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DEUTERON AND NEUTRON STRUCTURE FUNCTIONS AND EFFECT OF RELATIVISTIC FERMI MOTION

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Introduction

Usually the neutron structure function F_2^n was found from experimentally known proton and deuteron structure functions. The relativistic description of the deuteron is necessary to take into account the effect of the Fermi motion of nucleons with high momentum. The estimations of the nuclear effect in the deuteron based on the nonrelativistic wave functions give the order of the effect (1-3)$^o/_o$ for $x < 0.7$ [1-3]. The relativistic approach based on the formal replacement of the argument of the nonrelativistic wave function on some combination of light-cone variables x and $k_\perp$ gives the same result [4,5]. However from the theoretical point of view such relativisation procedure is more vulnerable so it is based on the supposition that the relativistic wave function depends on a single variable.

We used an alternative model of the relativistic deuteron proposed in our paper [6]. Our approach contrary to the approach [4] is based on the relativistic deuteron wave function (RDWF) with one nucleon on mass shell and therefore the RDWF is automatically dependent on one variable. It can be expressed via the vertex function $\Gamma_\alpha(x, k_\perp)$. The payment of this advantage is the symmetry violation between nucleons in the deuteron, i.e. by the replacement $x \leftrightarrow 1 - x$. This model has been successfully used for the descriptions of the deuteron electromagnetic form factors and some processes involving deuteron [6,7].

In the present paper the model of the relativistic deuteron [6] is used to calculate spin-independent structure function F_2^D. The dependence of the ratio $R_F^{D/N}(x, Q^2) = F_2^D(x, Q^2)/F_2^N(x, Q^2)$ of structure functions on x and Q^2 is investigated. This ratio characterizes the nuclear effect in the deuteron. It is shown that in our more consistent relativisation scheme the nuclear effect in the deuteron is noticeable and reaches 6$^o/_o$ for $x \simeq 0.7$. We find the neutron structure function F_2^n by comparing our calculated results for the deuteron SF and the ratio $R_F^{D/p}$ with available experimental data on F_2^D and F_2^p. The extracted neutron SF F_2^n somewhat differs from the usual one in the range of the middle x. On the basis of the obtained SF F_2^n the Gottfried sum rule (GSR) was verified. It is shown that the dominant contribution to GSR is due to the range of low Q^2 and high x and the nuclear effect for the Gottfried sum rule can be more than 10$^o/_o$ at $x < 0.7$ and $Q^2 > 0.8\ (GeV/c)^2$.

1 Structure Functions and Relativistic Wave Function of Deuteron

The deuteron structure functions $W_{1,2}(\nu, Q^2)$ are related to the imaginary part of the forward scattering amplitude of the virtual photon on the deuteron $W_{\mu\nu}$

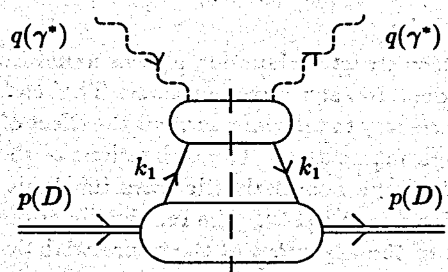

Figure 1: The imaginary part of the forward scattering amplitude of the $\gamma^* + D \to \gamma^* + D$ process in the RIA

by the standard relation

$$W^D_{\mu\nu} = -(g_{\mu\nu} - q_\mu q_\nu/q^2) \cdot W^D_1 + (p_\mu - q_\mu(pq)/q^2)(p_\nu - q_\nu(pq)/q^2) \cdot W^D_2/M^2. \quad (1)$$

Here q, p are momenta of photon and deuteron, M is the deuteron mass.

In the relativistic impulse approximation (RIA) the forward scattering amplitude of the virtual photon γ^* on deuteron $A^D_{\mu\nu}$ is defined via the similar scattering amplitude on the nucleon $A^N_{\mu\nu}$ (Fig.1) as follows

$$A^D_{\mu\nu}(q,p) = \int \frac{d^4k_1}{(2\pi)^4 i} Sp\{A^N_{\mu\nu}(q,k_1) \cdot T(s_1,k_1)\}. \quad (2)$$

In the expression (2) $T(s_1,k_1)$ is the amplitude of the $\bar{N} + D \to \bar{N} + D$ process and usual notations $Q^2 = -q^2 > 0$, $\nu \equiv (pq)$, $s_1 = (p-k_1)^2$ are used. The integration is carried out with respect to the active nucleon momentum k_1. The calculating of the imaginary part of the amplitude $A^D_{\mu\nu}$ gives us the possibility to put the nucleon spectator with momentum $k = p - k_1$ on mass shell.

Therefore the tensor $W^D_{\mu\nu}$ is expressed via the DNN vertex with one nucleon on mass shell. The vertex is described by the function $\Gamma_\alpha(k_1)$ and depends on one variable k_1. The vector index α characterizes the deuteron spin. Considering the relation between the RDWF and the vertex function $\Gamma_\alpha(k_1)$: $\psi_\alpha(k_1) = \Gamma_\alpha(k_1) \cdot (m + \hat{k}_1)^{-1}$, the expression for the tensor $W^{\alpha\beta}_{\mu\nu}$ can be written as

$$W^{\alpha\beta}_{\mu\nu} = \int \frac{d^4k}{(2\pi)^4 i} \delta(m^2 - k^2)\theta(k_0)\theta(p_+ - k_+)\, Sp\{w^N_{\mu\nu} \cdot \bar{\psi}^\alpha(k_1) \cdot (m + \hat{k}_1) \cdot \psi^\beta(k_1)\}. \quad (3)$$

Here θ - function and light-cone variables ($k_\pm = k_0 \pm k_3, k_\perp$) are used. The vertex function $\Gamma_\alpha(k_1)$ is defined via 4 scalar functions $a_i(k_1^2)$ ($i = 1-4$) and has the form [8]

$$\Gamma_\alpha(k_1) = k_{1\alpha}[a_1(k_1^2) + a_2(k_1^2)(m + \hat{k}_1)] + \gamma_\alpha[a_3(k_1^2) + a_4(k_1^2)(m + \hat{k}_1)]. \quad (4)$$

The scalar functions $a_i(k_1^2)$ have been constructed in the paper [9] in the form of a sum of pole terms. Some pole positions and residues have been found from the comparison in the nonrelativistic limit of our RDWF with the known nonrelativistic one. For the latter the Paris wave function [10] was taken. Other parameters were fixed from the description of the static characteristics of the deuteron (an electric charge - $G_e(0) = 1(e)$, magnetic - $G_m(0) = \mu_D(e/2M_D)$ and quadrupole - $G_Q(0) = Q_D(e/M_D^2)$ moments) in the relativistic impulse approximation.

The calculation of (3) in the light-cone variables gives the final expression for the deuteron SF $F_2^D \equiv \nu W_2^D$

$$F_2^D(\alpha, Q^2) = \int_\alpha^1 dx\, d^2k_\perp\, p(x,k_\perp) \cdot F_2^N(\alpha/x, Q^2). \quad (5)$$

The nucleon SF $F_2^N = (F_2^p + F_2^n)/2$ is defined by proton and neutron ones. The positive function $p(x,k_\perp)$ describes the probability that the active nucleon carries away the fraction of the deuteron momentum $x = k_{1+}/p_+$ and the transverse momentum $k_\perp$ in the infinite momentum frame. It is expressed via the vertex function $\Gamma_\alpha(k_1)$ as follows

$$p(x,k_\perp) \propto Sp\{\bar{\psi}_\alpha(k_1) \cdot (m + \hat{k}_1) \cdot \psi_\beta(k_1) \cdot \hat{q}/\nu \cdot \rho^{\alpha\beta}\}, \quad (6)$$

where $\rho^{\alpha\beta}$ is the deuteron polarization density matrix. Note that in the approach used the distribution function $p(x,k_\perp)$ includes not only usual S- and D-wave components of the deuteron, but the P-component too. The latter describes the contribution of the $N\bar{N}$-pair production. The contribution of this mechanism is small in the low momentum range ($x < 1$), but it may be considerable in the high momentum one ($x > 1$).

2 Neutron Structure Function

In the RIA the deuteron SF F_2^D is defined by (6) as a sum of the proton and neutron SF. We calculate the F_2^D using the RDWF [6] and the usual parametrization for the neutron SF $F_2^n = (1 - 0.75x)F_2^p$. For the proton one the parametrization of the NMC data [14] is used. Emphasize that the Q^2-dependence of the deuteron (5) and neutron SF is defined by the behaviour of the proton SF on Q^2. Calculated results of the ratio $R_F^{D/p} = F_2^D/F_2^p$ with the neutron SF show the discrepancy with experimental data [12-14] in the range $x \simeq (0.08-0.4)$. Good agreement between theory and experiment (the curve p in Fig.2) is observed when the neutron SF is parametrized as follows:

$$F_2^n(x,Q^2) = (1-0.75x)(1-0.15\sqrt{x}(1-x)) \cdot F_2^p(x,Q^2). \qquad (7)$$

Figure 2 shows the dependence of the ratio $R_F^{D/N,p} = F_2^D/F_2^{N,p}$ on x. The ratio $R_F^{D/p}$ is in good agreement with data [16]. The nuclear effect in the deuteron is described by $R_F^{D/N}(x,Q^2)$ (the curve N in Fig.2). The Q^2-dependence of this magnitude is weak. With the increasing x the effect of relativistic Fermi motion grows and the ratio $R_F^{D/N}$ reaches 6°/o for $x \simeq 0.7$. The dependence of the ratio $R_F^{D/N}$ on x resembles the nuclear EMC effect and in the range $x < 0.7$ is practically independent of Q^2. Our results differ from the estimation of the nuclear effect obtained in [1-5], where the latter one is not larger than (1-3)°/o and the ratio $R_F^{D/N}$ obtained in [1-4] has not a typical EMC-behaviour.

Thus, the obtained results allow us to conclude that the dependence of the ratio $R_F^{D/N}$ on x is the universal one in the range $x < 0.7$ and is defined by the structure of the RDWF. The following parametrization for the ratio $R_F^{D/N}$ was obtained in this range

$$R_F^{D/N}(x) = \sum_{i=0}^{6} a_i x^i. \qquad (8)$$

The coefficients a_i are presented in Table 1. The parametrization $R_F^{D/N}$ (8) can be used to extract the spin- independent neutron SF from known deuteron and proton experimental data and analyze the spin-dependent one $g_1^n(x,Q^2)$

$$F_2^n(x,Q^2) = 2 \cdot [R_F^{D/N}(x)]^{-1} \cdot F_2^D(x,Q^2) - F_2^p(x,Q^2). \qquad (9)$$

Figure 3 shows calculated results of the dependence of the F_2^n on x for $Q^2 = 4, 10, 80\ (GeV/c)^2$. We compare $F_2^D(x,Q^2)$ with experimental data [12-14] in Figure 4(a,b). These curves are in good agreement with data both in low and high x-range. Scaling factors for different values of x and the corresponding curves are shown on Figure 4(a).

Table 1. Coefficients a_i for the nuclear effect function $R_F^{D/N} = \sum_0^6 a_i x^i$

a_0	a_1	a_2	a_3	a_4	a_5	a_6
0.998642	-0.0512088	0.415614	-5.32661	17.598	-23.3847	11.1059

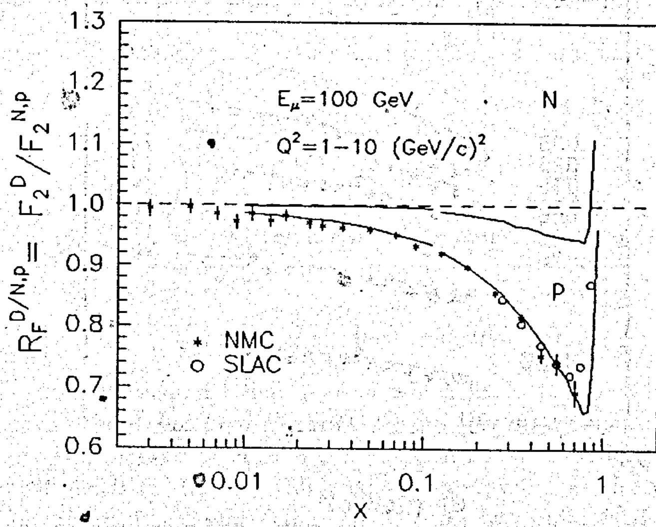

Figure 2: The ratio $R_F^{D/N,p}$ of the structure functions for the deep-inelastic lepton-deuteron scattering. Experimental data: o - [13], * - [16]

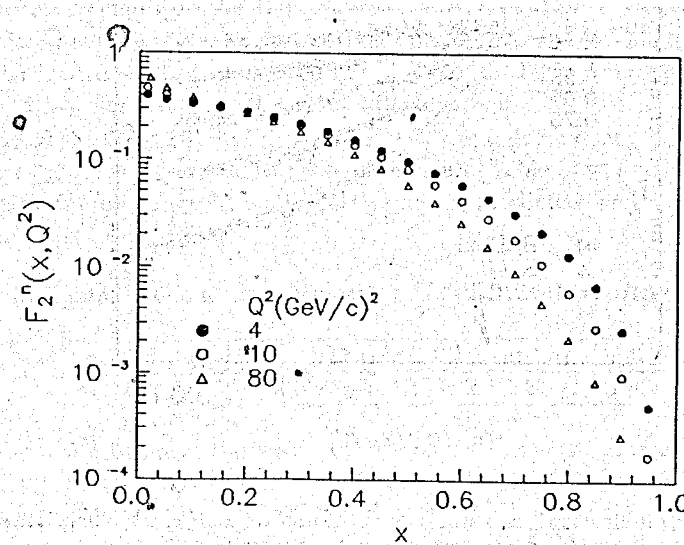

Figure 3: The neutron structure function $F_2^n(x,Q^2)$

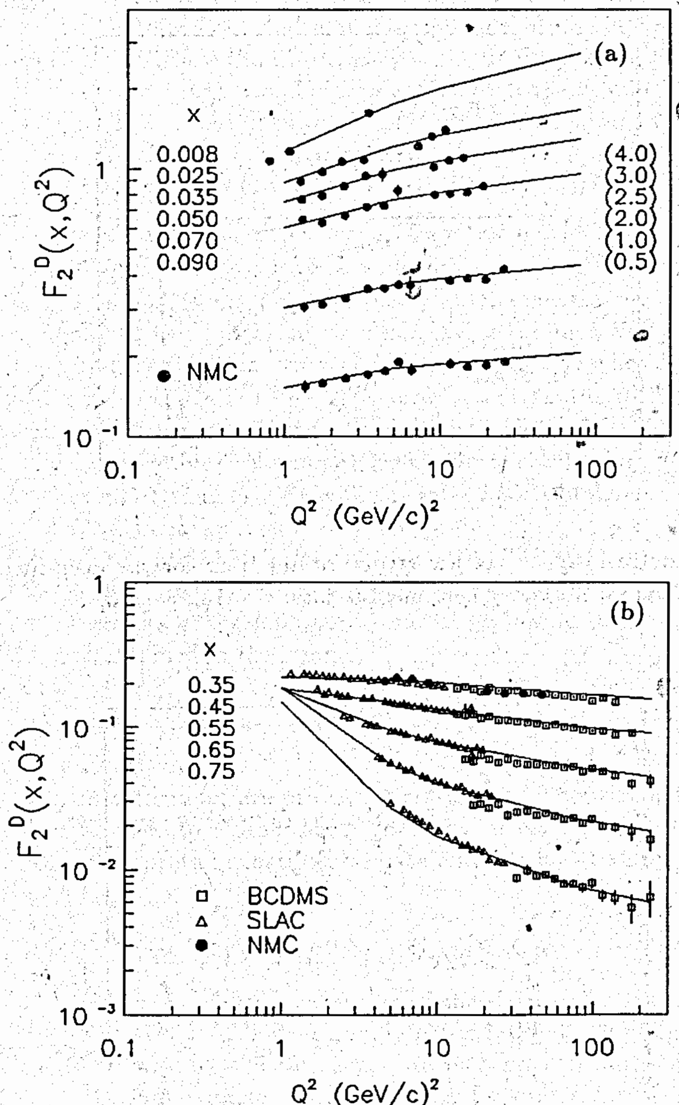

Figure 4: Deep inelastic deuteron structure function $F_2^D(x, Q^2)$. Experimental data: ▯ - BCDMS [12], △ - SLAC [13], ∘ - NMC [14]

3 Gottfried Sum Rule

The neutron SF obtained in the previous section can be used to verify the Gottfried sum rule (GSR) [15]:

$$\int_0^1 [F_2^p(x) - F_2^n(x)] \frac{dx}{x} = \frac{1}{3}. \tag{10}$$

In the parton model neutron and proton SF can be written via the valence (u_V, d_V) and sea ($\bar{u}, \bar{d}, s, \bar{s}$) parton distributions and the sum rule (10) can be written as follows

$$S_G = \frac{1}{3} + \frac{2}{3} \cdot \int_0^1 [\bar{u}(y) - \bar{d}(y)] dy. \tag{11}$$

It is generally assumed that $\bar{u} = \bar{d}$ which leads to $S_G = \frac{1}{3}$. As has been reported in [16] the value of S_G obtained from the measurements of F_2^D and F_2^p is considerably below the naive quark-parton model value of 1/3: $S_G = 0.240 \pm 0.016$. Usually this result is interpreted as the violation of the isospin symmetrical sea. This value for S_G was obtained for averaged $Q^2 = 4$ $(GeV/c)^2$ and the extrapolation procedure for the unmeasured low (0-0.004) and high (0.8-1.0) x-range was used.

It should be noted that the nuclear structure of a deuteron has not been taken into account during the extraction of the neutron SF F_2^n at all. The calculated results of the nuclear correction $\delta F_2^n/F_2^n = (1 - R_F^{D/N})(1 + F_2^p/F_2^n)$ for the neutron SF are presented in Table 2. The ratio $\delta F_2^n/F_2^n$ reaches $\simeq 19\ ^o/_o$ at $x \simeq 0.7$. It tends to fall with decreasing x.

Note that $R_F^{D/N}$ (8) and the ratio F_2^p/F_2^n are independent of Q^2 and therefore $\delta F_2^n/F_2^n$ is Q^2-independent too. The result (Table 2) was obtained for the incident muon energy $E_\mu = 10^5$ (GeV) and is predicted for the unmeasured kinematical range.

Table 2. The nuclear correction $\delta F_2^n/F_2^n$ for the neutron SF

x	10^{-3}	10^{-2}	10^{-1}	0.5	0.7
$\delta F_2^n/F_2^n$	2.83e-3	3.71e-3	1.30e-2	1.24e-1	1.88e-1

Figure 5(a,b) shows the results of the dependence of the difference $F_2^p(x, Q^2) - F_2^n(x, Q^2)$ and the Gottfried integral $S_G(x, Q^2) = \int_x^1 [F_2^p(y, Q^2) - F_2^n(y, Q^2)] dy/y$ on x for $Q^2 = 0.8, 1, 2, 4, 10$ $(GeV/c)^2$. One can observe the noticeable Q^2-dependence in the low x-range. The range of low Q^2 and large x gives a dominant contribution to $S_G(x, Q^2)$. With increasing Q^2 the magnitude of S_G decreases and the discrepancy with predictions in the quark-parton model grows. It could mean that the clouds of $q\bar{q}$- pairs of valence u and d quarks

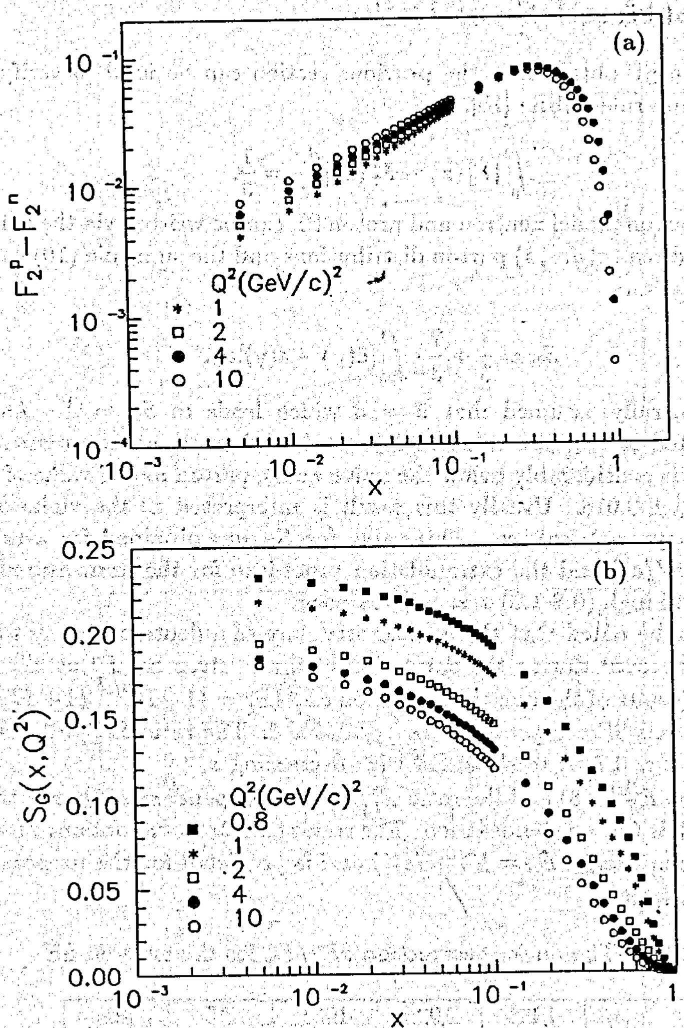

Figure 5: The difference $F_2^p(x,Q^2) - F_2^n(x,Q^2)$ (a) and the Gottfried integral $S_G(x,Q^2)$ (b)

are essentially different and with increasing Q^2 the violation of the flavour symmetry of the sea becomes large. We estimated the correction $(\delta S_G/S_G)$ for the Gottfried integral due to the nuclear effect. The results calculated for the incident muon energy $E_\mu = 10^5$ (GeV) are presented in Table 3. It is seen that the nuclear correction changes from $12^\circ/_\circ$ to $19^\circ/_\circ$ in the wide kinematical range.

Table 3. The nuclear correction $\delta S_G/S_G$ for the Gottfried integral

$Q^2 \setminus x$	10^{-3}	10^{-2}	10^{-1}	0.3	0.7
0.80	1.52e-1	1.48e-1	1.57e-1	1.63e-1	1.20e-1
4.00	1.54e-1	1.47e-1	1.61e-1	1.80e-1	1.26e-1
80.0	1.53e-1	1.39e-1	1.58e-1	1.86e-1	1.33e-1

Conclusion

We have found in the framework of the covariant approach in the light-cone variables using the deuteron model with one nucleon on mass shell that the effect of relativistic Fermi motion in deuteron gives rise to the depletion of the free nucleon to the deuteron SF ratio $R_F^{D/N}$ which reaches $6^\circ/_\circ$ at $x \simeq 0.7$. The proposed procedure of the extraction of the neutron SF $F_2^n(x,Q^2)$ takes into account correctly the relativistic deuteron spin structure and can be used to extract the spin-dependent neutron SF $g_1^n(x,Q^2)$ too. It has been shown that the correction due to the nuclear effect for the Gottfried integral is $(12 \div 19)$ $^\circ/_\circ$ in the range $x = 10^{-3} \div 0.7$ and $Q^2 = 0.8 \div 80$ $(GeV/c)^2$ and this effect must be taken into account to verify the Gottfried sum rule.

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References

[1] W.B.Atwood, G.B.West, Phys.Rev. **D7** (1981) 1080;

[2] A.Bodek, J.L.Ritchie, Phys.Rev. **D23** (1981) 1070; Phys.Rev. **D24** (1981) 1400.

[3] A.Bodek, A.Simon, Z.Phys. **C29** (1985) 231.

[4] L.L.Frankfurt, M.I.Strikman, Phys.Lett. **B76** (1978) 333; Nucl.Phys. **B181** (1981) 22.

[5] L.P.Kaptari, A.Yu.Umnikov, Phys.Lett. **B259** (1991) 155.

[6] M.A.Braun, M.V.Tokarev, Particles and Nuclei **22** (1991) 1237.

[7] M.V.Tokarev, Few Body Systems **4** (1988) 133.

[8] R.Blankenbecler, L.F.Cook, Phys.Rev. **119** (1960) 1745.

[9] M.A.Braun, M.V.Tokarev, Vestnik LGU **4** (1986) 7.

[10] M.Lacombe et al., Phys.Lett. **B101** (1981) 139.

[11] J.J.Aubert et al., Nucl.Phys. **B259** (1985) 189; Nucl.Phys. **B293** (1987) 740.

[12] A.C.Benvenuti et al. Phys.Lett. **B223** (1989) 485; Phys.Lett. **B237** (1990) 592.

[13] L.Whitlow et al., Phys.Lett. **B282** (1992) 475.

[14] P.Amaudruz et al., Preprint CERN-PPE/92-124 (1992).

[15] K.Gottfried, Phys.Rev.Lett. **18** (1967) 1174.

[16] P.Amaudruz et al., Phys.Rev.Lett. **66** (1991) 2712.

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