объединенный
институт
ядерных
исследований
дубна

E2-93-167

Kh.M.Beshtoev

CONTRIBUTION OF THE WEAK INTERACTION TO THE OSCILLATIONS OF K^0-MESONS

Submitted to "Il Nuovo Cimento A".

1993

1. Introduction

The standard approach to the description of the mixing or oscillations of quarks and leptons is the approach [1], in which the property of the mass matrix being not diagonal is utilized. In the preceding work [2] the conclusion was made that weak interaction gives no contribution to the masses of fermions (see Appendix), since only left-hand components of fermions take part in it. In this connection it is apparently necessary to analyze the influence of weak interaction on the oscillations and mixing of K^0-mesons and neutrinos.

2. Contribution of weak interaction to the oscillation of K^0 mesons

The interaction of $K^0 \rightarrow \bar{d}\,\gamma_5\, s$ and $\bar{K}^0 \rightarrow \bar{s}\,\gamma_5\, d$ mesons is the same owing to the interaction being identical both of s and $\bar{s}$ and of d and $\bar{d}$. Moreover, if the symmetry with respect to particles and antiparticles of the Kobayashi-Maskawa matrices [3] (in the 6-quark case CP is violated) is taken into account, then the transitions $K^0 \rightarrow \bar{K}^0$ and $\bar{K}^0 \rightarrow K^0$ will be symmetric. Contributions to these transitions will be given by the standard four-vertex diagram [4] and the diagram with exchange of the Z^0-boson [5] (Z^0 includes K^0 and $\bar{K}^0$ with the same probability, i.e. the mixing angle equals $\pi/4$). Since K^0- and $\bar{K}^0$-mesons consist of fermions (s, $\bar{s}$, d, $\bar{d}$), which owing to the left nature of the weak interaction cannot acquire mass through this interaction, their masses do not vary. As a result, we arrive at the conclusion that the mixing angle for K^0, $\bar{K}^0$ equals $\pi/4$.

The masses of K^0 and $\bar{K}^0$ are the same (the mixing angle equals $\pi/4$),then the mass matrix of K^0- and $\bar{K}^0$- mesons has the form of

$$\begin{vmatrix} m_{K^0 K^0} & m_{K^0 \bar{K}^0} \\ m_{\bar{K}^0 K^0} & m_{\bar{K}^0 \bar{K}^0} \end{vmatrix} .$$

Having in our mind the further using, we turn now to the matrix of widths for K^0-, $\bar{K}^0$- mesons

$$\begin{vmatrix} \Gamma_{K^0 K^0} & \Gamma_{K^0 \bar{K}^0} \\ \Gamma_{\bar{K}^0 K^0} & \Gamma_{\bar{K}^0 \bar{K}^0} \end{vmatrix} \qquad (1)$$

and we shall diagonalize this matrix bearing in mind that the mixing angle for K^0 and $\bar{K}^0$ is $\pi/4$:

$$\begin{vmatrix} \Gamma_{K_1^0 K_1^0} & 0 \\ 0 & \Gamma_{K_2^0 K_2^0} \end{vmatrix} \qquad (2)$$

where $$K_1^0 = \frac{K^0 + \bar{K}^0}{\sqrt{2}}, \quad K_1^0 = \frac{K^0 - \bar{K}^0}{\sqrt{2}}.$$

Then

$$\Gamma_{K^0 K^0} = \Gamma_{\bar{K}^0 \bar{K}^0} = \frac{\Gamma_{K_1^0 K_1^0} + \Gamma_{K_2^0 K_2^0}}{2},$$

$$\Gamma_{\bar{K}^0 K^0} = \Gamma_{K^0 \bar{K}^0} = \frac{\Gamma_{K_1^0 K_1^0} - \Gamma_{K_2^0 K_2^0}}{2}. \qquad (3)$$

The time (or length) of the $K^0 \longleftrightarrow \bar{K}^0$ oscillation is determined by the time of formation of K^0- and $\bar{K}^0$-mesons and,

correspondingly,
will be equal to

$$t^{-1}_{K^0 \longleftrightarrow \bar{K}^0} = \frac{\Gamma_{K_1^0 K_1^0} + \Gamma_{K_2^0 K_2^0}}{2}.$$

Owing to the masses of K^0 and $\bar{K}^0$ being equal and to their widths totally overlapping, such oscillation will be real [5].

At diagonalization of the matrix (1) we have come to the mass matrix for K_1^0 and K_2^0:

$$\begin{vmatrix} m_{K_1^0 K_1^0} & 0 \\ 0 & m_{K_2^0 K_2^0} \end{vmatrix} \qquad (4)$$

where

$$m_{K^0 K^0} = m_{\bar{K}^0 \bar{K}^0},$$

$$m_{\bar{K}^0 K^0} = m_{K^0 \bar{K}^0},$$

$$m_{K_1^0 K_1^0} = m_{K^0 K^0} - m_{K^0 \bar{K}^0}, \qquad (5)$$

$$m_{K_2^0 K_2^0} = m_{K^0 K^0} + m_{K^0 \bar{K}^0},$$

$$m_{K_1^0 K_1^0} + m_{K_2^0 K_2^0} = m_{K^0 K^0} + m_{\bar{K}^0 \bar{K}^0}.$$

When transition is performed to the K_1^0- and K_2^0-mesons there appears a mass difference, and, in this case, it is necessary to take into account the influence of this difference on the oscillation of the K_1^0- and K_2^0-mesons. We considered this question in ref. [5]. Therein it was shown that when the particles pass through vacuum, real transition of particles of one sort of definite mass m_1 into particles of another sort of mass m_2 may take place when permitted by the uncertainty relations, if the widths of these particles overlap their mass difference. This is possible owing to the second particle having time to become a real particle. This means that in the case of K^0, $\bar{K}^0$-mesons oscillation will occur without any suppression (the masses of the K^0- and $\bar{K}^0$ mesons are identical, and their widths totally overlap). At the same time that in the case of K_1^0- and K_2^0-mesons the overlapping of their widths due to their mass difference is very small, and, therefore, the oscillation between K_1^0 and K_2^0 is strongly suppressed (actually, so-called virtual oscillations will take place), and for transition of the K_1^0- and K_2^0-mesons to the corresponding mass surface interaction must be present for implementing the transition to the mass surface (since this mass difference is very small, for such a transition a relatively small external field is sufficient).

Now let us return to the issue of the mixing angle for K_1^0 and K_2^0, which is related to the overlapping of the mass difference by their widths that will be manifested as real $K_1^0 \longleftrightarrow K_2^0$. Actually, this phenomenon will be manifested as the appearance of a term violating CP parity. In the simplest case this mixing angle θ is

$$\begin{vmatrix} \Gamma_{K_1^0 K_1^0} & -\Gamma_{K_2^0 K_2^0} \\ -\Gamma_{K_2^0 K_2^0} & \Gamma_{K_2^0 K_2^0} \end{vmatrix} \qquad \delta \simeq 2\cdot\theta, \quad K_2^{0\prime} = K_2^0 - \theta K_1^0 \qquad (6)$$

$$\sin\theta = \frac{\Gamma_{K_2^0 K_2^0}}{\Gamma_{K^0{}_1 K^0{}_1}} \simeq 0.0016.$$

This angle practically coincides with the angle of the CP violating term observed experimentally [6], $\delta = 0.00327$. If $K_1^0 \longleftrightarrow K_2^0$ occurs through K^0, $\bar{K}^0$ ($K_1^0 \longleftrightarrow K^0$, $\bar{K}^0 \longleftrightarrow K_2^0$), then θ is ($\Gamma_{K^0 K^0}$ is taken from (3))

$$\sin\theta = \frac{2\,\Gamma_{K^0_2 K^0_2}}{\Gamma_{K^0_1 K^0_1}} \simeq 0.0032$$

and does not coincide with the experimental value.

On the other hand, for explanation of the CP violation one must, following ref. [7], make use of an interaction mechanism which, evidently, is equivalent to Kobayashi-Maskawa mechanism of CP violation.

Thus, owing to the contribution of weak interaction not causing a change in the quark masses, no quark mixing (oscillation) occurs in this case. But taking into account weak quark decays leads, upon violation of the flavour numbers by matrices of the Kobayashi-Maskawa type, to the appearance of oscillations of neutral K^0-mesons considered above.

3. Contribution of the weak interaction to neutrino oscillations

Since the weak interaction does not change the neutrino mass, the neutrino mass matrix will not be altered by the contribution of this interaction (see Appendix). Thus, the contribution of weak interaction [2] will not lead to any lepton mixing.

How can lepton mixing (oscillation) appear then?

Such mixing may be due to matrices of the Kobayashi-Maskawa type [3], or it may occur within the framework of direct violation of lepton numbers [8]. So the lepton mass matrix $\psi = (\psi_e, \psi_\mu, \psi_\tau)$ will be of the diagonal form

$$M_1 = \begin{vmatrix} m_e & 0 & 0 \\ 0 & m_\mu & 0 \\ 0 & 0 & m_\tau \end{vmatrix} . \qquad (7)$$

In the case of leptons the non-diagonal matrix is the matrix V^+, which mixes leptons,

$$\psi' = V^+ \psi ,$$

and the diagonalization of which leads to the appearance of new lepton states $\psi' = (\psi_1, \psi_2, \psi_3)$. In principle, this procedure is equivalent to the introduction of a non-diagonal mass matrix. Once more we stress that this non-diagonal mass matrix is not related to the weak interaction, but is generated by the Kobayashi-Maskawa matrix or by a mechanism of direct violation of lepton numbers [8].

The issue of which type (virtual or real) of oscillations occurs in this case is dealt with in ref. [5]. Thus, owing to its specific left-hand nature, weak interaction does not contribute to neutrino oscillations. Neutrino oscillations, which may occur owing to the existence of neutrino decay channels, i.e. to taking into account the neutrino decay widths, can be described by analogy with the oscillations of K^0-mesons considered above.

4. Conclusion

The contribution of weak interaction to the oscillations of K^0-mesons and neutrinos is studied. It is shown that, when quark decays via weak interaction are taken into account, there appears, upon violation of the flavour numbers by Kobayashi-Moskawa matrices, real $K^0 \longleftrightarrow \bar{K}^0$ vacuum oscillation with a mixing angle equal to $\pi/4$, while $K^0_1 \longleftrightarrow K^0_2$ vacuum oscillation will take place in conditions of strong suppression due to the difference between the masses of the K^0_1- and K^0_2-mesons (virtual oscillation will take place). Real $K^0_1 \longleftrightarrow K^0_2$ oscillation in vacuum will occur with a mixing angle θ equal to $\theta \simeq 0.0016$ or $\theta \simeq 0.0032$ (such oscillation leads to CP violation). The angle of CP violation measured experimentally [6] is $\delta \simeq 0.00327$. All these processes will proceed on a background of decays of the neutral K^0-mesons.

Transition from the K^0, $\bar{K}^0$ to the K^0_1, K^0_2 representation is implemented by diagonalization of the matrices of decay widths of the K^0, $\bar{K}^0$-mesons.

The conclusion is made that neutrino oscillations may take place owing to matrices of the Kobayashi-Maskawa type or to a mechanism of direct violation of the lepton numbers [8], i.e. outside the framework of weak interaction. An analysis of neutrino oscillations that may arise because of the existence of neutrino decay channels can be performed by analogy with the neutral K^0 mesons considered above.

Appendix

The Dirac equation for ψ_R, ψ_L has the form:

$$\begin{aligned} (E + \sigma_i H_i)\,\psi_L - M\,\psi_R &= 0 , \quad i=1,3 \\ (E - \sigma_i H_i)\,\psi_R - M\,\psi_L &= 0 , \quad E = \varepsilon - e A_4 \end{aligned} \qquad (A.1)$$

where $H_i = P_i - e A_i$, σ_i are the Pauli matrices. The Dirac equation for the same particle in new variables upon taking into account the interaction through A_μ will be of the form:

$$(E' + \sigma_i P'_i) \psi_L - M' \psi_R = 0 ,$$
$$(E' - \sigma_i P'_i) \psi_R - M' \psi_L = 0. \quad (A.2)$$

From (A.1) and (A.2) we obtain

$$((E - E') + \sigma_i (H_i - P'_i)) \psi_L = \Delta M \psi_R ,$$
$$((E - E') - \sigma_i (H_i - P'_i)) \psi_R = \Delta M \psi_L , \quad (A.3)$$

where $\Delta M = M - M'$, and that the contribution of the interaction via the field A_μ leads to the appearance of the mass ΔM, there being symmetry between the right and left components of the fermion.

Using (A.3) we can now consider the case when only the left component of the fermion (bearing in mind the weak interaction) takes part in the interaction. Then (A.3) can be rewritten in the form:

$$((E - E') + \sigma_i (H_i - P'_i)) \psi_L = 0$$
$$0 = \Delta M \psi_L . \quad (A.4)$$

Since ψ_L differs from zero, we have from (A.4) $\Delta M = 0$. That is, if only the left component of the fermion takes part in the interaction, then the mass of the fermion is not changed by the presence of the interaction. But it is not difficult to see that the same conclusion is obtained in the case when the field A_μ is a non-Abelian field.

References

1. S. M.Bilenky and S. T. Petcov, Rev.of Mod. Phys. 58 (1987) 679.
2. Kh. M. Beshtoev ,JINR, P2-93-44 ,Dubna,1993.
3. M. Kobayashi and K. Maskawa, Prog. Theor. Phys. 49 (1973) 652.

4. L. B. Okun , Leptons and Quarks , M., Nauka, 1990.
5. Kh. M. Beshtoev, JINR, P2-92-318 ,Dubna,1992.
6. Review of Particle Properties , Phys. Lett. B239 (1990).
7. L. Wolfenstein , Phys. Rev. Lett. 13 (1964).
   J. Liu and L. Wolfenstein , Nucl. Phys. B289 (1987) 1.
8. Kh. M. Beshtoev, JINR, P2-91-183 ,Dubna,1991.

Received by Publishing Department

on May 12, 1993.