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S.V.Goloskokov

IS THE ANOMALOUS BEHAVIOUR OF THE POMERON SPIN-FLIP PART POSSIBLE IN PERTURBATIVE QCD?

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1 Introduction

It is well known that the spin effects are absent in the massless QCD at high energies [1], but in hard processes, where the perturbative QCD in the $m \to 0$ limit must be applicable, the spin effects are observed [2]. The mass terms inclusion leads to a small decreasing with energy of the ratio of spin-flip and non-flip amplitudes

$$\frac{| T_{flip} |}{| T_{non-flip} |} \simeq \frac{m}{\sqrt{s}}.$$

It may be expected that spin effects in hard processes can be explained on the basis of axial anomaly not vanished as $m \to 0$.

Usually the axial anomaly is connected with the triangle graphs [3]. In different hard processes this anomaly is manifests itself through the box diagram [4]. These diagrams determine the gluons contribution to the spin-dependent structure functions of deep inelastic scattering which play an important role in solving of the EMC "Spin Crisis" (see e.g. [5]). Another known example is the box contribution to the photon-photon scattering. The axial anomaly results in a nonzero cross-section for the real longitudinal photon interactions in this reaction [6].

The anomaly can lead to the mass divergences of the diagrams are compensated with the terms proportional to small masses in the diagrams numerator. As a result, there appears a mass-independent contribution producing the helicity-flip in the massless limit. This generalized approach was applied to the search of axial anomaly [7] and an anomalous pole was shown to exist in spin-dependent cross-sections of e^+e^- annihilation.

So the axial anomaly manifestation can be found in different processes. In [8] it was supposed that this anomaly can manifest itself in the high energy hadron scattering at fixed momenta transfer $|t|$ too. In this case the kinematical smallness $\sim |t|$ in the helicity-flip amplitudes can be compensated by the anomalous singularity of the diagram in the $|t| \to 0$ limit similar to the mass compensation as $m \to 0$. This effect can lead to nonvanishing spin-flip amplitudes for $|t| = 0$ ("anomalon" contribution).

In this paper we shall search for this anomalous behavior in the spin-flip amplitudes of the quark-quark subprocess in perturbative QCD. In the second part of the paper it will be shown that the mass and momentum-transfer singularities exist in the axial and mass terms of the gluon-ladder diagrams for quarks on the mass shell. The off-mass-shell effects will be analyzed in the γq high-energy reaction in the third part.

2 Anomaly in the quark-quark scattering

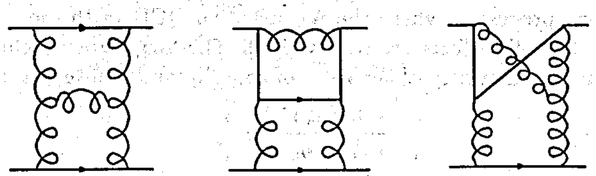
Let us investigate the spin effects in the mass-shell quark-quark scattering as $s \to \infty, |t| - fixed$ in perturbative QCD. We use the following definitions of the initial quark momenta

$$p_1 = p + r, \;\; p_2 = p' - r,$$

and

$$s \simeq 2(pp'), \; t = \frac{r^2}{4} = -\Delta^2, \; (pr) = (p'r) = 0.$$

It can be shown that we can calculate the imaginary parts of diagrams only in the case of t-channel pomeron exchange. Diagrams, Fig.1, determine the amplitudes with the spin-flip in the upper quark line [9].



(a) (b) (c)

Fig.1 The α_s^3 contribution to the spin-flip amplitude.

The imaginary parts of the spin-non-flip matrix element in the down quark line of diagrams, Fig.1(a)-(b), can be written as follows (the light-cone variables [10] are used here)

$$\mathrm{Im}\langle T_i(s,t)\rangle = c_i \frac{\alpha_s^3}{(2\pi)^2 s} \int_{s_0/s}^{1} \frac{dx}{x(1-x)} \int d^2k_\perp d^2l_\perp$$
$$N(k_\perp, l_\perp, r, x) \left[(G_i^1 G_i^2) \prod (F(l \pm r)) \right]. \quad (1)$$

where c_i is a color factor, $G_i^1 G_i^2$ are the corresponding functions of t-channel propagators in the up part of the graph, $F(l \pm r)$ are the gluon propagators from the down part of the graph, $N(k_\perp, l_\perp, r, x)$ is the matrix structure in the up quark line matched with the gluon block. For diagram, Fig.1a, the functions G and F have the form

$$G_a^{1,2} = -\frac{1-x}{x^2 m^2 + (1-x)\lambda^2 + [k_\perp \pm (1-x) r_\perp]^2};$$
$$F(l \pm r) = -\frac{1}{\lambda^2 + [l_\perp \pm r_\perp]^2}, \quad (2)$$

where we introduce the mass λ into the gluon propagators. It follows from (2) that in the $\lambda \to 0$ limit we have infrared singularity only from the gluon propagators $F(l \pm r)$ in the down part of the graph. All other propagators have no divergences in this limit.

The numerators of the diagrams can be decomposed over independent matrix structures

$$N(k_\perp, l_\perp, r, x) = \sum_{k=0}^{4} N_i^k \hat{\Gamma}_k. \quad (3)$$

We use the following definitions of these matrix structures

$$\hat{\Gamma}_0 = \frac{\not{p}'}{s}, \; \hat{\Gamma}_1 = I, \; \hat{\Gamma}_2 = \not{p}, \; \hat{\Gamma}_3 = \frac{i\epsilon^{\alpha\beta\gamma\rho} p_\alpha p'_\beta r_\gamma \gamma_\rho \gamma_5}{s}, \; \hat{\Gamma}_4 = \frac{i m \sigma^{\alpha\beta} p'_\alpha r_\beta}{s}. \quad (4)$$

We shall analyze here the contributions of the structures $\hat{\Gamma}_3, \hat{\Gamma}_4$ which are determined by the antisymmetric part in the quark line and lead only to the spin-flip effects [9]. The axial $\hat{\Gamma}_3$ term will be called the anomalous term because it does not contain the quark mass; and $\hat{\Gamma}_4$, the normal term. For the diagram, Fig.1a, we find

$$N_a^3 = 20 s^2 x(1-x); \quad N_a^4 = -20 s^2 x. \quad (5)$$

It is easy to see that in the $\langle T_a \rangle$ and $\langle T_b \rangle$ the integrals over $d^2k_\perp$ and $d^2l_\perp$ are factorized completely and the amplitudes can be written in the form

$$\langle T_{a,b}(s,t)\rangle = A^{2g}(s,t) \sum B_{a,b}^k \hat{\Gamma}_k = A^{2g}(s,t) \hat{B}_{a,b}, \quad (6)$$

where

$$A^{2g}(s,t) = 4is\alpha_s^2 c_2 \int d^2l_\perp \prod (F(l \pm r)), \quad c_2 = \frac{8}{36} \quad (7)$$

is the born two-gluon high-energy amplitude $O(\alpha_s^2)$ with $(F(l \pm r))$ determined by (2). The A^{2g} amplitude has an infrared divergence in the $\lambda \to 0$ limit.The amplitudes $\hat{B}_{a,b}$ containing the integration over $d^2k_\perp$ are free from the singularities at $\lambda = 0$.

The factorization (6) is a very important property of the helicity amplitudes. Really the magnitudes of polarization, spin correlation parameters are quadratic in the helicity amplitudes in the numerator and denominator. As a result, the infrared singularities of A^{2g} must cancel at the physical spin-dependent observables. We hope that this result is justified not only in the α_s^3 order of perturbative QCD.

Let us analyzed the form of the obtained B_a^k amplitude in the $\lambda = 0$ and $s \to \infty$ limit. After integration over d^2k we have

$$B_a^k = \int_0^1 \frac{dx\, f^k(x)}{\Delta\sqrt{\Delta^2(1-x)^2 + 4m^2x^2}} \ln \frac{\sqrt{\Delta^2(1-x)^2 + 4m^2x^2} + \Delta(1-x)}{\sqrt{\Delta^2(1-x)^2 + 4m^2x^2} - \Delta(1-x)} =$$
$$2\sum_{n=0}^{\infty} \frac{\Delta^{2n}}{(2n+1)} \int_0^1 \frac{dx\,(1-x)^{2n+1} f^k(x)}{[\Delta^2(1-x)^2 + 4m^2x^2]^{n+1}}, \quad (8)$$

It easy to show using (1,5) that the functions $f^3(x)$ and $f^4(x)$ in (8) have a nonzero limit for $x = 0$. For the $n = 0$ term in (8) one can obtain

$$B_a^k(n=0) = \frac{\pi f^k(0)}{m\Delta} + g^k(m, \Delta). \quad (9)$$

Here $g^k(m, \Delta)$ are smooth functions for $m, \Delta \to 0$. They are determined by the terms $\sim x^m$ $(m \neq 0)$ in the numerator of (8). Of the same form are all the terms with $n \neq 0$ in (8). As a result, the behavior (9) is true for the whole integral. So, we obtain the mass and momentum-transfer singularities for the anomalous and the normal terms in the spin-flip amplitude.

Considering that

$$\langle \hat{\Gamma}_3 \rangle_{flip} = \langle \hat{\Gamma}_4 \rangle_{flip} = \frac{m\Delta}{2} \qquad (10)$$

we conclude that the $\langle \hat{\Gamma}_3 \rangle_{flip}$ and $\langle \hat{\Gamma}_4 \rangle_{flip}$ contributions to the spin flip amplitude do not depend on m and Δ for small masses and momenta transfer.

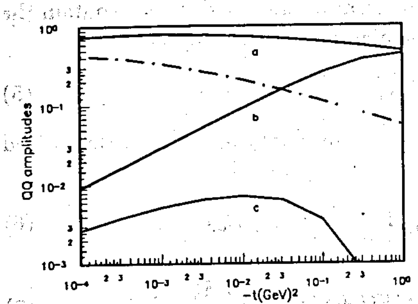


Fig.2 The $\hat{\Gamma}_3$ contributions to the spin-flip amplitudes $\langle \hat{B} \rangle_{flip}$ for diagrams, Fig.1a-c, full curve - for $m = 0.33 GeV$; dotted line - $\langle \hat{B}_a \rangle_{flip}$ for $m = 0.005 GeV$.

The results of calculations of the $\hat{\Gamma}_3$ contribution to the spin-flip amplitudes $\langle \hat{B}_i \rangle_{flip}$ determined in (6) for diagrams, Fig.1a-c, are shown in the Fig.2. It can be seen that these terms in the amplitude, Fig.1a, really has an anomalous behavior. The amplitude, Fig.1b,c, has a normal behavior because of another form of the t-channel propagators. However, from (5) it follows that

$$f^3(0) = -f^4(0). \qquad (11)$$

So the anomalous terms from $\langle \hat{\Gamma}_3 \rangle_{flip}$ and $\langle \hat{\Gamma}_4 \rangle_{flip}$ are cancelled and the amplitude, Fig.1a, together with all other amplitudes have a normal behavior for the on-mass-shell quarks. Note that this cancellation of the normal and the anomalous divergences for large masses is well known [4, 7]. Here these terms are cancelled for all masses.

3 Spin effects in γq scattering

Note that because the masses are different in nature, the divergences cancel out. Really, we have in $\hat{\Gamma}_4$ -the mass from the numerator of the internal quark propagator (m_{int}), in (2,9) -the mass determined by the quark propagator pole (m_{pole}), in (10)- the mass of the external quark (m_{ext}). If these masses have different values, the matrix elements $\langle \hat{B}_a \rangle_{flip}$ will have an anomalous term proportional to $(m_{ext} - m_{int})/m_{pole}$. So it is very important to investigate what will happen with the anomalous and

normal terms if we take into account the off-mass-shell quarks because the obtained compensation can be destroyed in this case [10]. To answer this question, we shall investigate here γq elastic scattering at high energies. The anomalous and normal terms analyzed in the previous section will appear in this reaction if gluons will interact with a single quark in the loop, (Fig.3). This graph corresponds to the standard pomeron contribution to the γq high-energy scattering.

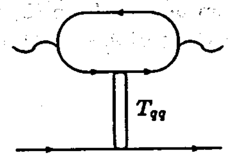


Fig.3 The pomeron contribution to high-energy γq scattering.

In calculations we shall keep only the $\hat{\Gamma}_3$ and $\hat{\Gamma}_4$ structures in the quark-loop. The result can be represented in the form

$$T^{\gamma q}_{\lambda 2+}(s,t) = \alpha\, A^{2g}(s,t)\, \Phi^{\gamma q}_{\lambda 2+}(t). \qquad (12)$$

Here $\Phi^{\gamma q}_{\lambda 2+}(t)$ is the amplitude with the positive initial and arbitrary final polarization, $\alpha = 1/137$. As in the previous section, the factorization of the high-energy $(A^{2g}(s,t))$ and large-distance effects $(\Phi^{\gamma q}(t))$ is performed in (12).

A simple two-gluon exchange contribution to the spin-non-flip amplitude, the functions $\Phi^{\gamma q}_{-+}(t)$ for diagrams, Fig1a,b, are shown in Fig.4. We can see that the spin effects are not very small as compared to the non-flip contribution from the born diagram. However both diagrams, Fig1a,b, have the same behaviour at small $|t|$. So, the anomalous terms are absent in this case.

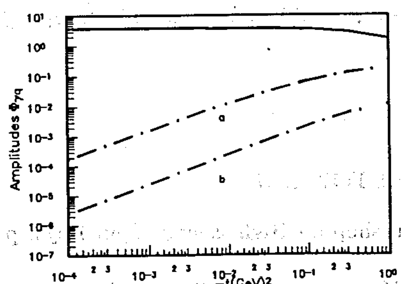


Fig.4 The amplitudes $\Phi^{\gamma q}(t)$ for $m = 0.33 GeV$ determined in (12): full curve - born two-gluon contribution to the $\Phi^{\gamma q}_{++}(t)$, dotted line - $\Phi^{\gamma q}_{-+}(t)$ amplitudes for diagrams, Fig1a,b.

We have analyzed what may happen to the anomaly. It was shown that the contributions of $\hat{\Gamma}_3$ and $\hat{\Gamma}_4$ do not compensated each other in the spin-flip amplitude of γq scattering. The absence of the anomalous behavior in the $\Phi^{\gamma q}_{-+}(t)$ amplitude is caused by the changes in the t-channel gluon propagators from the off-mass-shell effects in the quark loop. This can be shown using (8). Really, if the external quarks are not on the mass-shell, we have the following change in (8)

$$(\Delta^2(1-x)^2+4m^2x^2) \Longrightarrow (\Delta^2(1-x)^2+4m^2x-4p^2x(1-x)). \qquad (13)$$

As a result, for $p^2 \neq m^2$ the quadratic divergence $\sim 1/x^2$ in the $B_a^k(n=0)$ integral e.g. at small momenta transfer will change to the liner one $\sim 1/x$. This leads to vanishing the pole $1/m$ in (9). For small quark masses we have the same result for $\Phi^{\gamma q}(t)$ as in Fig.4 but on another scale. Really, it can be seen from (8) that in the $s \to \infty$ limit the scattering amplitude depends on the variable $\Delta^2/4m^2$. The γq amplitude depends on the $\Delta^2/4m^2$ variable too.

4 Conclusion

So the analysis of the pomeron spin properties at small quark masses and momenta transfer in the α_s^3 order of perturbative QCD shows that the anomalous behaviour found in the individual contributions (9) is absent in the full spin-flip amplitudes of qq and γq scattering. The reasons for disappearing the anomaly are different in these reactions.

In the qq subprocess the cancellation of the singularities in the anomalous and normal terms is obtained in accordance with [4, 7]. In γq scattering the off-mass-shell effects destroy the anomalous behaviour of the amplitudes at small quark masses and momenta transfer.

This does not signify that we prove the absence of the anomaly in elastic scattering. May be it is determined by nonperturbative contributions (see [8] e.g.). It is possible that the anomalous terms found here can manifest itself in more complicated diagrams or in other processes.

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