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DETERMINING OF THE NUCLEAR COMPOSITION OF PRIMARY COSMIC RAYS FROM THE EXPERIMENTAL DISTRIBUTIONS OF MULTIPLE MUONS IN ATMOSPHERIC SHOWERS

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INTRODUCTION

Protons and nuclei passing through the Atmosphere generate multiple muons [1]. The content of such muons in atmospheric cascades depends strongly on the primary nuclear composition of the cosmic rays, so measurements and theoretical calculations of the distribution of multiple muons in the Atmosphere are an essential task for the determination of the primary composition.

A METHOD FOR DETERMINING THE NUCLEAR COMPOSITION OF PRIMARY COSMIC RADIATION

1. There exists an opinion [2] that single and double muons present in a cascade in the Atmosphere determine the nuclear composition of the primary cosmic radiation if the depth at which the experimental device is situated and its dimensions are taken into account.

The above opinion is evidently based on the assumption that the multiple muons in a cascade exhibit a Poisson distribution,

$$W(m) = \frac{\bar{n}^m}{m!} e^{-\bar{n}}.$$

If $\bar{n}$ and $\bar{N}$, respectively, are the mean numbers of muons from a proton and from an average nucleus α, then one may set (the superposition model is satisfactorily applied, when the nuclei penetrate the Atmosphere to large depth [3])

$$\bar{N} = \bar{n}\,\alpha \tag{1}$$

and

$$\frac{W_p(2)}{W_p(1)} = \frac{\bar{n}}{2}, \quad - \frac{W_\alpha(2)}{W_\alpha(1)} = \frac{\bar{N}}{2}.$$

Then

$$\frac{W_\alpha(2)/W_\alpha(1)}{W_p(2)/W_p(1)} = \alpha\,. \tag{2}$$

2. A straightforward computation of the multiple muons involving the threshold $E_\mu^{thresh} = 200$ GeV [4] has revealed that the assumption concerning the

Poisson distribution of multiple muons is unjastified (the distribution of secondary pions produced in strong interaction events is described by KNO formula or by negative binomial distribution [5]). Therefore, we shall consider the multiple muons a proton to be distributed according to the formula

$$1 = W(0) + W(1) + W(2) + \dots \tag{3}$$

and to be normalized to unity. If α is the average atomic number of primary cosmic rays, then

$$1 = 1^{\alpha} = (\sum_{n=0} W_p(n))\,\alpha\,. \tag{4}$$

From (4) one obtains

$$\frac{W_\alpha(2)}{W_\alpha(1)} = \frac{(\alpha - 1)\,(W_p(1))^2/2 + W_p(2)\,W_p(0)}{W_p(0)\,W_p(1)} =$$

$$= \frac{(\alpha - 1)W_p(1)}{2W_p(0)} + \frac{W_p(2)}{W_p(1)}\,. \tag{5}$$

From (3) and (5) it is not possible to obtain a simple relation similar to (2) for determining the average atomic number of the primary cosmic radiation.

Further we shall show expression (5) also to be incorrect, since a mixture of nuclei cannot be represented by its mean nuclear number α.

3. Formula (5) holds only when the primary cosmic rays consist of nuclei of nuclear number α, i.e.

$$I_p(E) = I_0\,E_p^{-\gamma} \longrightarrow I_\alpha(E) = I_0' E_\alpha^{-\gamma}$$

$$E_p = E_\alpha/\alpha\,, \quad I_0' = I_0/\alpha^{1-\gamma}. \tag{6}$$

Now consider the primary cosmic rays to consist of a mixture of five nuclei: p, A^4, A^{14}, A^{26}, A^{56}. In this case the integral spectrum of the primary cosmic radiation will be written in the form:

$$I(E) = I_0\,(\sum_i d(A^i)\,E_0^{-\gamma}),\ \ (i = 1, 4, 14, 26, 56)$$

or, upon passing to equal energies per nucleon,

$$I(E) = I_0\,E_N^{-\gamma}\,(\sum_i d(A^i)\,(A^i)^{-\gamma})\,.$$

In this case the distributions for single-muon and two-muon groups will be determined by formulae

$$W(1) = \sum_i d(A^i)\, W_{A^i}(1)$$

$$W(2) = \sum_i d(A^i)\, W_{A^i}(2), \quad (2)/(1) = W(2)/W(1). \tag{7}$$

In (7) the respective distributions $W_{A^i}(1)$ and $W_{A^i}(2)$ for single and double muons can be expressed for each $A^i i \geq 1$ through $W_{A^1}(1)$ and $W_{A^1}(2)$. These formulae, however, contain five coefficients $d(A^i)$, and therefore the ratio (2) / (1) cannot determine the average atomic number $\overline{A}$ of the primary cosmic rays. For obtaining $\overline{A}$ one must know the values of these five coefficients. Then ($d(A^i)$ — see below)

$$\overline{A} = \frac{\sum_i d(A^i)\,(A^i)^2}{\sum_i d(A^i)\, A^i}. \tag{8}$$

4. The methods described above do not permit determination of the nuclear composition of the primary cosmic radiation. In what way can one determine the nuclear composition of the primary cosmic rays, making use of the distribution of multiple muons? If the distributions of multiple muons from different nuclei were identical, then the problem of finding the primary nuclear composition would remain insolvable. But the distributions of multiple muons from different nuclei differ strongly (see Table 1). Thaking advantage of this fact one may develop the following method [4] for finding the nuclear composition of the primary cosmic radiation.

To this end we took the integral energy spectrum of primary cosmic rays in the form:

$$I(E_0) = I_0 E_0^{-\gamma}\,(d(p) + d(A^4) + d(A^{14}) + d(A^{26}) + d(A^{56})). \tag{9}$$

Here $d(p) + d(A^4) + d(A^{14}) + d(A^{26}) + d(A^{56}) = 1$, E_0 is the energy per nucleus, $\gamma = 1.75$. Upon transition to equal energies per nucleon, formula (9) acquires the following form:

$$I(E_N) = IE_N^{-\gamma}\,(d(p) + d(A^4)/4^\gamma + d(A^{14})/14^\gamma +$$
$$+ d(A^{26})/26^\gamma + d(A^{56})/56^\gamma)). \tag{10}$$

The distribution of multiple muons obtained with the aid of formula (10) will be written as follows:

Table 1*

n/A	p	A^4	A^{14}	A^{26}	A^{56}
0	0.9535	0.8622	0.7846	0.5851	0.0
1	0.0411	0.0994	0.1413	0.1839	0.1886
2	0.0043	0.0222	0.0635	0.0818	0.0953
3	$8.00\cdot10^{-4}$	$8.40\cdot10^{-3}$	$3.07\cdot10^{-2}$	$5.01\cdot10^{-2}$	$4.42\cdot10^{-2}$
4	$2.00\cdot10^{-4}$	$3.90\cdot10^{-3}$	$1.53\cdot10^{-2}$	$2.62\cdot10^{-2}$	$2.95\cdot10^{-2}$
5	$7.60\cdot10^{-5}$	$1.60\cdot10^{-3}$	$1.09\cdot10^{-2}$	$1.96\cdot10^{-2}$	$3.05\cdot10^{-2}$
6	$2.96\cdot10^{-5}$	$8.40\cdot10^{-4}$	$8.00\cdot10^{-3}$	$1.18\cdot10^{-2}$	$2.06\cdot10^{-2}$
7	$2.54\cdot10^{-5}$	$5.17\cdot10^{-4}$	$3.90\cdot10^{-3}$	$9.50\cdot10^{-3}$	$2.35\cdot10^{-2}$
8	$1.10\cdot10^{-5}$	$3.01\cdot10^{-4}$	$4.00\cdot10^{-3}$	$6.10\cdot10^{-3}$	$5.80\cdot10^{-3}$
9	$0.58\cdot10^{-5}$	$1.76\cdot10^{-4}$	$2.70\cdot10^{-3}$	$6.30\cdot10^{-3}$	$1.37\cdot10^{-2}$
10	$0.51\cdot10^{-5}$	$1.06\cdot10^{-4}$	$3.00\cdot10^{-3}$	$5.90\cdot10^{-3}$	$1.18\cdot10^{-2}$
12	$0.30\cdot10^{-5}$	$0.42\cdot10^{-4}$	$1.80\cdot10^{-3}$	$3.70\cdot10^{-3}$	$9.80\cdot10^{-3}$
14	$1.20\cdot10^{-6}$	$0.42\cdot10^{-4}$	$7.50\cdot10^{-4}$	$1.90\cdot10^{-3}$	$5.40\cdot10^{-3}$
16	$0.90\cdot10^{-6}$	$3.10\cdot10^{-5}$	$4.00\cdot10^{-4}$	$1.30\cdot10^{-3}$	$3.50\cdot10^{-3}$
18	$0.30\cdot10^{-6}$	$1.20\cdot10^{-5}$	$2.88\cdot10^{-4}$	$1.20\cdot10^{-3}$	$5.00\cdot10^{-3}$
20	$1.70\cdot10^{-7}$	$9.70\cdot10^{-6}$	$1.41\cdot10^{-4}$	$7.90\cdot10^{-4}$	$2.70\cdot10^{-3}$
25	$1.50\cdot10^{-7}$	$6.60\cdot10^{-6}$	$9.50\cdot10^{-5}$	$4.45\cdot10^{-4}$	$2.30\cdot10^{-3}$
30	$1.50\cdot10^{-7}$	$2.20\cdot10^{-6}$	$4.90\cdot10^{-5}$	$3.73\cdot10^{-4}$	$1.30\cdot10^{-3}$
40	—	$1.20\cdot10^{-6}$	$2.00\cdot10^{-5}$	$9.80\cdot10^{-5}$	$5.40\cdot10^{-4}$
50	—	$3.70\cdot10^{-7}$	$8.90\cdot10^{-6}$	$4.60\cdot10^{-5}$	$2.90\cdot10^{-4}$

*In Table 1 the results are presented of a simulation of the distributions of the flux of multiple muons with a threshold $E_\mu^{thresh} \geq 230$ GeV for an infinite plane. The simulation was performed using the program described in ref. [4] for a primary integral spectrum with the exponent $\gamma = 1.75$ for p, A^4, A^{14}, A^{26}, A^{56}. The simulation was performed for primary nuclei within the energy range from 300 to $5\cdot10^5$ GeV/nucl.

$$W(N) = d(p)\,W(N,p) + d(A^4)\,W(N,A^4)/4^\gamma + d(A^{14})\,W(N,A^{14})/14^\gamma +$$

$$+ d(A^{26})\,W(N,A^{26})/26^\gamma + d(A^{56})\,W(N,A^{56})/56^\gamma =$$

$$= d(p)W'(N,p) + d(A^4)W'(N,A^4) + d(A^{14})\,W'(N,A^{14}) +$$

$$+ d(A^{26})\,W'(N,A^{26}) + d(A^{56})\,W'(N,A^{56}). \tag{11}$$

Table 2*

n/A	p	A^4	A^{14}	A^{26}	A^{56}
1	1.0	0.704	0.296	0.214	0.10
2	$2.61\cdot10^{-3}$	$1.98\cdot10^{-2}$	$7.48\cdot10^{-3}$	$4.90\cdot10^{-3}$	$1.95\cdot10^{-3}$
3	$2.89\cdot10^{-3}$	$3.61\cdot10^{-3}$	$2.14\cdot10^{-3}$	$1.68\cdot10^{-3}$	$8.17\cdot10^{-4}$
4	$7.37\cdot10^{-4}$	$1.20\cdot10^{-3}$	$1.11\cdot10^{-3}$	$7.60\cdot10^{-4}$	$5.62\cdot10^{-4}$
5	$2.60\cdot10^{-4}$	$4.65\cdot10^{-4}$	$5.49\cdot10^{-4}$	$3.80\cdot10^{-4}$	$3.40\cdot10^{-4}$
6	$9.64\cdot10^{-5}$	$2.05\cdot10^{-4}$	$3.60\cdot10^{-4}$	$2.25\cdot10^{-4}$	$1.94\cdot10^{-4}$
7	$4.47\cdot10^{-5}$	$7.61\cdot10^{-5}$	$2.55\cdot10^{-4}$	$1.35\cdot10^{-4}$	$1.31\cdot10^{-4}$
8	$2.15\cdot10^{-5}$	$4.57\cdot10^{-5}$	$1.97\cdot10^{-4}$	$8.59\cdot10^{-5}$	$9.57\cdot10^{-5}$
9	$1.45\cdot10^{-5}$	$3.10\cdot10^{-5}$	$1.70\cdot10^{-4}$	$5.97\cdot10^{-5}$	$7.88\cdot10^{-5}$
10	$1.42\cdot10^{-5}$	$2.18\cdot10^{-5}$	$1.49\cdot10^{-4}$	$4.35\cdot10^{-5}$	$5.98\cdot10^{-5}$
12	$1.10\cdot10^{-5}$	$1.16\cdot10^{-5}$	$1.27\cdot10^{-4}$	$2.80\cdot10^{-5}$	$3.90\cdot10^{-5}$
14	$4.0\cdot10^{-6}$	$6.30\cdot10^{-6}$	$9.21\cdot10^{-5}$	$1.42\cdot10^{-5}$	$2.45\cdot10^{-5}$

*In Table 2 the results are given of a simulation of the distribution of multiple muons with E_μ^{thresh} = 230 GeV and taking into account the geometry of the underground scintillation telescope of INR Russian Academy of Sciences [4]. The single muons are reconstructed from computations of item 2 for an infinite plane. The simulation has been performed for the primary integral spectrum with $\gamma = 1.75$ for p, A^4, A^{14}, A^{26}, A^{56}. The simulation was performed for primary nuclei within the energy range from 300 to $5\cdot10^5$ GeV/nucl.

In Table 2 the numerical values of W' are given $N = 1$—14 (for solution of the equations the values $N = 2 - 8$ are used).

The unknown quantities in expression (11) are $d(p)$, $d(A^4)$, $d(A^{14})$, $d(A^{26})$, $d(A^{56})$, which can be found by solving the set of five equations.

To this end we find the values $W(N)$, $W(N+1)$, $W(N+2)$, $W(N+3)$, $W(N+4)$ in (11) for all $0 \le d(p) \le 1$, $0 \le d(A^4) \le 1 - d(p)$, $0 \le d(A^{14}) \le$ $\le 1 - d(p) - d(A^4)$, $0 \le d(A^{26}) \le 1 - d(p) - d(A^4) - d(A^{14})$, $0 \le d(A^{56}) \le$ $\le 1 - d(p) - d(A^4) - d(A^{14}) - d(A^{26})$ with a 0.01 step. Then from known experimental values for $N-$, $(N+1)-$, $(N+2)-$, $(N+3)-$, $(N+4)$-multiple events [6] of $A(N)$, $A(N+1)$, $A(N+2)$, $A(N+3)$, $A(N+4)$ we calculate $F(\ldots)$ for all the values of $d(p)$, $d(A^4)$, $d(A^{14})$, $d(A^{26})$, $d(A^{56})$ with a 0.01 step:

$$F(d(p), d(A^4), d(A^{14}), d(A^{26}), d(A^{56})) =$$

$$= (A(N) - W(N, d(p), d(A^4), d(A^{14}), d(A^{26}), d(A^{56})))^2 / A^2(N) +$$
$$+ (A(N+1) - W(N+1,...))^2 / A^2(N+1) +$$
$$+ (A(N+2) - W(N+2,...))^2 / A^2(N+2) +$$
$$+ (A(N+3) - W(N+3,...))^2 / A^2(N+3) +$$
$$+ (A(N+4) - W(N+4,...))^2 / A^2(N+4).$$

Precisely the values of $d(p)$, $d(A^i)$, for which the lowest value of $F(...)$ is achieved, will be the sought solution of the set of equations.

The following values (for equal energies per nucleus) were obtained for $d(p)$, $d(A^4)$, $d(A^{14})$, $d(A^{26})$, $d(A^{56})$:

$$d(p) = 0.41$$
$$d(A^4) = 0.11$$
$$d(A^{14}) = 0.25$$
$$d(A^{26}) = 0.15$$
$$d(A^{56}) = 0.08.$$

Reduction to equal energies per nucleon yields:

$$d(p) = 0.96979$$
$$d(A^4) = 0.02302$$
$$d(A^{14}) = 0.00584$$
$$d(A^{26}) = 0.00118$$
$$d(A^{56}) = 0.00017.$$

The following mean atomic number will correspond to the obtained chemical composition:

$$\bar{A} = \frac{\sum_i d(A^i)\,(A^i)^2}{\sum_i d(A^i)\,A^i} = 3.3$$

for primary nuclei of energies from 300 to $5 \cdot 10^5$ GeV/nucl.

SUMMARY

The mesured nuclear composition of cosmic rays at 1 GeV/nucl is [7] $d(p) = 0.939$, $d(A^2) = 0.055$, $d(A^9) = 0.0009$, $d(A^{14}) = 0.0035$, $d(A^{28}) =$ $= 0.0011$, $d(A^{56}) = 0.0003$, and $\bar{A} = 3.5$.

From the results presented in this article the conclusion may be drawn that within the primary energy range from 1 to $5 \cdot 10^5$ GeV/nucl the primary composition and, correspondingly, the quantity $\bar{A}$, varies weakly.

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REFERENCES

1. Kessler D., Nare R. — Nuovo Cim., 1957 5, p.1540.
   Zatsepin G.T. — Proc. of Seminar on $\nu - e$ Problem, Moscow, 1972, p.240.
2. Chudakov A.E. — Proc. 16th ICRC, 1979, v.3, p.192.
3. Dedenko L.G., Zatsepin G.T. — Proc. Moscow Cosmic Ray Conf., 1960, v.2, p.201.
   Gaisser T.K. et al. — Phys. Rev., 1982 D25, p.2341.
4. Arkhestov G.Kh., Beshtoev Kh.M. — Preprint INR, П-507, 1986; Yad. Phys., 1988, v.47, p.753.
   Arkhestov G.Kh., Beshtoev Kh.M. — Preprint INR, П-0656, 1990.
5. Alner G.J. et al. — Phys. Lett., 1986, B167, p.476.
   Grishin V.G. — Quarks and Hadrons in the Interactions of High Energy Particles. Moscow, Energoatomizdat, 1988.
6. Budko E.V. et al. — Izv. Akad. Nauk SSSR, Ser. fiz. 1985 49, p.1373.
7. Meyer P., Minagawa G. — Proc. 15th ICRC, Plovdiv, 1977, v.1, p.248.
   Bakatanov V.N. et al. — Preprint INR, П-0430, 1985; Pis'ma JETP, 1985 49, p.307.

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