered fragments the inclusive cross section of the reaction $A(p,n)_\Delta B$ can be written as [5, 7]

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n} = \int d\vec{Q}[\Phi_N^A(\vec{Q})]^2 \frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_1,\vec{Q})}{d\Omega_n}, \quad (7)$$

where $[\Phi_N^A(\vec{Q})]^2$ is the momentum distribution of the nucleons in nucleus A, participating in the charge-exchange reaction.

The total momentum distribution function $[\Phi_N^A(\vec{Q})]^2$ can be expressed through the partial distributions of protons $[\Phi_p^A(\vec{Q})]^2$ and neutrons $[\Phi_n^A(\vec{Q})]^2$,

$$[\Phi_N^A(\vec{Q})]^2 = [\Phi_p^A(\vec{Q})]^2 + \frac{1}{3}[\Phi_n^A(\vec{Q})]^2, \tag{8}$$

where $\frac{1}{3}$ is the isotopic weight factor for the creation of a Δ^+ isobar on the neutron. The effective number of protons (neutrons) participating in the process $A(p,n)_\Delta B$ is defined by the integrated impuls distributions,

$$\bar{N}_N^A = \int d\vec{Q}[\Phi_N^A(\vec{Q})]^2, \tag{9}$$

with distorted waves folded in (see eq. (13) below). In the plane wave approximation the effective numbers are equal to $\bar{N}_p^A(PW) = Z$, ($\bar{N}_n^A(PW) = \frac{1}{3}N$), where Z(N) is the number of the protons (neutrons) in nucleus A.

The presence of the momentum of an intranuclear nucleon $\vec{Q}$ among the arguments of the cross section of charge-exchange on a nucleon in the nuclear medium $d\sigma_{pN\to n\Delta^{++}}(\vec{P}_i,\vec{Q})/d\Omega_n$ points to the possible necessity to include **the effects of going** off mass shell. However, in the investigated region of energies, $T_i > 0.6$ GeV, the influence of the off-mass-shell effects can be neglected as the momentum of the incident nucleon $\vec{P}_i$ and the transferred momentum $\vec{q}$ obey the conditions $|\vec{P}_i| \gg P_F$ and $|\vec{q}| \gg P_F$, where P_F is the Fermi momentum. The point is that the strength of the interaction depends on the momentum of the incident proton $\vec{P}_i$ and on that of an intranuclear nucleon $\vec{Q}$ by $(P_i^2 + Q^2)^{1/2}$, so that the total correction for the Fermi motion of the nucleons and for their binding does not exceed 3.5%. In addition, the off mass shell effects influence the A-dependence of the integral inclusive cross section of $A(p,n)_\Delta B$, that we are interested in, rather weakly. These considerations allow us to use the following approximation

$$\frac{d\sigma_{pN\to n\Delta^{++}}(\vec{P}_i,\vec{Q})}{d\Omega_n} \approx \frac{d\sigma_{pN\to n\Delta^{++}}(\vec{P}_i,\vec{Q}=0)}{d\Omega_n} = \frac{d\sigma_{pN\to n\Delta^{++}}(\vec{P}_i)}{d\Omega_n}\Big|_{free}. \tag{10}$$

As we intended to show, in this approximation (7) becomes factorized, as follows

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n} = \bar{N}\frac{d\sigma_{pp\to n\Delta^{++}}(\vec{P}_i)}{d\Omega_n}\Big|_{free}, \tag{11}$$

where the quantity $\bar{N}$ (we now drop sub (super) scripts N (A)) has the simple expression,

$$\bar{N} = \int d\vec{Q}[\Phi_N^A(\vec{Q})]^2 = (Z + \frac{1}{3}N)\int d\vec{r}\rho(r)f^2(b,z) = (Z + \frac{1}{3}N) < f^2 > . \tag{12}$$

Here $< f^2 >$ is the effective factor of absorption, $\rho(\vec{r})$ is the one nucleon density with the normalization $\int d\vec{r}\rho(\vec{r}) = 1$ and $f^2(b,z)$ the Glauber factor of absorption

$$f^2(b,z) = [(1 - \frac{\sigma_{pN}^{tot} - \sigma_{pN}^{el}}{A}T_-(b,z))(1 - \frac{\sigma_{nN}^{tot} - \sigma_{nN}^{el}}{A}T_+(b,z))]^A \equiv \sum_{N_p=0}^{A}\sum_{N_n=0}^{A} f_{N_pN_n}^2(b,z)$$

$$f^2_{\lambda_p \lambda_n}(b,z) = \sum_{\lambda_p=0}^{A} \sum_{\lambda_n=0}^{A} \begin{pmatrix} A \\ \lambda_p \end{pmatrix} (1 - \frac{\sigma^{tot}_{pN}}{A} T_-(b,z))^{A-\lambda_p} (\frac{\sigma^{el}_{pN}}{A} T_-(b,z))^{\lambda_p}$$

$$\begin{pmatrix} A \\ \lambda_n \end{pmatrix} (1 - \frac{\sigma^{tot}_{nN}}{A} T_+(b,z))^{A-\lambda_n} (\frac{\sigma^{el}_{nN}}{A} T_+(b,z))^{\lambda_n}. \qquad (13)$$

Here σ^{tot}_{pN} (σ^{tot}_{nN}) is the total cross section of proton (neutron)-nucleon scattering and σ^{el}_{pN} (σ^{el}_{nN}) the corresponding elastic scattering cross section. The thickness functions $T_\pm$ are given by the standard forms

$$T_+(b,z) = A \int_z^\infty d\xi \rho([b^2 + \xi^2]^{1/2}). \qquad (14)$$

$$T_-(b,z) = A \int_{-\infty}^z d\xi \rho([b^2 + \xi^2]^{1/2}). \qquad (15)$$

Formula (13) represents the decomposition of the absorption factor over the number of quasielastic collisions λ_p (λ_n) of ejectile-protons (outgoing neutrons) with the nucleons of nucleus A (B). It allows us to represent the effective numbers in the physically transparent form

$$\bar{N} = (Z + \frac{1}{3} N) \int d\vec{r} \rho(\vec{r}) \sum_{\lambda_p=0}^{A} \sum_{\lambda_n=0}^{A} f^2_{\lambda_p \lambda_n}(b,z). \qquad (16)$$

$$\bar{N}_{\nu_p \nu_n} = (Z + \frac{1}{3} N) \int d\vec{r} \rho(\vec{r}) \sum_{\lambda_p=0}^{\nu_p} \sum_{\lambda_n=0}^{\nu_n} f^2_{\lambda_p \lambda_n}(b,z). \qquad (16a)$$

where the latter will be referred to as partial effective numbers. Each of the partial sums describes the contribution to the total cross section from definite groups of final states of the system $B + \Delta$. For example, $\bar{N}_{00}$ corresponds to generating the state (ΔN^{-1}) *in the nucleus* B *in the reaction; the* $\bar{N}_{10}$ *corresponds to the process where the* incident proton excites at the beginning a state (1p-1h) in nucleus A and only after the charge exchange process the state (ΔN^{-1}) is generated. Generally if $\nu_t + \nu_N = i$ this means that in the reaction $A(p,n)_\Delta B$ one generates the excited state (ip-ih)+(ΔN^{-1}).

Assuming charge symmetry allows us to simplify expression (13) somewhat since it implies equality of the elementary cross sections $\sigma_{pN} = \sigma_{nN} = \sigma_{NN}$, and consequently instead of two functions $T_\pm$ it is possible to introduce one function only

$$T(b) = T_-(b,z) + T_+(b,z) = A \int_{-\infty}^{\infty} d\xi \rho([b^2 + \xi^2]^{1/2}). \qquad (14a)$$

In the case of large mass numbers $A \gg 1$ we obtain the well-known eikonal approximation

$$f^2_{eik} = exp[-(\sigma^{tot}_{NN} - \sigma^{el}_{NN}) T(b)]. \qquad (17)$$

In obtaining (17) from (13) we have used the smooth energy dependence of the cross sections $\sigma_{pN}(T_p)$ and $\sigma_{nN}(T_n)$, because, strictly speaking $\sigma_{pN}(T_p) = \sigma_{nN}(T_n)$ only at $T_p = T_n$.

If the nucleus is registered in the ground state as was for example done in [12], than in the absorption factors (13) and (17) we have formally to put $\sigma^{el}_{NN} = 0$. In that case, formula (17) corresponds to the eikonal approximation for the elastic scattering in the *optical model.*

By inserting (17) in (12) we can write the effective absorption factor $< f^2 >$ in the eikonal approximation in the following form

$$< f^2 > = \frac{2\pi}{A} \int db\, b\, T(b)\, e^{-\sigma T(b)}. \qquad (18)$$

where $\sigma = \sigma_{NN}^{tot}$ for the exclusive and $\sigma = \sigma_{NN}^{tot} - \sigma_{NN}^{el}$ for inclusive reactions, respectively. The integral (18) can be estimated by the saddle point method

$$< f^2 > = [(2\pi)^{3/2} b_0]/[A\sigma^2 e \mid T'(b_0) \mid]. \qquad (19)$$

where b_0 is root of the equation

$$T(b_0) = \sigma^{-1}. \qquad (20)$$

At the energies $T_p > 0.6$ GeV the values of σ lie in the interval $\sigma \approx$20-40 mbarn ($\sigma_{NN}^{tot} \approx$40 mbarn, $\sigma_{NN}^{el} \approx$10-20 mbarn). In this case $b_0 \approx R_A$, $\mid T'(b_0) \mid \approx 1/(\sigma a)$, where R_A is the nuclear radius, a is the diffuseness of the nuclear surface. An approximate expression for $< f^2 >$ may now be written as

$$< f^2 > \approx \frac{(2\pi)^{1/2} R_A a}{\sigma A e}. \qquad (21)$$

From (21) we obtain immediately that $< f^2 > \propto A^{-2/3}$ and consequently, that the A-dependence $\overline{N}$ goes as

$$\overline{N} \propto A^{\alpha}. \qquad (22)$$

where $\alpha \approx 1/3$ for $(p,n)_\Delta$ and $(^3He,t)_\Delta$ reactions (see Fig.2).

Table 1. The A- and θ-dependencies of experimental cross sections $d\sigma[A(p,n)_\Delta B]/d\Omega$, [mb/sr] $\equiv \sigma(\theta)$ (in lab. system) and effective numbers $\overline{N}^{exp}$ at T_p = 1 GeV [11]. The values $\overline{N}^T$ are the results of calculations at $\theta_n = 0$.

Target	$\overline{N}^T$	$\sigma(4°)$	$\sigma(7.5°)$	$\sigma(11.3°)$	$\overline{N}^{exp}(4°)$	$\overline{N}^{exp}(7.5°)$	$\overline{N}^{exp}(11.3°)$
H	1.00	42.7	31.0	20.7	1.00	1.00	1.00
D	0.61	52.1	44.7	29.6	1.22	1.14	1.13
^7Li	2.15	123.8	90.5	-	2.90	2.92	-
^9Be	2.50	177.1	132.1	81.2	1.15	1.27	3.92
^{10}B	2.77	165.8	118.3	73.2	3.88	3.82	3.54
^{11}B	2.74	159.8	117.5	71.3	3.71	3.79	3.59
^{12}C	2.96	162.3	122.9	81.3	3.80	4.96	3.92
^{16}O	3.15	220.6	159.9	118.0	5.17	5.16	5.67
^{19}F	3.11	246.6	169.4	117.9	5.78	5.16	5.9[illegible]
^{24}Mg	1.65	254.3	153.6	128.4	5.96	1.95	5.91
^{25}Mg	1.65	243.6	179.6	122.5	5.70	5.79	5.92
^{26}Mg	1.65	263.6	207.1	128.8	6.17	6.68	6.22
^{27}Al	4.84	255.8	191.1	133.7	5.99	6.16	6.16
^{40}Ca	5.90	331.5	228.2	162.1	7.76	7.36	7.83
^{44}Ca	5.86	344.0	245.1		8.06	7.90	-
Cu	6.79	400.2	297.9	209.9	9.37	9.61	10.14
^{116}Sn	8.13	554.7			12.99	-	-
^{124}Sn	8.06	533.2			12.49	-	-
^{181}Ta	9.02	611.5	451.8	303.3	14.32	14.67	14.66
Pb	9.27	588.4	481.6	324.5	13.78	15.53	15.68

The series (16) converges sufficiently fast, about 90% of the final value $\overline{N}$ is given by the partial sum $\overline{N}_{11}$ [25]. This result justifies using the completeness approximation to obtain (7) and gives creditability to expressions (21,22). For the calculations of the values $\overline{N}$ we used experimental cross sections σ^{tot}_{NN} and σ^{el}_{NN} from [13]. In [25] we conclude that in the considered region of energies the T_p-dependence of the effective numbers is smooth which substantiate the approximation $\sigma_{pN}(T_p) \approx \sigma_{nN}(T_n)$ employed above.

4. COMPARISON WITH DATA FOR REACTION $A(p,n)_\Delta B$

We now turn to an analysis of the experimental data on the reaction $A(p,n)_\Delta B$ in the terminology of effective numbers and with reference to eq.(11). Table 1 contains the θ_n dependence (three angles) of effective numbers $\overline{N}^{exp}$, extracted according to formula (11) on the basis of experimental cross sections [14] for a number of nuclei and for the elementary process at $T_p = 1$ GeV. From the table it is seen that within the experimental errors the effective number approximation describes the angular dependence of cross sections of charge-exchange. The observed anomalies, the value $\overline{N}^{exp}$=4.95 for the nucleus ^{24}Mg ($\theta_n = 7.5°$) and also $\overline{N}^{exp}$=13.78 for Pb ($\theta_n = 4°$), most likely reflect statistical hoist of the corresponding experimental data more than a presence of dynamical or structural factors in the reaction mechanism. An analogous situation was described in Refs.[5, 7] in the analysis of the inclusive process (p,pd) (The experimental number of the deuterons in Pb, $\overline{N}^{exp} = 19 \pm 2.5$, measured at $T_p = 1$ GeV, was excluded because it deviated from the systematics). We conclude that $\overline{N}^{exp}(\theta_n)$, within experimental uncertainties, does not depend on θ_n. Thus, the angular dependence of cross sections $d\sigma[A(p,n)_\Delta B]/d\Omega_n$ testifies in favour of the approximation of effective numbers. Physically, this means that the process takes place on the nuclear periphery, i.e. in the region where the density of nucleons is small, and consequently, all NN and ΔN interactions in the nuclei are close to free interactions.

Table 2. The A-dependence of cross sections $d\sigma[A(p,n)_\Delta B]/d\Omega_n \equiv \sigma(\theta_n)$ [mb/sr] and effective numbers for reaction $A(p,n)_\Delta B$ at $T_p = 0.8$ GeV [15, 16] at $\theta = 0°$. The values $\overline{N}^T$ are the results of our calculations.

Target	A	Z	$\sigma(0°)$	$\overline{N}^{exp}$	$\overline{N}^T$	$\Delta N = \overline{N}^{exp} - \overline{N}^T$	$K = \frac{\overline{N}^{exp}}{\overline{N}^T}$	$< f^2_{33} >$
H	1	1	33.0 ∓ 3.0	1.0	1.0	0	1.00	---
Al	27	13	271.4 ∓ 2.0	8.2	6.0	2.2	1.36	0.34
Ti	47.9	22	372.1 ∓ 2.7	11.3	7.9	3.4	1.43	0.26
Cu	63.5	29	425.0 ∓ 3.2	12.9	8.9	4.0	1.45	0.22
W	183.9	74	695.5 ∓ 5.6	21.1	12.3	8.8	1.71	0.11
Pb	207.2	82	695.4 ∓ 5.5	21.1	12.6	8.5	1.70	0.10
U	238	92	767.9 ∓ 6.3	23.3	13.0	10.3	1.80	0.09

While the angular behaviour seems to be accounted for by the factorized form (11), the magnitude of the extracted effective numbers $\overline{N}^{exp}$ deviates from what is obtained theoretically, $\overline{N}^T$, by the approach described above. In Tables 2 and 3 we list the results of data processing performed in [15, 16] (at $T_p = 0.8$ GeV at $\theta = 0°$) and in [11] (at $T_p = 1$ GeV at $\theta = 4°$). From the tables it is seen that $\overline{N}^{exp}$ systematically exceeds $\overline{N}^T$ by about a factor 1.5, which clearly points to the insufficiency of the approximation of effective numbers for the description of integral cross section of the reaction $A(p,n)_\Delta B$.

Table 3. The same as in table 2 but for T_p=1 Gev at $\theta_n = 4°$ [11]

Target	$\sigma(4°)$	$\overline{N}^{exp}$	$\overline{N}^T$	$\Delta N = \overline{N}^{exp} - \overline{N}^T$	$K = \frac{\overline{N}^{exp}}{\overline{N}^T}$	$< f^2_{33} >$
H	42.7 ∓ 4.3	1.0	1.0	0	1.00	-
D	52.1 ∓ 5.2	1.2	0.6	0.6	2.0	0.15
^7Li	123.8 ∓ 3.8	2.9	2.2	0.7	1.35	0.50
^9Be	177.1 ∓ 5.3	4.2	2.5	1.7	1.66	0.11
^{10}B	165.8 ∓ 5.0	3.9	2.8	1.1	1.40	0.12
^{11}B	159.8 ∓ 4.7	3.7	2.7	1.0	1.36	0.39
^{12}C	162.3 ∓ 4.8	3.8	3.0	0.8	1.28	0.37
^{16}O	220.6 ∓ 11.0	5.2	3.2	2.0	1.64	0.30
^{19}F	246.6 ∓ 12.4	5.8	3.1	2.7	1.86	0.25
^{24}Mg	254.3 ∓ 17.5	6.0	4.7	1.3	1.29	0.29
^{25}Mg	243.6 ∓ 17.0	5.7	4.7	1.0	1.23	0.28
^{26}Mg	263.6 ∓ 17.6	6.2	4.7	1.5	1.33	0.28
^{27}Al	255.8 ∓ 7.5	6.0	4.8	1.2	1.24	0.27
^{40}Ca	331.5 ∓ 23.2	7.8	5.9	1.9	1.38	0.22
^{44}Ca	341.0 ∓ 24.2	8.1	5.9	2.2	1.38	0.21
Cu	400.2 ∓ 16.2	9.4	6.8	2.6	1.38	0.17
^{116}Sn	554.7 ∓ 28.0	13.0	8.1	1.9	1.60	0.11
^{124}Sn	533.2 ∓ 26.9	12.5	8.1	1.1	1.55	0.11
^{181}Ta	611.5 ∓ 26.1	14.3	9.0	5.3	1.59	0.08
Pb	588.4 ∓ 23.8	13.8	9.3	1.5	1.49	0.08

The employment of "optical" absorption factors instead of Glauber's gives even a further deterioration of the description. In this case the ratio $K = \overline{N}^{exp}/\overline{N}^T$ becomes about 2.

5. EXTENSION OF THE THEORY

This situation may be qualitatively understood if we consider the A-dependence of the quantity $\Delta N = \overline{N}^{exp} - \overline{N}^T$. It follows from Tables 2 and 3 that ΔN grows, on the whole, according to the law A^α, where α=0.6-0.8. If we assume (remember a previous conclusion) that the effect of going off mass-shell in the cross section $d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_t, \vec{Q})/d\Omega_n$ is not very large, we may consider expression (11) to be valid for description of that part of the cross section of the reaction $A(p,n)_\Delta B$ where a realistic Δ^{++} or a Δ^+-isobar is generated (see diagram 1, Fig.1a). By definition, diagram 1 contains only the direct charge-exchange process,but in the presence of the nuclear medium,implying some renormalization of the 2-body interaction. However, within the developed formalism, the cross section of diagram 1 includes both the direct and exchange (Pauli principle) terms. The latter term corresponds physically to excitation of

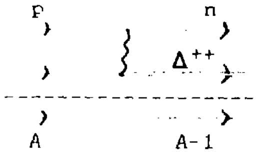

Fig.1 a

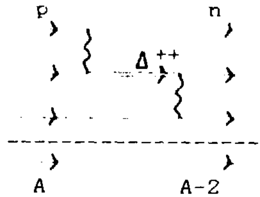

Fig.1 b

Fig.1 (a)This process in the shorthand notation will be denoted by $(p,n)_{\Delta}^{\pi N}$ with emphasize on reality- principal observability- of a produced Δ-isobar whose de-excitation takes place through decay into a pion and a nucleon. (b)This process may be referred to as mesonless Δ-isobar de excitation [17, 25]. We will denote it by $(p,n)_{\Delta}^{NN}$, thus emphasizing that a *virtual Δ-isobar is involved through charge-exchange on one of the* intranuclear nucleons.

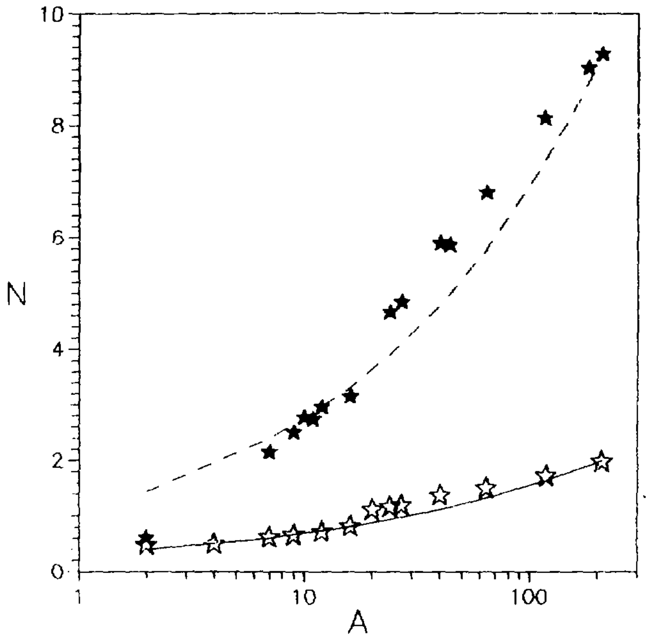

Fig.2 Mass dependence of the effective number N for (p,n) (black stars, dashed line correspondent approximation $N \approx 1.1A^{0.40}$) and ($^3He,t$) (white stars, solid line correspondent approximation $N \approx 0.31A^{0.35}$) reactions, respectively.

the Δ-isobar in the incident particle and plays an important role. A similar comment can be made for diagram 2 (see Fig.1b) discussed below. Taken this into account the observed cross section should be written as the sum

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n} = \frac{d^{(1)}\sigma_{A(p,n)_\Delta B}}{d\Omega_n} + \frac{d^{(2)}\sigma_{A(p,n)_\Delta B}}{d\Omega_n}. \quad (23)$$

The first term corresponds to the approximation of effective numbers (11); the second term is connected with the process of exchange of virtual mesons, in which a virtual Δ-isobar also is present (see diagram 2), bringing in effects from 3-body and higher order forces.

The cross section corresponding to diagram 2 can be written as

$$\frac{d^2\sigma_{A(p,n)_\Delta B}}{d\Omega_n} \approx \int d\vec{P}\int d\vec{Q}'\int d\vec{Q}\phi(\vec{P},\vec{Q},\vec{Q}')[\Phi_N^{A-1}(\vec{Q}')]^2\frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q}')}{d\vec{P}}$$

$$|\,G(E_\Delta)\,|^2\,[\Phi_N^A(\vec{Q})]^2\frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q})}{d\Omega_n}. \quad (24)$$

Here $\varphi(\vec{P},\vec{Q},\vec{Q}')$ is the kinematic factor, $G(E_\Delta)$ is Green's function describing the propagation of the Δ-isobar with energy E_Δ in the intermediate state, $\vec{P}$ the impuls of the fast nucleon created as result of the second act of the charge-exchange. $\vec{Q}$ and $\vec{Q}'$ are the momenta (Fermi motion) of nucleons in the target-nucleus involved in the first and second act of charge-exchange, respectively. To obtain expression (24) we have carried out averaging over the spin projection of the Δ and used the approximation of completeness. As in the case of the single-step charge-exchange, the inequality $|\,\vec{P}_i\,|\gg P_F$ allows one to ignore the $\vec{Q}$-dependence of cross section $d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q})/d\Omega_n$. Thus, the cross section of the process $(p,n)_\Delta^{NN}$ is also factorized,

$$\frac{d^2\sigma_{A(p,n)_\Delta B}}{d\Omega_n} = \Delta\overline{N}\cdot\frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q})}{d\Omega_n}\,|_{free}. \quad (25)$$

where

$$\Delta\overline{N} = \int d\vec{P}\int d\vec{Q}'\int d\vec{Q}\phi(\vec{P},\vec{Q},\vec{Q}')[\Phi_N^{A-1}(\vec{Q}')]^2\frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q}')}{d\vec{P}}$$

$$|\,G(E_\Delta)\,|^2\,[\Phi_N^A(\vec{Q})]^2. \quad (26)$$

To neglect the off-mass-shell effects in formula (26) is inadmissible because the momenta $|\,\vec{P}\,|$ and P_F are commensurable quantities. The region of integration in (24) and (26) is determined by the width of the Δ-peak and the Fermi momentum, by the properties of Green's function and momentum distribution $[\Phi_N^A(\vec{Q})]^2$. As final result we have found a modified factorized form for (23),

$$\frac{d\sigma_{A(p,n)_\Delta B}}{d\Omega_n} = [\overline{N}+\Delta\overline{N}]\frac{d\sigma_{p+p\to n+\Delta^{++}}(\vec{P}_i,\vec{Q})}{d\Omega_n}\,|_{free}, \quad (27)$$

$$\overline{N}+\Delta\overline{N} = \kappa_1 A^{1/3} + \kappa_2 A^{2/3}. \quad (27a)$$

Expression (27) indicates that the angular spectra of neutrons from the charge-exchange channel $(p,n)_\Delta^{NN}$ coincide in form with analogous spectra of neutrons from the charge exchange channel $(p,n)_\Delta^{\pi N}$. This result allows us to understand the experimental data from Ref. [14] and it also points to the impossibility in principle to separate the contribution of the channel $\Delta \to \pi N$ to the charge-exchange cross section from the contribution of the channel $\Delta N \to NN$ on the basis of merely angular spectra of neutrons.

The leading A-dependence of the contribution from diagram 2 follows directly from the 2-step character of the process. Direct calculation in the plane wave approximation shows that the functions $\varphi(\vec{P},\vec{Q},\vec{Q}')$ and $|G(E_\Delta)|^2$ depend little on A, while $[\Phi_N^A(\vec{Q})]^2 \propto A^{1/3}$ as discussed previously. Thus, $\Delta N \propto A^{1/3}A^{1/3} = A^{2/3}$ which is in full agreement with the experimental data [14, 15, 16] (see also expression (27a) and Fig.3). In Fig.4 we present the energy spectrum of neutrons $d\sigma[^{12}C(p,n)_\Delta]/d\Omega_n dT_n$ at T_p=1 GeV compared with theory (we used the plane wave approximation for the fast nucleon created as result of the second act of charge-exchange). The theoretical spectra were calculated for $\theta_n = 4°$. As follows from our analysis, the channel described by diagram 2 improves the fit to the data in the Δ-region in agreement with the conclusions of Ref. [18] devoted to $(e,e')_\Delta$.

In Fig.5 we present separate contributions from the channels $(p,n)_\Delta^{\pi N}$ and $(p,n)_\Delta^{NN}$ extracted from experimental data [14]. The comparison substantiates that for low mass numbers A diagram 1 (Fig.1a) dominates, while diagram 2 (Fig.1b) plays an increasing role with increasing mass number, as is qualitatively expected. For large

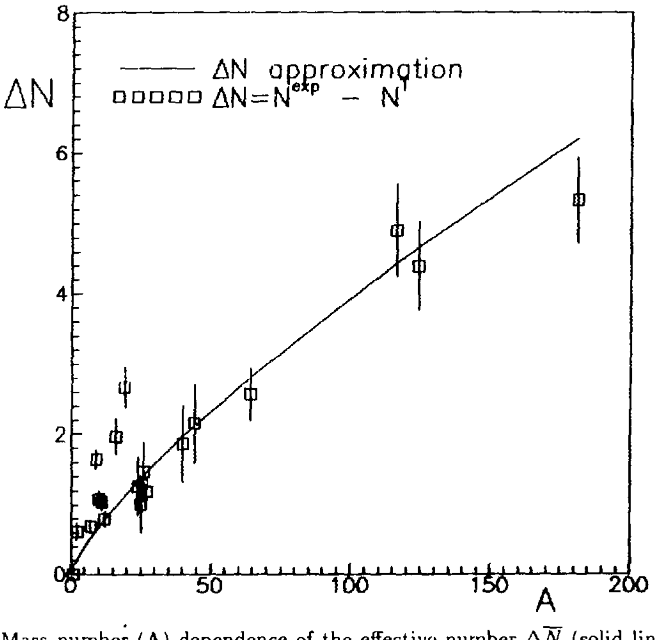

Fig.3 Mass number (A) dependence of the effective number $\Delta\overline{N}$ (solid line). The dots are the differences between experimental data [14] and calculated by the formula (11).

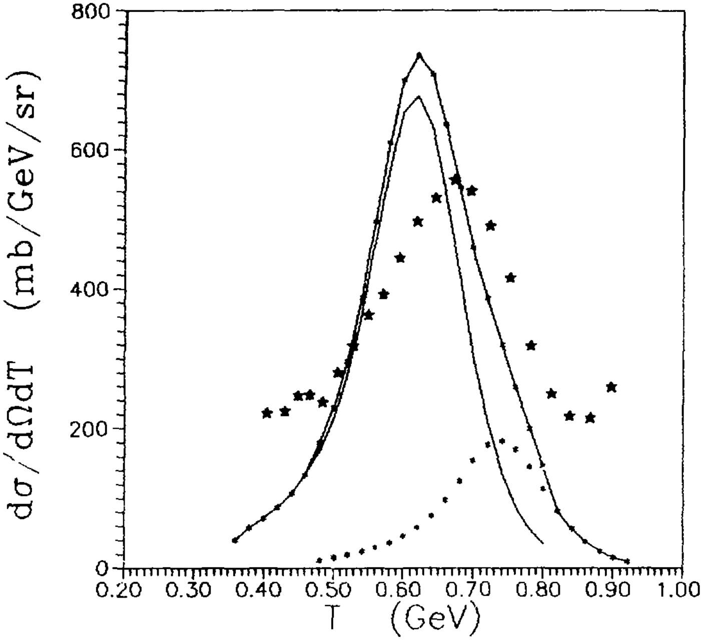

Fig.4 The energy spectrum of neutrons from the reaction $^{12}C(p,n)_{\Delta}$ at T_p=1 GeV. The experimental data [14] at $\theta_n = 4°$ are denoted by stars. The solid curve is the contribution of diagram 1; the dotted, diagram 2; and solid line with centered symbols is their sum. The theoretical spectra were calculated at $\theta_n = 1°$. The effects of channels coupling was not taken into account.

mass number ($\geq$ 100) the two contributions are of comparable magnitude. For an additional discussion for $^{12}C(^3He,t)$, see in Ref. [19].

In ref. [24] $(t,^3He)$ charge-exchange reactions at P_t = 9.1 GeV/c were investigated. The comparison of the different topologies gives us an important information about reaction mechanisms. From table 1 one can see that the topology (0,0) has an enhancement in comparison with (1,0) and (0,1) topologies (following Ref.[21] we introduce notations: (0,0) - topology $(0p,0\pi^-)$, (0,1) - topology $(0p,1\pi^-)$, (1,0) - topology $(1p,0\pi^-)$ etc.). Less than 25% of the (0,0)-events for the ^{24}Mg target at 9 GeV/c correspond to the quasielastic charge-exchange reaction (Ref.[21]). This fact can be considered as a direct indication to the important contribution of the two-step incoherent reaction mechanism (mesonless Δ-isobar de-excitation [25]) and one-step coherent π^0 production to the (0,0)-channel. There are no alternative reaction mechanisms for events with (0,0)-topology kinematically permitted. The experimental discrimination of these mechanisms for the (0,0)-channel was not carried out in [21].

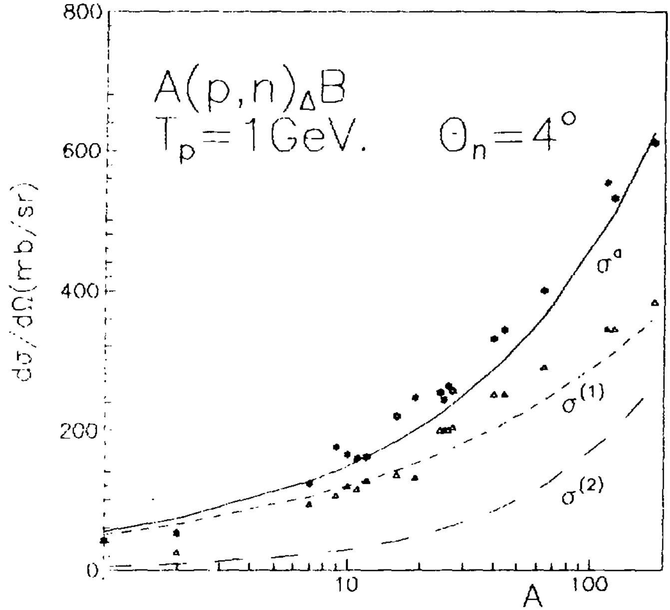

Fig.5 Partial contributions $\sigma^{(1)}$ and $\sigma^{(2)}$ from the processes $(p,n)_\Delta^N$ and $(p,n)_\Delta^{NN}$ pictured in diagram 1 and 2 of Fig.1. $\sigma^{(a)}$ is their sum. The full dots are experimental data from Ref.[14]. The partial contributions $\sigma^{(1)}$ and $\sigma^{(2)}$ were obtained by a χ^2 fit using the formula $d\sigma/d\Omega_n = d\sigma^{(1)}/d\Omega_n + d\sigma^{(2)}/d\Omega_n = \bar{\kappa}_1 A^\alpha + \bar{\kappa}_2 A^{2\alpha}$, ($\bar{\kappa}_1=50$, $\bar{\kappa}_1=5.1$, $\alpha=0.38$), where the first (second) term corresponds to diagram 1 (2). Triangles are theoretical calculations correspondent effective number approximation (1).

Table 4. Data on the topology of events, i.e. the quantity of the negative (N_-) and positive (N_+) particles produced in the charge-exchange reaction $(t,{}^3He)$ on ${}^{24}Mg$ target at 9 GeV/c. The value of the final ${}^3He$ momentum are given in **brackets** (GeV/c).

N_-	N_+	${}^{24}Mg(t,{}^3He)$
0	0	673 (8.915)
1	0	568 (8.735)
1	1	132 (8.562)
0	1	212 (8.803)
0	2	52 (8.639)

The formalism described above has also been applied to compute the cross section of **the reaction** A(p,n)B in the quasielastic region for nuclear excitation, i.e. in a high momentum region of the neutron spectrum in which an incident proton either knocks out a neutron or suffers a charge-exchange on target-nucleons without excitation of the Δ isobar.

Table 5. The A-dependence of effective numbers $\overline{N}^{exp}_{QCE}$ and $\overline{N}^{T}_{QCE}$ for the reaction A(p,n)B in the quasielastic region for $T_p = 1$ GeV at $\theta_n = 4^\circ$ [11]

Target	$\overline{N}^{exp}_{QCE}$	$\overline{N}^{T}_{QCE}$
^{12}C	2.23	1.80
^{16}O	2.72	1.88
^{27}Al	3.22	3.02
^{40}Ca	3.39	3.41
^{116}Sn	6.62	5.53
Pb	9.79	6.92

From Table 5 it is seen (for details see [8]) that also in this region of excitation of a nucleus, the proposed approach describes the integral cross section of reaction A(p,n)B with reasonable accuracy.

6. COMPARATIVE ANALYSIS OF (p,n) AND $(^3He,t)$ REACTIONS ON NUCLEI

The experimental information on the inclusive cross sections of the (p,n) reactions on nuclei for the quasielastic and the Δ-isobar excitation region is far from sufficient. The most comprehensive tabulation is that of Ref.[11] which presents experimental data on neutron production in 1 GeV proton interactions with different nuclei at angles 4°, 7.5° and 11.3°. Neutron spectra at $\theta = 0°$ from p-p and p-d collisions at T_p=647 and 800 MeV incident energies were measured in Ref.[15], and systematics of $\theta = 0°$ neutron production by 800 MeV protons on targets with $27 \leq A \leq 238$ have been reported in Ref. [16]. Neutron spectra at $\theta = 0°$ from p-p collisions have also been measured for T_p=647, 771 and 805 MeV [20]. We have employed these results above.

A systematic experimental study of Δ-isobar excitations in nuclei started with experiments of the inclusive type $(^3He,t)$ in Dubna [17] at beam energies $T_{^3He}$ from 800 MeV/A up to 5.23 GeV/A and in Saclay [21] at energies 500, 667 and 767 GeV/A, near the threshold of the Δ-isobar production. In these experiment one measured differential cross sections of the charge- exchange reactions on free proton targets and nuclear targets as functions of the energy $Q = (E_{^3He} - E_t)$ transferred to the target at an angle.

We have calculated the quantity

$$\overline{\sigma} = \frac{C(p_i,p_f)}{3F(t)} \frac{d^2\sigma[A(^3He,t)B]}{pdQd\Omega}, \qquad (28)$$

where $F(t) = \exp(-27.736 \mid t \mid)$ is the square of magnetic transition formfactor for $(^3He,t)$ and the factor $C(p_i,p_f) = \sigma(pp \to pn\pi^+)\mid_{p_f} / \sigma(pp \to pn\pi^+)\mid_{p_i}$ (see Fig.6) is introduced to compensate the energy dependence of the cross sections and to bring the experimental data at different energies to the same one, which is chosen to be 800 MeV/A. Here $\sigma(pp \to pn\pi^+)\mid_{p_f}$ $(\sigma(pp \to pn\pi^+)\mid_{p_i})$ is the cross section of the elementary process at impuls $p_f(p_i)$, and $d^2\sigma A(^3He,t)B/pdQd\Omega$ the invariant cross section of charge-exchange reactions on nuclei. Fig.7 shows the corresponding experimental cross sections for (p,n) and $(^3He,t)$ reactions on a proton-target, reduced to the momentum 1.47 GeV/c/A, as explained before. Taking into account the experimental uncertainties

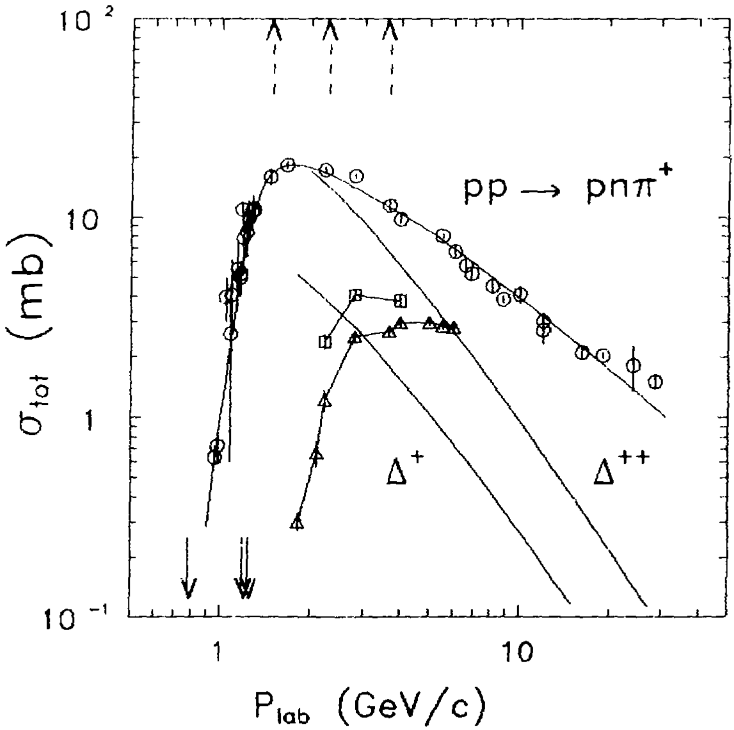

Fig.6 Momentum dependence of the total cross sections of the reaction $pp \to pn\pi$.Data are taken from the compilation [27, 28]. Full line represents the VerWest and Arndt parametrization [29] (up to lab momentum 3 GeV/c) and approximation given in [31]. The single arrow indicates the pion production threshold; the double arrow indicates the Δ -production threshold. The dashed arrows indicate the momenta per nucleon in Dubna experiments [17] for the $(^3He, t)$ charge exchange reaction with Δ excitations.

of the cross sections there is remarkable agreement between the different reactions for a number of beam energies, not only in the shape of the Δ-peak, but also in absolute values. Some deviation is , however, present at high $Q \approx 500$ MeV, which is, partly due to the interaction in the final state between the detected neutrons and protons from the reaction $pp \to pn\pi^+$ at E_p=800 MeV. In that case for $Q \approx 500$ MeV the neutron is at rest in the CM and the proton momentum is about 30 MeV/c. This characteristic kinematic region moves to higher Q with increasing beam energies and at 1 GeV it escapes from the region of interest for us (see Fig.8).

It is interesting to stress the general properties of the discussed results. Let us compare the energy dependence of the total cross sections for $pp \to n\Delta^{++}$ and deuteron knock-out reactions (see Fig 9). The deuteron knock-out reaction at high transfer

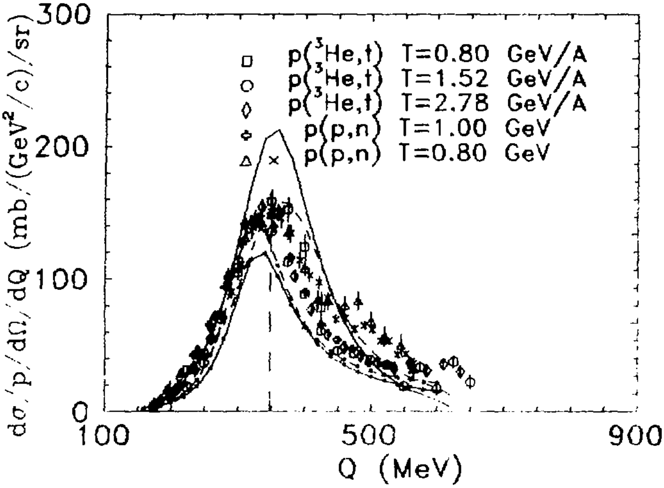

Fig.7 "Reduced" cross sections of the inclusive p(p,n) and $p(^3He,t)$ charge exchange reactions, when the effects of the formfactor and the energy dependence of the "elementary" reaction cross section are removed as explained in the text. The dashed line marks the Δ-peak position in the p(p,n) data. The $p(^3He,t)$ data are taken from Ref.[17]. p(p,n) data are from Refs. [14, 20, 23]. Theoretical curves were obtained with OSET set of vertex parameters (see details in [26]). Solid line - T_p=0.8 GeV, dashed line - T_p=1.0 GeV, solid-dotted line T_p=1.52 GeV, dashed-dotted line - T_p=2.78 GeV.

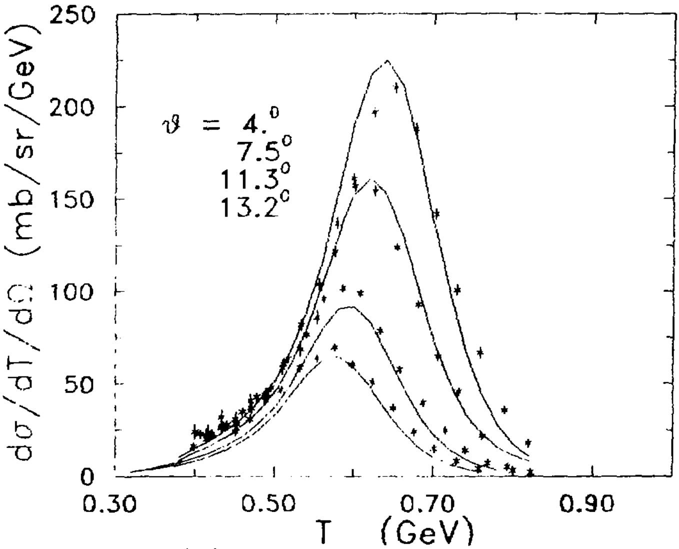

Fig.8 Results of our calculations of the angular dependence of the double differential cross section for the $(p,n)_\Delta$ reaction at T=1 GeV with OSET set of vertex parameters (see [26]). *Effects of energy resolution are taken into account.*

momentum as a rule is interpreted as a result of the short range nucleon nucleon correlation. It is seen from Fig.9 that nucleon nucleon correlation and Δ isobar excitation in principle can simulate each other.

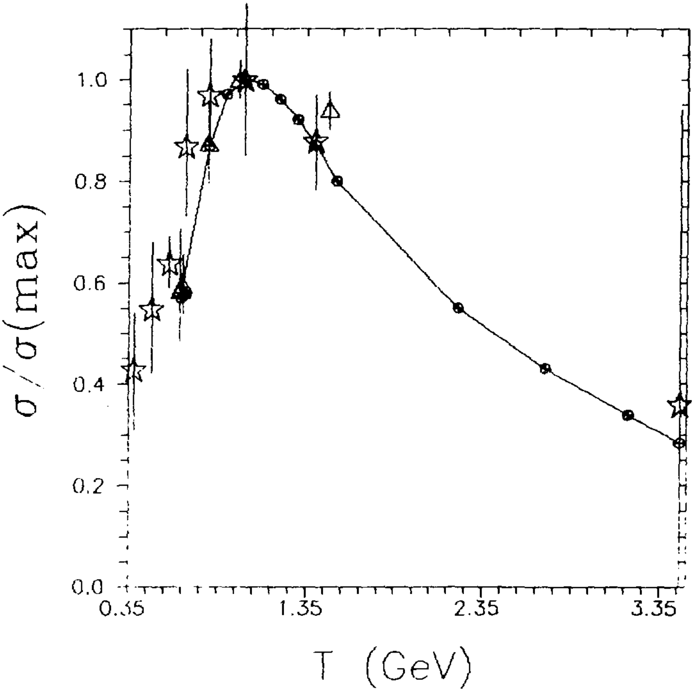

Fig.9 Comparative analysis of the energy dependence of the total cross sections for the reactions $pp \to pn\pi$ (triangles), $pp \to n\Delta^{++}$ (circles) and differential cross section of the inclusive deuteron knock-out reaction $A(p,p'd)B$ at $\Theta = 0$ (stars).All cross sections are normalized for unity at maximum.

7. SUMMARY

We have outlined basic features of a theory which accounts rather well for (p,n) data at T_p=1 GeV on a wide range of nuclei:

1) The A-dependence of the peak for the quasielastic charge exchange process (QCE) is

$$\{\frac{d\sigma A(p,n)B}{d\Omega_n}\}_{QCE} \sim A^{\frac{1}{3}}, \qquad (29)$$

which argues in favor of a one-step reaction mechanism and indicates that the peripherical region gives the main contribution to the cross section due to a strong volume absorption. The maximum is shifted to lower Q values compared with elastic scattering of protons on a neutron (that is on a deuteron, experimental data for the free p+n →

n + p scattering are not available, only data for the inverse reaction n + p → p + n exist). This shift is trivially associated with the binding energy of the knock out neutrons in the target nucleus. The width of the quasielastic peak is almost completely associated with the Fermi motion of the target nucleons. The angular distributions of the reaction $A(p,n)B_{QE}$ are practically independent of the target nucleus.

2) The A dependence of the charge-exchange reaction $A(p,n)_\Delta B$ in the Δ-excitation region is

$$\{\frac{d\sigma A(p,n)_\Delta B}{d\Omega_n}\} \sim aA^{\text{[illegible]}} + bA^{\text{[illegible]}}. \tag{30}$$

The first term is again associated with the single step reaction mechanism while the second is due to a multi-step (but "direct") mechanism. The maximum is shifted by about 30-40 MeV toward the region of high neutron momentum which indicates the presence of some reaction mechanism which compensates the binding effects of nucleons in the nuclei. The width of Δ in nuclei is about 1.5 times larger than the width of the decay of the free Δ-isobar. It increases strongly with increasing A and cannot be explained fully by the Fermi-motion of nucleons.

Our main conclusion is that both the inclusive cross sections of the reactions (p,n) on a number of targets and $({}^3He,t)$ on C are found to be proportional to the cross section of the corresponding processes on a free proton:

$$\frac{d^2\sigma[A(a,b)_\Delta B]}{pdQd\Omega} = N_{eff}F(t)\frac{d^2\sigma(p+p\to n+\Delta^{++})}{pdQd\Omega}, \tag{33}$$

where (a,b) is (p,n) or $({}^3He,t)$. This is a theoretical result, supported by available data. The increased complexity of the projectile (ejectile) is contained in the extra factor F(t). This suggests that all NN and ΔN interactions in the nucleus are close to free interactions. The shift of the Δ-peak in C relative to the free nucleon target is the same in both reactions and is equal to about 30-40 MeV independent of the type and energy of the projectile (see discussion in Ref. [25, 30]). It means that the reaction mechanism for the (p,n) and $({}^3He,t)$ is in principle almost the same, the process takes place on the nuclear periphery and all NN and ΔN interactions in the nucleus are close to free interactions.

We expect that the same conclusion applies to other types of light ion charge-exchange reactions. If that is true, possibilities open up for extraction of information about formfactors for more exotic nuclei such as the radioactive nuclei $({}^6He, {}^6Li)$, $({}^{11}Li, {}^{11}Be)$.

More experimental data for charge-exchange reactions induced by different projectiles is desirable, in particular of the exclusive type. Perspective of performing exclusive studies of charge-exchange reactions are challenging, including spin observables, and constitute the next logical step in understanding the basic mechanism of such reaction.

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