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E2-93-211

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DEUTERON SPIN-DEPENDENT STRUCTURE FUNCTION $g_1^D(x, Q^2)$ AND EFFECT OF RELATIVISTIC FERMI MOTION

Submitted to «Physics Letters B»

1993

Introduction

Recently SMC collaboration has reported on the first measurement of the spin-dependent deuteron structure function $g_1^D(x,Q^2)$ [1]. Therefore it is important to estimate the nuclear effect in the deuteron in order to extract correctly the neutron spin-dependent SF g_1^n from deuteron and proton experimental data. Such analysis is necessary to verify Bjorken sum rule (BSR) [2]. Usually the nuclear effect is described by different deuteron models [3-6] and the estimation of this effect is model-dependent. Therefore the study of the effect of relativistic Fermi motion in different relativistic approaches is actual and can give important information on the nucleon and deuteron structure.

In our previous paper [7], the effect of relativistic Fermi motion in deuteron for the unpolarized deep-inelastic muon-deuteron scattering has been considered. It has been shown that in our relativisation scheme the nuclear effect in the deuteron is more than one obtained in [4,18]. It reaches 6% for $x \simeq 0.7$ and should be taken into account to verify the Gottfried sum rule (GSR).

In the present paper the model of the relativistic deuteron in the light-cone variables [6] is used to consider the deep-inelastic scattering of polarized muons off vector polarized deuterons and estimate the nuclear effect in this process. The covariant approach in the light-cone variables [6] is based on the relativistic deuteron wave function (RDWF) with one nucleon on mass shell. The RDWF is dependent on one variable - the virtuality of nucleon $k_1^2(x,k_\perp)$ and can be expressed via the vertex function $\Gamma_a(x,k_\perp)$. The latter is asymmetric to the replacement $x \leftrightarrow 1-x$. This model has been successfully used for the descriptions of the deuteron electromagnetic form factors and some processes involving deuteron [6]. We calculated the deuteron SF $g_1^D(x,Q^2)$ and results are compared with SMC experimental data [1]. The dependence of the ratio $R_g^{D/N}(x,Q^2) = g_1^D(x,Q^2)/g_1^N(x,Q^2)$ of structure functions on x and Q^2 is investigated. This ratio characterizes the nuclear effect in the vector polarized deuteron. It is shown that the nuclear effect is practically independent of x and Q^2 in the wide kinematical range $x = 10^{-3}-0.7$, $Q^2 = 1-80\ (GeV/c)^2$ and reaches $\sim 9\%$. The BSR is verified using the Carlitz - Kaur model [8] for the nucleon spin-dependent SF $g_1^{p,n}$. The estimations of the nuclear effect for the neutron SF g_1^n and Bjorken integral $S_{Bj}(x,Q^2)$ are obtained. The correction $\delta S_{Bj}/S_{Bj}$ is more than 10% at $x > 10^{-3}$ and $Q^2 > 0.8\ (GeV/c)^2$ and should be taken into account to extract g_1^n from the deuteron and proton data and verify the BSR.

1 Model of Relativistic Deuteron

The deuteron spin-dependent structure functions $G_{1,2}(\nu,Q^2)$ are related to the imaginary part of the forward scattering amplitude of the virtual photon

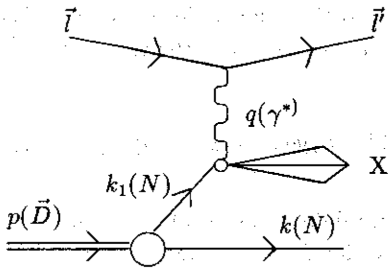

Figure 1: The Feynman amplitude of the deep-inelastic lepton-deuteron scattering in the RIA

on the deuteron $G_{\mu\nu}$ by the standard relation

$$G^D_{\mu\nu} = i\epsilon_{\mu\nu\alpha\beta}q^\alpha\{s^\beta M G^D_1(\nu, Q^2) + [s^\beta(qp) - p^\beta(sq)]M^{-1}G^D_2(\nu, Q^2)\}. \quad (1)$$

Here q, p are momenta of photon and deuteron, M is the deuteron mass. The vector s describes the deuteron spin and satisfies the following conditions: $s^2 = -1, (ps) = 0$.

In the relativistic impulse approximation (RIA) the forward scattering amplitude of the polarized virtual photon γ^* on the vector polarized deuteron $A^D_{\mu\nu}$ is defined via the similar scattering amplitude on the nucleon $A^N_{\mu\nu}$ as follows

$$A^D_{\mu\nu}(q,p) = \int \frac{d^4k_1}{(2\pi)^4 i} Sp\{A^N_{\mu\nu}(q,k_1) \cdot T(s_1,k_1)\}. \quad (2)$$

In the expression (2) $T(s_1, k_1)$ is the amplitude of the $\bar{N} + D \to \bar{N} + D$ process and usual notations $Q^2 = -q^2 > 0$, $\nu \equiv (pq)$, $s_1 = k^2 = (p - k_1)^2$ are used. The integration is carried out with respect to the active nucleon momentum k_1. According to [9] the integral (2) is calculated in the light-cone variables ($k_\pm = k_0 \pm k_3, k_\perp$). The peculiar points of the integrand (2) on the plane of the complex variable k_- are due to the peculiarities of the nucleon virtualities k_1^2 and k^2. Some of the peculiarities are due to the propagators $\sim (m^2 - k_1^2)^{-1}$, $(m^2 - k^2)^{-1}$. The others are related with the vertex DNN and the amplitude $A^N_{\mu\nu}$. The integral is not zero if the region of the integration on k_+ is restricted

$$0 < k_\pm < p_\pm - k_\pm. \quad (3)$$

Taking into account only a nucleon pole in the unitary condition for the amplitude $T(s_1, k_1)$ and the relation between the RDWF and the vertex function $\Gamma_\alpha(k_1)$: $\psi(k_1) = \Gamma_\alpha \cdot (m + k_1)^{-1}$, the expression (2) can be written as

$$G^{\alpha\beta}_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4 i} \delta(m^2 - k^2)\theta(k_0)\theta(p_+ - k_+)\ Sp\{G^N_{\mu\nu} \cdot \bar{\psi}^\alpha(k_1) \cdot (m + \hat{k}) \cdot \psi^\beta(k_1)\}. \tag{4}$$

Here θ - function and light-cone variables are used. The vertex function $\Gamma_\alpha(k_1)$ is defined via 4 scalar functions $a_i(k_1^2)$ $(i = 1 - 4)$ and has the form [10]

$$\Gamma_\alpha(k_1) = k_{1\alpha}[a_1(k_1^2) + a_2(k_1^2)(m + \hat{k}_1)] + \gamma_\alpha[a_3(k_1^2) + a_4(k_1^2)(m + \hat{k}_1)]. \tag{5}$$

The scalar functions $a_i(k_1^2)$ have been constructed in the paper [6] in the form of a sum of pole terms. Some pole positions and residues have been found from the comparison in the nonrelativistic limit of our RDWF with the known nonrelativistic one. For the latter the Paris wave function [11] was taken. The other parameters were fixed from the description of the static characteristics of the deuteron (an electric charge - $G_e(0) = 1(e)$, magnetic - $G_m(0) = \mu_D(e/2M)$ and quadrupole - $G_Q(0) = Q_D(e/M^2)$ moments) in the relativistic impulse approximation.

The calculation of (4) in the light-cone variables gives the final expression for the deuteron spin-dependent SF g_1^D

$$g_1^D(\alpha, Q^2) = \int_\alpha^1 dx\ d^2k_\perp\ \Delta p(x, k_\perp) \cdot g_1^N(\alpha/x, Q^2). \tag{6}$$

The nucleon spin-dependent SF is defined as $g_1^N = (g_1^p + g_1^n)$. The function $\Delta p(x, k_\perp)$ describes the helicity distribution for the active nucleon that carries away the fraction of the deuteron momentum $x = k_{1+}/p_+$ and the transverse momentum $k_\perp$ in the infinite momentum frame. It is expressed via the vertex function $\Gamma_\alpha(k_1)$ as follows

$$\Delta p(x, k_\perp) \propto Sp\{\bar{\psi}^\alpha(k_1) \cdot (m + \hat{k}) \cdot \psi^\beta(k_1) \cdot \hat{q} \cdot \sigma^{\mu\nu} \cdot \rho^v_{\alpha\beta} \cdot \epsilon_{\mu\nu\gamma\delta} p^\gamma s^\delta\}, \tag{7}$$

where $\rho^v_{\alpha\beta}$ is the vector part of the deuteron polarization density matrix. Note that in the approach used the distribution function $\Delta p(x, k_\perp)$ includes not only usual S- and D-wave components of the deuteron but the P-wave component too. The latter describes the contribution of the $N\bar{N}$-pair production. The contribution of this mechanism is small in the low momentum range ($x < 1$), but it may be considerable in the high momentum one ($x > 1$).

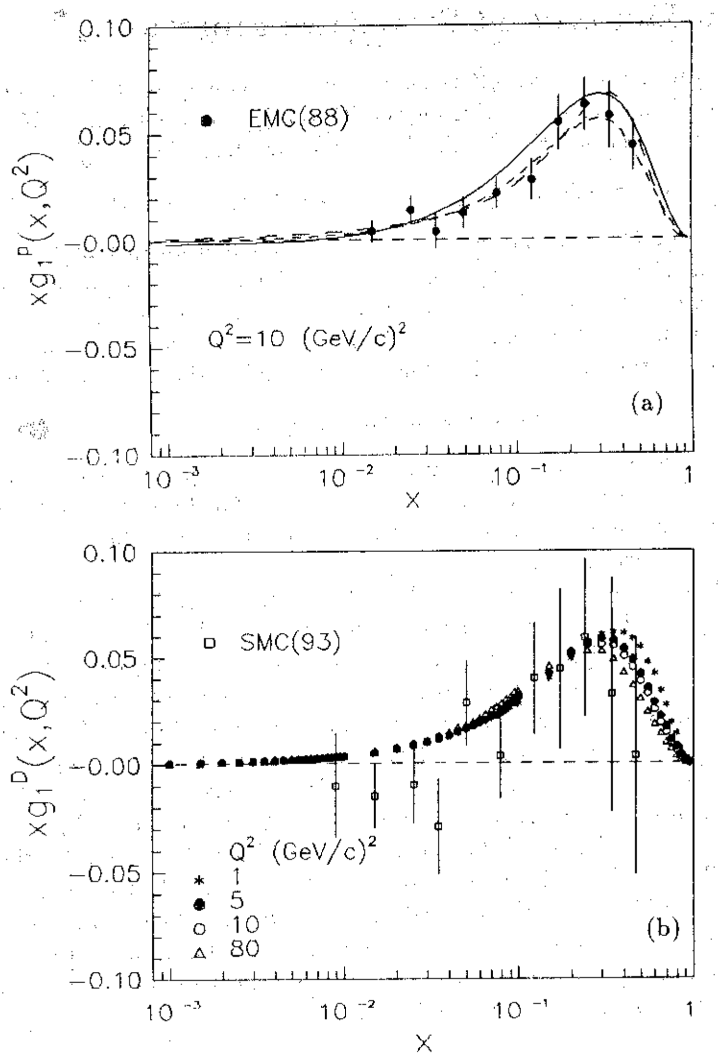

Figure 2: Deep-inelastic spin-dependent proton (a) and deuteron (b) structure functions. Experimental data: • - EMC [14], □ - SMC [1]. Theoretical results have been obtained with parton distributions taken from: - - - [14], — — [15], ···· [17]

2 Spin-Dependent Structure Function $g_1^D(x, Q^2)$

In the RIA the deuteron SF g_1^D is defined by (6) as a sum of the proton and neutron SF. We calculate g_1^D using the RDWF [6] and the Carlitz – Kaur model [8] for the nucleon SF $g_1^{p,n}$. The latter ones are defined by the formulas:

$$g_1^p = cos(\theta) \cdot [4u_v - 3d_v]/18, \; g_1^n = cos(\theta) \cdot [u_v - 2d_v]/18. \qquad (8)$$

The structure functions are proportional to the momentum distribution of the valence quarks (u_v, d_v) in the high x-range. Their behaviour in the low x-range is regulated by the "spin dilution factor" $cos(\theta)$. It is a measure of transfer of spin from valence quarks to gluons and $q\bar{q}$ pairs and is significant at low x. We use the model [12] in which this factor has the following form

$$cos(\theta) = [1 + R_0 x G(x, Q^2)]^{-1}. \qquad (9)$$

The parameter R_0 was obtained by fitting data on g_1^p [13]. The parton distributions (u_v, d_v, G) were taken from [14-16]. It should be noted that the Q^2-dependence of the g_1^D calculated in the present paper is defined by the behaviour of the parton distribution on Q^2.

Figure 2(a) shows the results for the xg_1^p obtained with different parton distributions [14,15,17] and experimental data [13]. Note that the proton and neutron SF in the model [17] have the negative singular asymptotic $g_1^{p,n} \sim (-1)/(x ln^2(x))$ at $x \to 0$. It is related to anomalous exchange of instanton configurations between quarks. Large experimental errors do not allow one to choose between different proton models. It is necessary to measure g_1^p with higher accuracy in the lower x-range to discriminate proton models. The dependence of the deuteron SF $xg_1^D(x, Q^2)$ on x for $Q^2 = 1, 5, 80 \; (GeV/c)^2$ is shown in Figure 2(b). The result for the deuteron SF is similar to that for the proton one. The weak Q^2-dependence of the xg_1^D is observed. We compare our results with SMC data [1]. The calculated points lie above zero. Four left experimental points are systematically displaced in the negative region. Taking into account large errors the agreement between calculated results and experimental data should be considered good.

3 Nuclear Effect in Deuteron

The nuclear effect in the deuteron for the spin-dependent SF is described by the ratio $R_g^{D/N}(x, Q^2) = g_1^D(x, Q^2)/g_1^N(x, Q^2)$. Figure 3 (curve 1) shows the dependence of the $R_g^{D/N}$ on x and $Q^2 = 1 - 80 \; (GeV/c)^2$. The parametrizations of the parton distributions are taken from [14-16]. The ratio $R_g^{D/N}$ is practically constant and independent of parton distributions in the wide kinematical range. It should be noted that $R_g^{D/N}$ is independent of nucleon model

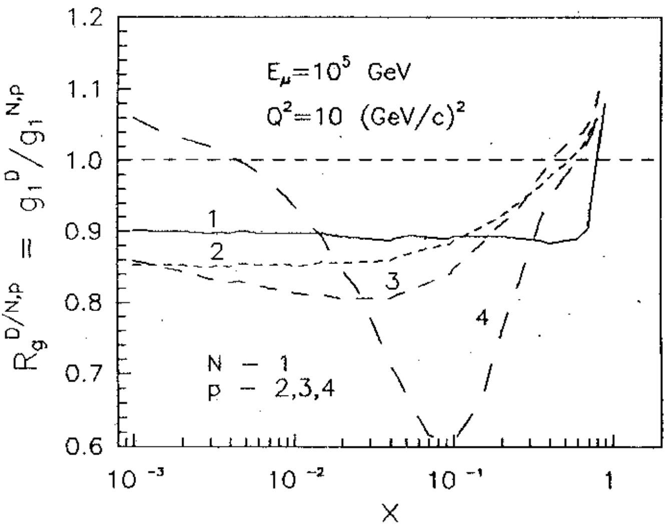

Figure 3: The ratio $R_g^{D/N,p} = g_1^D/g_1^{N,p}$ of the spin-dependent structure functions for the deep-inelastic lepton-deuteron scattering. Parton distributions are taken from: 2 - [14], 3 - [15], 4 - [16]

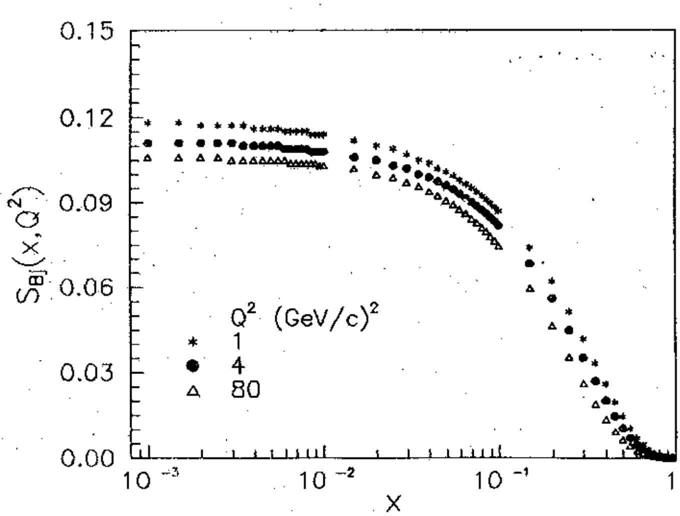

Figure 4: The Bjorken integral $S_{Bj}(x, Q^2)$

[8,17] too. The effect of relativistic Fermi motion is approximately equal to $\sim 9\ {}^{o}/_{o}$ in the range $x = 10^{-3} - 0.7$. The estimation of the nuclear effect in deuteron based on another relativistic approach [5] has been obtained in papers [18,19]. It is consistent with our results. The function $R_g^{D/N}$ can be approximated as follows $R_g^{D/N}(x, Q^2) = 0.892 \pm 0.002$. This parametrization can be used to extract the neutron SF g_1^n from experimentally known deuteron and proton ones

$$g_1^n(x, Q^2) = 2 \cdot [R_g^{D/N}(x)]^{-1} \cdot g_1^D(x, Q^2) - g_1^p(x, Q^2). \tag{10}$$

Thus, the obtained results allow us to conclude that the dependence of the ratio $R_g^{D/N}$ on x is the universal one in the range $x < 0.7$ and is defined by the structure of the RDWF. The results clearly demonstrate that the neutron SF g_1^n extracted from deuteron and proton data is modified by the nuclear medium.

Figure 3 (curves 2,3,4) shows the dependence of the ratio $R_g^{D/p} = g_1^D/g_1^p$ on x for $Q^2 = 10\ (GeV/c)^2$. This ratio, in contrast to $R_g^{D/N}$, is strongly dependent both on parton distributions [14-16] and nucleon models [8,17].

It should be noted that the nuclear structure for g_1^D in [1] was taken into account as follows $\Gamma_1^D = \Gamma_1^N(1 - 1.5 \cdot w_D)$, where $\Gamma_1^{D,N} = \int_0^1 g_1^{D,N}(x)dx$, w_D is the probability of the D-wave in deuteron. The nuclear correction $\delta g_1^n = (1 - R_g^{D/N})(g_1^p + g_1^n)$ calculated for the neutron SF is presented in Table 1. The results were obtained for the incident muon energy $E_\mu = 10^5\ (GeV)$. It is seen that this correction can be very large.

Table 1. The nuclear correction δg_1^n for the neutron SF

$Q^2 \setminus x$		10^{-3}	10^{-2}	10^{-1}	0.7
0.8	g_1^n	-4.19e-2	-2.25e-2	-7.30e-3	7.09e-3
	δg_1^n	7.36e-2	3.97e-2	2.74e-2	4.99e-3
4.00	g_1^n	-2.27e-2	-1.50e-2	-3.41e-3	4.45e-3
	δg_1^n	4.21e-2	3.12e-2	2.94e-2	3.05e-3
80.0	g_1^n	-1.64e-2	-1.26e-2	5.26e-4	2.52e-3
	δg_1^n	3.01e-2	2.95e-2	3.42e-2	1.69e-3

The performed analysis of the nuclear correction for the neuteron SF enables one also to consider the influence of the nuclear effect on the Bjorken sum rule [2]:

$$\int_0^1 [g_1^p(x) - g_1^n(x)]dx = \frac{1}{6}\frac{g_A}{g_V}. \tag{11}$$

Here g_A, g_V are axial-vector and vector coupling constant of neutron β-decay. The ratio g_A/g_V is determined by the experiment to be $\simeq 1.254$. As was shown in the previous section, the nuclear effect is $\sim 9\ ^o/_o$ in the range $x = (10^{-3} \div 0.7)$. We used the result on ratio $R_g^{D/N}$ and equation (10) to estimate the nuclear effect for the Bjorken integral $S_{Bj}(x, Q^2) = \int_x^1 [g_1^p(y, Q^2) - g_1^n(y, Q^2)]dy$.

Figure 4 shows the results of the dependence of the Bjorken integral $S_{Bj}(x, Q^2)$ on x for $Q^2 = 1, 4, 80\ (GeV/c)^2$. The weak Q^2-dependence in the low x-range is observed. It should be noted that the obtained value of $S_{Bj}(x, Q^2)$ at $x = 10^{-3}$ is rather lower than expected from the BSR.

We estimated the correction $\delta S_{Bj}/S_{Bj}$ for the Bjorken integral due to the nuclear effect. The results obtained for the incident muon energy $E_\mu = 10^5\ (GeV)$ are presented in **Table 2.** It is seen that the nuclear correction changes from $11^o/_o$ to $16^o/_o$ in the wide kinematical range.

Table 2. The nuclear correction $\delta S_{Bj}/S_{Bj}$ for the Bjorken integral

$Q^2 \setminus x$	10^{-3}	10^{-2}	10^{-1}	0.7
0.80	1.17e-1	1.17e-1	1.22e-1	1.61e-1
4.00	1.18e-1	1.18e-1	1.23e-1	1.62e-1
80.0	1.18e-1	1.18e-1	1.24e-1	1.63e-1

Conclusion

We have considered the deep-inelastic muon-deuteron scattering in the framework of the covariant approach in the light-cone variables. The spin-dependent deuteron structure function $g_1^D(x, Q^2)$ has been calculated and compared with SMC data. It has been shown that the effect of the relativistic Fermi motion in deuteron described by the ratio $R_g^{D/N}$ is $\sim 9^o/_o$. It is an important argument that the nuclear medium alters considerably the spin structure of the free nucleon. The procedure of the extraction of the neutron SF $g_1^n(x, Q^2)$ takes into account correctly the relativistic deuteron spin structure and can be used to analyze other experimental data. The correction for the Bjorken integral due to the nuclear effect was derived to be $(11 - 16\ ^o/_o)$ in the range $x = 10^{-3} - 0.7$ and $Q^2 = 1 - 80\ (GeV/c)^2$. It should be taken into account to verify the Bjorken sum rule.

Acknowledgement

The author would like to acknowledge helpful discussion with G.I.Smirnov and thank A.M.Baldin and Yu.A.Panebratsev for support of the present work. This work has been partially supported by the Russian Foundation of Fundamental Research under Grant No. 93-02-3961.

References

[1] B.Adeva et al., SMC collaboration, Phys.Lett. **B302** (1993) 533.

[2] J.D.Bjorken, Phys.Rev. **148** (1966) 1467; Phys.Rev. **D1** (1970) 1376.

[3] F.Gross, Phys.Rev. **D10** (1974) 223.

[4] L.L.Frankfurt, M.I.Strikman, Phys.Rep. **5** (1981) 215.

[5] V.A.Karmanov, Particles and Nuclei **19** (1988) 525.

[6] M.A.Braun, M.V.Tokarev, Vestnik LGU **4** (1986) 7; M.A.Braun, M.V.Tokarev, Particles and Nuclei **22** (1991) 1237.

[7] M.A.Braun, M.V.Tokarev, Preprint JINR, E2-93-184.

[8] R.Carlitz, J.Kaur, Phys.Rev.Lett. **38** (1977) 673; J.Kaur, Nucl.Phys. **B128** (1977) 219.

[9] P.V.Landshoff, J.C.Polkinghorn, R.D.Short, Nucl.Phys. **B28** (1971) 225.

[10] R.Blankenbecler, L.F.Cook, Phys.Rev. **119** (1960) 1745.

[11] M.Lacombe et al., Phys.Lett. **B101** (1981) 139.

[12] J.Qiu et al., Phys.Rev. **D41** (1990) 65.

[13] J.Ashman et al., Phys.Lett., **B206** (1988) 364; Nucl.Phys. **B328** (1989) 1.

[14] M.Gluck, E.Reya, A.Vogt, Z.Phys. **C48** (1990) 471.

[15] J.G.Morfin, W.K.Tung, Z.Phys. **C52** (1991) 13.

[16] D.W.Duke, J.F.Owens, Phys.Rev. **D30** (1984) 49.

[17] A.E.Dorokhof, N.I.Kochelev, Yu.A.Zubov, Argonne Report PHY-7056-TH-92; Inter.Jour.Mod.Phys. **A8** (1993) 603.

[18] I.I.Bazhanskij et al., in: Proc. VIII Inter. Seminar on High Energy Physics Problems (1986), Dubna, JINR D1, 2-86-668, **1** (1987) 318.

[19] L.P.Kaptari, A.I.Titov, A.Yu.Umnikov, Particles and Nuclei **22** (1991) 839.

Received by Publishing Department
on June 9, 1993.