объединенный
институт
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исследований
дубна

E2-93-188

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THE MODEL OF QUANTUM SCALAR DIPOLE-TYPE FIELD

Submitted to "Nuovo Cimento A"

1993

1 Introduction

The neutral scalar fields are the fundamental components of the recent models of weak interactions. The scalar objects which play the role of Higgs-bosons are needed for the observed masses of gauge vector bosons to appear. Another obvious candidate for a (pseudo)-scalar coupling between the quark bound systems and leptons would be a Higgs-like particle X-boson or the exotic partner of a graviton. Recently, several theoretical issues involved have been connected with X-boson interaction effects inside a quark bound system. In the Standard Model (SM) the X-boson coupling with a quark q_1 and an antiquark $\bar{q}_2$ is characterized by the following Lagrangian density

$$L_X = -g_X X q_1 \bar{q}_2,$$

where X denotes the X-boson field with mass m_X , the coupling constant g_X is defined in a standard way, $g_X = 2^{1/4} G_F^{1/2} \tilde{m} b$. Here G_F is the Fermi coupling constant, the quark mass $\tilde{m}$ is equal to the constituent quark mass $\hat{m}_q$ in the case of equal masses, otherwise $\tilde{m}$ is replaced by the reduced mass and b is at present an unknown parameter. We suppose more complete searches, both for long range and short distances, the Yukawa-type interactions mediated by the exchange of the hypothetical light scalar X-boson. According to SM, where the coupling of the X-boson to quarks is proportional to the quark mass, both for heavy and heavy-light quark bound system (HLQBS), the Yukawa- type coupling constant is large and the X-boson exchange contribution to the short-distance effect can no be neglected as compared with the QCD contribution due to the one-gluon exchange [1].

Since the dominant interaction inside the HLQBS is short range ($R_c \sim 0.1 \div 0.2\ fm$) on the QCD Λ -scale ($\Lambda = 200 \div 100\ MeV$, respectively and R_c is the QCD characteristic scale on which the nonperturbative fluctuations dominate), it is an ideal probe for new interaction effects arising from the exchange of the low-mass X-boson. It seems, this effect will diminish the masses of HLQBS and increase the magnitude of the ground state vertex function of HLQBS. The contribution of X-boson exchange to the short-distance part of the potential V_X , arising from the Yukawa coupling $g_Y^2 = \sqrt{2} G_F (\beta \tilde{m})^2$ of the X-boson to a quark, is given by (in the $\mathbf{R}^3$ -space)

$$V_X(r) \sim r^\alpha \exp(-m_X r) g_Y^2/(4\pi), \quad (1)$$

where α is the c-number, the parameter β is rather arbitrary, depending on the ratio of the vacuum expectation values of X (Higgs) doublets and on the mixing of two neutral X (Higgs) bosons. The interaction (1) is most important for HLQBS and small m_X , i.e.

$$m_X \langle R_c \rangle < O(1) ,$$

and taking into account the short range $R_c \sim 0.1 \div 0.2\ fm$, we can obtain the following mass interval for the hypothetical X-bosons

$$m_X \sim (1 \div 2)\ \text{GeV}$$

at decreasing Λ from 0.2 GeV to 0.1 GeV, respectively. The scalar bosons considered here are neutral CP-even objects, whose interactions with the fermions (and bound states) are known and whose masses are the free model parameters.

The finite temperature phase transitions in the model-like gauge-Higgs systems or pure scalar system theories (one of them is considered below) play an important role in scenarios of the early universe. The latter, as is assumed, was in a hot symmetric state at a high enough temperature. Due to a universe expansion and its cooling the so-called critical temperature passed through the electroweak phase transition breaking the symmetry spontaneously. The considered here model is expected to describe the main features of symmetry restoration correctly. We have presented our analysis in the context of generalized functions, which are the well defined distributions on:

- the space of complex Schwartz test functions on $\mathbf{R}^n$, $\mathbf{S}(\mathbf{R}^n)$;

-the space $\mathbf{S}_0(\mathbf{R}^n)$ of the generalised functions $F(z)$, where $\mathbf{S}_0(\mathbf{R}^n)$= $\{ F(z) \in \mathbf{S}(\mathbf{R}^n)$ and $F(0) = 0 \}$;

-the space of the moderate growth of distributions on $\mathbf{R}^n$, $\mathbf{S}'(\mathbf{R}^n)$. The article is organized as follows. Sec.2 reviews the basic Lagrange formalism for the system of two scalar neutral fields in the context of standard quantum field theory. Following the paper [2] we represent the interaction Lagrangian in terms of the partial normal-ordered scalar field operators. We mean that this scheme is valid only for the quadratic field factors in terms of the normal-ordered operators. For simplicity, we have presented our analysis based on a σ -like model. We compute the derivative relation of the mass with respect to the nonzero temperature in the scalar theory in Sec.3. Finally, in Sec.4 , we present our conclusions.

2 The Basic Formalism

The main feature of the subject presented here is that the quatization is performed according to the canonical formalism. As was noted in [3], the role of the scalar field of dipole-type in four dimensions is held in two dimensions by the simple pole field. The reason for considering such a problem in quantum field theory is that the analogy of behaviour between two and four dimensions has to be found at the level of Green functions.

Let us consider the $O(N)$ symmetric theory of a scalar field $\chi = \{\chi_1, \cdots, \chi_N\}$ with mass m_0 interacting with a massless fermion (quark) field Q by the

constant g. The simple σ -like model Lagrangian density looks like:

$$L = L_X + L_\Lambda + L_Q \ , \tag{2}$$

where

$$L_X \equiv L_\chi(z) = \frac{1}{2}\partial_\mu\chi(z)\partial^\mu\chi(z) + \frac{1}{2}m_0^2\chi^2(z) - \sqrt{\lambda}m_0\chi^3(z) - -\frac{\lambda}{4N}\chi^4(z) \ , \tag{3}$$

$$L_\Lambda \equiv L_\Lambda(z) = \partial_\mu\chi(z)\partial^\mu\Lambda(z) + \frac{1}{2}\Lambda^2 \ , \tag{4}$$

$$L_Q \equiv L_Q(z) = \bar{Q}(z)\left[i\hat{\partial} - g\hat{\partial}\chi(z)\right]Q(z) \ ,$$

$\chi^2(z) = \sum_{i=1}^N \chi_i^2(z)$, $\Lambda^2(z) = \sum_{j=1}^N \Lambda_j^2(z)$ and z are the 2h dimensional coordinates. The last two terms in (3) define the potential of self interaction of the field $\chi(z)$ by the unknown coupling constant λ. In the scheme described by the Lagrangian density (3) m_0 and λ are given as input parameters through a renormalization procedure to be shown below. The scalar neutral dipole-type field $\chi(z) \equiv \chi^*(z)$ obeys the equation

$$\Box^h\chi(z) = 0 \ , \tag{5}$$

$$(\Box \equiv \frac{\partial^2}{\partial z_1^2} + \cdots + \frac{\partial^2}{\partial z_m^2} - \frac{\partial^2}{\partial z_0^2} \ ; m+1 = D)$$

and canonical commutator relation on $\mathbf{R}^D$ at $D = 4$ [4]

$$[\chi(z),\ \chi(z')] = 2\pi \int d_4p\,\delta'(p)\exp\left[-ip(z-z')\right] = = \frac{1}{8\pi^3}\varepsilon(z^0)\Theta(z^2) \ ,$$

where $\delta'(p)$ is the well defined generalized function $\delta'(p) = \varepsilon(p^0)\delta'(p^2)$ from $\mathbf{S}'(\mathbf{R}_4)$, $\delta'(p) = 0$ at $p < 0$.

Note that eq. (5) obeys locality and Poincare covariance. Since the dimension of the χ -field is equal to unity (in the mass units) it could be represented as a free subcanonical massless scalar field to be compared with the canonical standard free scalar field with the mass dimension. Formally, the field $\chi(z)$ can be obtained from the quantization procedure of the system of two scalar neutral fields: $\chi(z)$ -field and an additional one- $\Lambda(z)$ with the Lagrangian density (4). The Euler-Lagrange (EL) equations

$$\Box^{h-1}\chi(z) = \Lambda(z) \ , \tag{6}$$

$$\Box\Lambda(z) = 0 \tag{7}$$

allow us to obtain eq. (5). The fundamental solution of eq.(5) is the generalized function $\Delta^c(z)$, obeying the equation

$$\Box^{h-1}\Delta^c(z) = \delta(z) \ ,$$

which is invariant under the linear transformation and saving the quadratic form $-z^2 = z_0^2 - z_1^2 - \cdots - z_m^2$. The solutions of the classical EL eqs. (6), (7) and

$$[i\hat{\partial} - g\hat{\partial}\chi(z)]Q(z) = 0$$

have the following form

$$Q(z) = \exp[-ig\chi(z)]Q_0(z) , \qquad (8)$$

where $\chi(z)$ obeys eqs. (5),(6) and Q_0 is the solution of the free Dirac eq. In quantum case the solution (8) becomes

$$Q(z) =: \exp[-ig\chi(z)] : Q_0(z) ,$$

where the scalar field $\chi(z)$ is realized in the pseudohilbert space $\mathbf{H}$ and $Q_0(z)$ is the free Dirac field, acting in the Fock space $\mathbf{F}$. Here we consider the local normal ordered function

$$: \{exp[-ig\sum_{i=1}^{N} n_i\chi(z_i)]\} :$$

as the generalized function of the moderate growth, where n_i are arbitrary integer numbers. To understand : exp (...) : it should be very instructive to consider the regularized field $\chi^{(r)}(z)$ as a smooth operator function of $\chi(z)$

$$\chi^{(r)}(z) = \int\{a(p)\exp[-ip(z-ir/2)] + a^*(p)\exp[ip(z+ir/2)]\}d_np , \quad (9)$$

$a(p)$ and $a^*(p)$ are the covariant operators of creation and annihilation, respectively, obeying the commutation relations

$$[a(p), a(q)] = [a^*(p), a^*(q)] = 0 ,$$

$$[a(p), a^*(q)] = (2\pi)^4\delta(p-q)(2\pi)\Theta(p^0)\delta(p^2-m^2)$$

for the scalar field with a mass m, r is a vector from an open upper light cone in Minkowski space $V^+ = \{z \in \mathbf{R} : z^0 > \mid z \mid = [\sum_{i=1}^{3}(z^i)^2]^{1/2}\}$. The definition of the local normal ordered exponential function of the regularized field $\chi^{(r)}(z)$ is as follows [4]

$$: \exp[\pm ig\chi^{(r)}(z)] := \frac{\exp[\pm ig\chi^{(r)}(z)]}{\langle 0 \mid \exp[\pm ig\chi^{(r)}(z)] \mid 0\rangle} .$$

The fermion (quark) field $Q(z)$ obeys the renormalized quantum field eq.

$$\{i\hat{\partial} - g\gamma^\mu N[\partial_\mu\chi(z)]\}Q(z) = 0 ,$$

which is an analog of the classical equation. Here N denotes normal ordering defined as a limit of

$$N[\partial_\mu\chi(z)]Q(z) = \partial_\mu[\chi(z) + igw(z-z')]Q(z') \text{ as } z' \to z ,$$

where the two-point Wightman function $w(x-y)$ is introduced in $\mathbf{R}^4$ as

$$w(z) = \langle 0 \mid \chi(z)\chi(0) \mid 0\rangle = -\frac{1}{(4\pi)^2}\ln[-\mu^2z^2 + i\Theta(z^0)] ,$$

formed in the time-ordered $w^c(z)$-function

$$w^c(z-z') = \langle 0 \mid T\chi(z)\chi(z') \mid 0\rangle = \Theta(z^0-z^{0\prime})w(z-z') + \Theta(z^{0\prime}-z^0)w(z'-z) ,$$

which obeys the eq.

$$\Box^\nu w^c(z) = \delta_{2\nu}(z) , \nu = 1, 2, \ldots$$

in 2ν -dimension. Under the dilatation transformation $z \to az$ the Wightman function $w(z)$ acquires the additional term, i.e.

$$w(z) \to w(az) = w(z) - \frac{1}{2(2\pi)^2}\ln a , \; a > 0 .$$

It could be interpreted as a spontaneous symmetry breaking. Therefore, this is an important point for the special role of the scalar dipole-type field $\chi(z)$.

Now we introduce the local gauge transformations of the χ-field as

$$\chi(z) = \chi'(z) + \alpha(z) ,$$

where $\alpha(z)$ is a smooth real solution of the eq. $\Box\alpha(z) = 0$, and $\alpha(z)$ belongs to the space $\mathbf{S}(\mathbf{R}^3)$ of complex Schwartz test functions on $\mathbf{R}^3$ at any fixed z^0; such a transformation forms the Abelian A_0 group of symmetry, $\alpha \in A_0$. The local gauge transformation of $Q(z)$ -field looks like

$$Q(z) = \exp[-ig\alpha(z)]Q(z) ,$$

but $Q_0(z)$ and $\Lambda(z) = \Box\chi(z)$ are lived as the A_0 -invariant functions. If we consider the α -dependent function (at $\alpha \in A_0$), which is a generator of the local gauge transformation and α is an arbitrary solution of the D'Alambert-like eq.

$$\lambda(\alpha) = \int_{z_0=const} d^3\vec{z}\,[\alpha(z)\,\partial_0\,\Lambda(z) - \Lambda(z)\,\partial_0\,\alpha(z)] ,$$

then there exists the following relation

$$\chi(z) \to \exp[i\lambda(\alpha)]\chi(z)\exp[-i\lambda(\alpha)] = \chi(z) + \alpha(z) .$$

In the case of a fast decreasing $\alpha(z)$-function at the space infinity the generators of local gauge transformations obey the following relations using the Poisson brackets (PB):

$$\{\lambda[\alpha(z)], Q(z')\} = ig\alpha(z)Q(z') ,$$
$$\{\lambda[\alpha(z)], \bar{Q}(z')\} = -ig\alpha(z)\bar{Q}(z') ,$$
$$\{\lambda[\alpha(z)], \Lambda(z)\} = 0 .$$

To understand the role of an additional scalar field $\Lambda(z)$, obeying eq. (7), it is very instructive to remind some relations with this field. We restrict here only by:

- the equal-time PB

$$\{\Lambda(z), Q(z')\}_{z^0=z^{0'}} = \{\Lambda(z), \bar{Q}(z')\}_{z^0=z^{0'}} = 0 ,$$
$$\{\Lambda(z), \Lambda(z')\}_{z^0=z^{0'}} = 0 ,$$
$$\{\partial_0\Lambda(z), Q(z')\}_{z^0=z^{0'}} = -ig\delta^{(3)}(\vec{z}-\vec{z}')Q(z')$$

and

- the PB at any time

$$\{\Lambda(z), Q(z')\} = -igD_0(z-z')Q(z') ,$$
$$\{\Lambda(z), \bar{Q}(z')\} = igD_0(z-z')\bar{Q}(z') ,$$

where
$$D_0(z-z') = 2\pi i \int d_n p\varepsilon(p^0)\delta(p^2)\exp\left[-ip(z-z')\right] .$$

To present the nearely real physical picture, we suppose that the field $\Lambda(z)$ is a real physical one, but nonobservable. The observable field would be the conserved current $j_\mu(z) = -\partial_\mu\Lambda(z)$. In the case of a massless scalar field at the same time with solution (9), which can be considered as the Fock notion of a massless scalar field, there is a class of the solutions $\chi(z)$, parametrizing by a real number c. To treat the case of the spontaneous symmetry breaking we introduce the new scalar dipole-type field $\chi'(z)$ as a result of the shift of the Fock solution by a constant c:

$$\chi'(z) = \chi(z) + c \qquad (10)$$

with the nontrivial Wightman functions at $c \neq 0$

$$w'(z-z') = \langle 0 \mid \chi'(z)\chi'(z') \mid 0\rangle = w(z-z') + c^2$$

and
$$\langle 0 \mid \chi'(z) \mid 0\rangle = c .$$

At the same time $\Lambda'(z) = \Lambda(z)$ with the trivial Wightman function

$$\langle 0 \mid \Lambda(z_1)\ldots\Lambda(z_N) \mid 0\rangle = 0 , \quad N \geq 1.$$

The gauge transformation (10) leads to the fact that the conserved gauge invariant current $\xi^\mu(z)$

$$\xi^\mu(z) = \partial^\mu\chi'(z) ,$$
$$\partial_\mu\xi^\mu(z) = 0$$

should be represented as a real observed field. Using the gauge transformation (10) we divide the total Lagrangian density (2) as

$$L(\chi, \Lambda, Q\ ; c) \rightarrow L_0(\chi, \Lambda\ ; c) + L_{int}(\chi, Q; c) ,$$

where

$$L_0(\chi, \Lambda, \ ; c) = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \partial_\mu\chi\partial^\mu\Lambda - \frac{\lambda}{N}(c\chi)^2 + \frac{1}{2}\Lambda^2 + \frac{1}{2}\mu^2\chi^2 , \qquad (11)$$

$$\begin{aligned} L_{int}(\chi, Q\ ; c) = & -\frac{\lambda}{4N}(\chi^2)^2 - \sqrt{\lambda}m_0\chi\chi^2 - \frac{\lambda}{N}(c\chi)\chi^2 + \\ & + \frac{1}{2}(m_0^2 - \mu^2 - 2\sqrt{\lambda}m_0c - \frac{\lambda}{N}c^2)\chi^2 - 2\sqrt{\lambda}m_0(c\chi)\chi + \\ & + (m_0^2 - 2\sqrt{\lambda}m_0c - \frac{\lambda}{N}c^2)(c\chi) - \sqrt{\lambda}m_0c^2\chi - \frac{\lambda}{4N}(c^2)^2 - \\ & - \sqrt{\lambda}m_0c^3 + \frac{1}{2}m_0^2c^2 + \bar{Q}(i\hat{\partial} - g\hat{\partial}\chi)Q , \end{aligned} \qquad (12)$$

where μ is an arbitrary massive parameter. Let us introduce the partial normal ordering procedure [2] for the quadratic field term $\chi^2(z)$ by means of

$$\chi^2(z) =: \chi(z)^2 : + N\Delta(z, z') \quad , \quad \text{as } z \rightarrow z' , \qquad (13)$$

where
$$\Delta(z, z') = N^{-1}w(z-z') . \qquad (14)$$

Since the l.h.s. of (13) is of $O(N)$, the magnitude of $\Delta(z, z')$ is of $O(1)$. Substituting (13) into (12) with taken account of (14) the Lagrangian density L_{int} becomes:

$$\begin{aligned} L_{int}(\chi, Q; c) = & -\frac{\lambda}{4N}(:\chi^2:)^2 - (\sqrt{\lambda}m_0 + \frac{\lambda}{N}c)\chi : \chi^2 : + \\ & + \frac{1}{2}\delta\mu^2 : \chi^2 : - \sqrt{\lambda}m_0N\Delta(z, z')\chi + [(m_0 - 3\sqrt{\lambda}c)m_0 - \lambda(\frac{c^2}{N} + \Delta)](c\chi) + \\ & + \frac{1}{2}(m_0^2 - \mu^2 - 6\sqrt{\lambda}m_0c - \frac{\lambda}{N}c^2)N\Delta - (\frac{\lambda}{4N}c^2 - \sqrt{\lambda}m_0c + \frac{1}{2}m_0^2)c^2 + \\ & + \bar{Q}(i\hat{\partial} - g\hat{\partial}\chi)Q ; \end{aligned}$$

where

$$\delta\mu^2 = m_0(m_0 - 6\sqrt{\lambda}c) - \mu^2 - \lambda\left(\frac{c^2}{N} + \Delta\right).$$

In fact our estimations are somewhat formal since $\Delta(z, z')$ include both the divergence in the short-distance limit and the infrared divergence when the scalar field is the massless one. The physical masses should be extracted directly from the unperturbative Lagrangian density $L_0(\chi, \Lambda; c)$ (11), which is rewritten in a more convenient form

$$L_0(\chi, \Lambda; c) = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi + \partial_\mu\chi\partial^\mu\Lambda + \frac{1}{2}\Lambda^2 +$$

$$+\frac{1}{2}\mu^2\chi^a\left(\delta_{ab} - \frac{\alpha_a\alpha_b}{\alpha^2}\right)\chi^b + \frac{1}{2}\left(\mu^2 - \frac{\lambda}{3N}c^2\right)\chi^a\frac{\alpha_a\alpha_b}{\alpha^2}\chi^b .$$

It is clear that the field χ acquires the masses:

$$\begin{aligned} m_1^2 &= -\mu^2, \\ m_2^2 &= -\mu^2 + \tfrac{\lambda}{3N}c^2 . \end{aligned}$$

Since the theory must be independent of the additional mass squared μ^2, we can fix it so that $\delta\mu^2 = 0$, i.e.

$$m_0 = 3\sqrt{\lambda}c\left\{1 + \sqrt{1 + \frac{1}{N}(\frac{\mu}{3c})^2(c^2 + N\Delta)}\right\} . \qquad (15)$$

The $\Delta(z)$-factor in (15) is a well- defined distribution of a moderate growth in the space $\mathbf{S}'(\mathbf{R}^D)$ of Schwartz functions on $\mathbf{R}^D$ [5] For the scalar fields with an arbitrary D the $\Delta(z)$-factor can be obtained from the fundamental solution $\Delta^c(z)$ of eq.(6) and looks as [6]

$$\Delta(z) = (-1)^h\frac{\Gamma(\frac{D}{2} - h + 1)}{4^{h-1}N\Gamma(h-1)\pi^{\frac{D}{2}}}(-z^2 + i\varepsilon z^0)^{-\frac{D}{2}+h-1} , h < 1 + \frac{D}{2} . \qquad (16)$$

For even D and $h \geq \frac{D}{2} + 1$ the $\Delta(z)$-function becomes

$$\Delta(z) = (-1)^{\frac{D}{2}}\frac{(-z^2 + i\varepsilon z^0)^{-\frac{D}{2}+h-1}}{4^{h-1}N\Gamma(h - \frac{D}{2})\Gamma(h-1)}\ln(-M^2z^2 + i\varepsilon z^0) , \qquad (17)$$

where M is an arbitrary parameter with dimension one in mass units,which is introduced as an infrared regularization parameter. The distributions $(-z^2 + io)^{-\frac{D}{2}+h} = (-z^2 - io)^{-\frac{D}{2}+h}$ are solutions of eq.(5), if D is even and $h \geq D/2$. To calculate the physical X-boson masses, let us restrict ourselves within a system without massless particles, i.e.$\mu^2 \neq 0$ and $c = 0$, otherwise $\Delta(z)$ brings us an infrared divergence. Taking into account again

that $\delta\mu^2 = 0$ and due to a reality of the physical X-boson mass the latter is given by $m_X^2 = \mu^2$, i.e.

$$m_X^2 = m_0^2 - \lambda\Delta(z) \qquad (18)$$

as a function of the bare parameters m_0^2 and λ. This time Δ is a function of μ^2 also. The mass relation (18) indicates the change of a magnitude of X-boson mass squared up to physical one, m_X^2. The level of this effect depends both on signs and magnitudes of λ and $\Delta(z)$-factor. The normalization point $m_X = m_0$ can be obtained both from the long-distance case at $h - 1 < D/2$ and for short rang, if $h - 1 > D/2$ (see formulae (16) and (17), respectively). Thus, for the $\chi(z)$-fields, obeying the eq. $\Box^{D/2}\chi(z) = \Lambda(z)$, the theory leads to decreasing the real X-boson mass in the case of four-dimensional space and a positive value of λ at long distances:

$$m_X^2 = m_0^2 - \frac{\lambda}{16N}\lim_{\delta\to 0}(-z^2 + i\varepsilon z^0)^\delta \ln\left[\,|-M^2z^2| + i\pi\Theta(z^2M^2)\right] .$$

Taking the limit $N \to \infty$ we find that $m_X^2/m_0^2 \sim 1 - \lambda/N$ tends to 1. Therefore, the $O(N)$ invariant scalar theory is trivial in the large N limit.

3 The Nonzero Temperature

In this section we extend our scheme to include the temperature effect in the scalar field mass squared. Generally, the scheme is the following: let us fix the bare parameters through the physical quantities as functions of the temperature T. As for the physical mass m_X (18) we suppose again that $c = 0$. This time since the parameter μ^2 in $\delta\mu^2$ depends on T, the Δ -factor is the function of μ^2 and T also. Then $\mu^2(T)$ is determined by

$$m_0^2 - \mu^2(T) - \lambda\Delta(\mu^2, T) = 0 . \qquad (19)$$

To consider the behaviour of $m_X(T)$ at $T \neq 0$, let us remind both the facts that the temperature-dependent contribution to the free energy of the ultrarelativistic scalar particles with mass m at the temperature T is proportional to $(m^2/24)T^2[1 + O(m/T)]$ [7] and the expression for $m_X^2(\chi) = (3\lambda/N)\chi^2 - m_0^2$ in the model with the Lagrangian density under the gauge transformation (10). Therefore, we rewrite the Lagrangian density (3) in the following form ($T \gg m_0$):

$$L_\chi(z) = \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \lambda_T(T^2 - T_c{}^2)\chi^2 - \sqrt{\lambda}m_0\chi^3 - \frac{\lambda}{4N}\chi^4 + \cdots ,$$

where $\lambda_T = \lambda/(4N)$, $T_c = 2m_0\sqrt{N/\lambda}$ is the critical temperature and dots denote the omitted terms, which have no dependence on $\chi(z)$. Finally, from

(19) the expression for the temperature-dependent X-boson mass looks like

$$m_X^2(T) = \lambda_T(T_c{}^2 - T^2) - \lambda\Delta . \tag{20}$$

Differentiating eq. (20) with respect to T, we can obtain

$$\frac{dm_X^2(T)}{dT} \cong -\left(2\lambda_T T + \lambda\frac{\partial\Delta}{\partial T}\right)\left[1 + \lambda\frac{(-1)^{D/2-1}(-z^2+io)^{-D/2+h-1}}{4^{h-1}Nm_X^2(T)\Gamma(h-\frac{D}{2})\Gamma(h-1)}\right]^{-1}.$$

Therefore, near $T = 0$, the sign of $dm_X^2(T)/dT$ depends on that of the magnitude of m_X^2, λ and relation of $h-1 \geq D/2$. Due to decreasing $m_X^2(T)$ with increasing T up to $m^2(T_0) = 0$, the symmetry should be restored at $T_0 < T_c$, where (at $D = 4$)

$$T_0 \cong 2\sqrt{N\left[\frac{m_X^2}{\lambda} + \Delta(\mu^2)\right]}\left\{1 - \frac{\lambda\Delta(\mu^2, T)}{2\left[m_X^2 + \lambda\Delta(\mu^2)\right]} + \cdots\right\},$$

if $h - 1 = D/2$. Since at the critical temperature $m_X^2(T_c) = -\lambda\Delta$, the magnitude of $m_X^2(T_c)$ depends both on signs of λ and $\Delta(\mu^2, T)$. Supposing that he scalar self-coupling $\lambda \sim 0.1$ one can obtain $T_c \sim 300\ GeV$ (see $T_c \approx 350\ GeV$ [8]) at $N = 1$ and at the input of the SM parameter $m_0 \sim 44\ GeV$. But for small $\lambda \sim 0.0156$ [9] we find $T_c \sim 680\ GeV$. For the positive both λ and Δ-factor, $T_0 < T_c$ and the symmetry should be restored before the phase transition. But, if $\lambda < 0$ at $\Delta > 0$ or for positive λ at $\Delta < 0$, $T_0 > T_c$.

4 Conclusions

We have given the formulation of an approach based on the $1/N$ expansion for studying an arbitrary order dimension system of two scalar neutral fields, $\chi(z)$-field and an additional one-$\Lambda(z)$. The basic idea has been to relate the existence of a new type interaction mediated by the exchange of a hypothetical scalar X-boson and the decrease of the physical X-boson mass m_X in the case of a special choice of D and h at a fixed value of the scalar self-coupling λ. It has been mentioned that our scheme is valid if the bare mass squared m_0^2 of a scalar dipole-type field is larger than $\lambda\Delta(z)$. We restricted our consideration to a subsystem of pure scalar theory, based on the SM of a Higgs-boson-like system.

In the case of $h - 1 = D/2$ m_X^2 will decrease with increasing $\lambda\Delta(z)$, if $\lambda > 0$ in four-dimension space-time. The physical mass m_X turns to the bare mass m_0 at fixed λ in both the cases, when $h - 1 < D/2$ at long and short distances, if $h - 1 > D/2$.

The finite temperature scalar (Higgs)-Yukawa model at a fixed value of the scalar self-coupling λ is investigated. The derivative relation of the

physical X-boson mass m_X with respect to the nonzero temperature T has also been presented. It was shown that the symmetry should be restored at the temperature $T_0 \cong T_c\left(1 - \frac{\lambda\Delta}{2m_0^2}\right)$. The problem of matching the reality which involves the estimations both of m_X^2 and dm_X^2/dT clearly shows the need of physical input of the scalar self-coupling λ .

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Received by Publishing Department
on May 26, 1993.