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STOCHASTIC SPACE-TIME, CONFINEMENT POTENTIAL AND PARTICLE PROPAGATOR

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I. Introduction

In previous paper /1/ we have introduced stochastic space-time induced by random strings, the behaviour of which is described by a probability distribution

$$P[y]=\frac{1}{N}\exp\Big[-\frac{1}{2}\int\limits_{M_1}\int\limits_{M_2} d^2\sigma_1\, d^2\sigma_2 \sqrt{g_1}\sqrt{g_2}\,\times \\ \times\, y^{\mu}(\sigma_1)\, D^{-1}_{\mu\nu}(\sigma_1-\sigma_2)\, y^{\nu}(\sigma_2)\Big], \tag{I}$$

where M is a two-dimensional surface known as the string world-sheet, $y^{\mu}(\sigma)$ $(\sigma=\sigma^a \equiv \{\sigma^1=\sigma, \sigma^2=\tau\})$ are coordinates of strings, N is a constant chosen so that $P[y]$ is normalized to unity and $D^{-1}_{\mu\nu}$ is the inverse of the two-point correlation

$$\langle y^{\mu}(\sigma^1)\, y^{\nu}(\sigma^2)\rangle_y = D^{\mu\nu}(\sigma_1-\sigma_2).$$

Further, we have assumed that due to the presence of the random string field $y^{\mu}(\sigma)$ space-time begins to fluctuate and its topology structure gives rise to changes in physical quantities at short distances. This means that the structure of space-time is distorted in the immediate neighbourhood of a particle, which leads to the concept of confinement and to the problem of reformulating general theory of gravitation to allow for it. In other words, stochastic strings (particles) alter the geometry of space-time, and that alteration in turn affects the behaviour of particles. In this paper, we would like to show that this sefl-referential nonlinear property of gravity is responsible for the appearance of the nonlocal interaction and the confinement force between quarks. It turns out that the force-transmitting quanta become spread-out objects, propagator of which possesses unusual properties.

As shown in /1/ the conform-like transformation of coordinates

$$\xi^{\mu}=x^{\mu}\exp\Big\{\frac{1}{2\sqrt{\pi\alpha'}}\int\limits_M d^2\sigma\sqrt{g}\,R\,U_{\mu}(\sigma)\,y^{\mu}(\sigma)\Big\} \tag{2}$$

leading to the passage from the usual local inertial system of reference ξ^{α} with the Minkowski metric $\eta_{\alpha\beta}$ to the quasilocal system of "averaged" (point-like) string coordinates x^{μ}:

$$y^{\mu}(\sigma)=\frac{1}{\sqrt{g}R}\,\delta(\sigma^1)x^{\mu}(\sigma^2)+\eta^{\mu}(\sigma^1,\sigma^2), \tag{3}$$

gives rise an induced gravitation with the metric tensor

$$G_{\mu\nu}(x,y)=\Lambda^2\Big[\eta_{\mu\nu}+\mathcal{E}_{\mu\nu}(x)+\frac{1}{4}\mathcal{E}^{\rho}_{\mu}(x)\mathcal{E}_{\nu\rho}(x)\Big], \tag{4}$$

where

$$\mathcal{E}^{\nu}_{\mu}(x)=\frac{1}{\sqrt{\pi\alpha'}}\,x^{\nu}(\tau)U_{\mu}(\tau)$$

$$\Lambda=\exp\Big\{\frac{1}{2}\,\frac{1}{\sqrt{\pi\alpha'}}\int_M d^2\sigma\sqrt{g}\,R\,U_{\mu}(\sigma)y^{\mu}(\sigma)\Big\}.$$

Here R is the Ricci curvature scalar of the manifold M, $U_{\mu}(\sigma)$ is a vector

$$-\eta^{\mu\nu}U_{\mu}(\tau)U_{\nu}(\tau)=1$$

depending on the time-like variable $\sigma^2=\tau$ only.

Carrying out an averaging procedure in (4) over random variables $y^{\mu}(\sigma)$ with the probability distribution (I) and the white noise covariance

$$D^{\mu\nu}(\sigma_1-\sigma_2)=-\eta^{\mu\nu}\frac{\lambda^2}{\sqrt{g}R}\,\delta^2(\sigma_1-\sigma_2), \tag{5}$$

we have

$$\langle G_{\mu\nu}(x,y)\rangle_y=\Big[\eta_{\mu\nu}+\mathcal{E}_{\mu\nu}(x)+\frac{1}{4}\mathcal{E}^{\rho}_{\mu}(x)\mathcal{E}_{\nu\rho}(x)\Big]\exp\Big\{\frac{\lambda^2}{2\pi\alpha'}\int_M d^2\sigma\sqrt{g}R\Big\}, \tag{6}$$

where

$$Euler(M)=\frac{1}{4\pi}\int_M d^2\sigma\sqrt{g}\,R$$

is a topological invariant known as the Euler characteristic of M.

If M has genus N (i.e., if M is homeomorphic to a sphere with N handles, or to a connected sum of N tori), then

Euler (M) = 2 − 2N.

Thus, the physical space-time at large distances is obtained by a over equivalent topological structures

$$G_{\mu\nu}(x)=\sum_N\langle G_{\mu\nu}(x,y)\rangle_y=\Big[\eta_{\mu\nu}+\mathcal{E}_{\mu\nu}(x)+\frac{1}{4}\mathcal{E}^{\rho}_{\mu}(x)\mathcal{E}_{\nu\rho}(x)\Big]\times I, \tag{7}$$

where the multiplier factor

$$I=e^{4\Delta}/(1-e^{-4\Delta}),\qquad \Delta=\lambda^2/\alpha'$$

has appeared due to the topological structure of space-time at short distances. Parameter λ in (5)-(7) is some constant dimension of length. Here we distinguish two possibilities:

(a) $\lambda^2=G^2$, where G is the Newtonian constant,
(b) $\lambda^2\sim\alpha'$, where α' is the inverse string tension (a size of the string).

The former means that fluctuation of the string coordinates takes place at the Planck scale, while the second case means that coordinates obey random properties in a domain characterized by the size of the string.

2. Energy and Potential of Quarks in an Induced Gravitation

By using metric (4) [or (6),(7)] we can easily construct the theory of induced "gravity" caused by random strings and reanalize its consequences. Here we concentrate on some interesting facts. First, when the quark particle moves in the constant fictitious "field" $\mathcal{E}_{\mu\nu}(\vec{x})$ in (4) its averaged energy is defined as /2/

$$E_f=\frac{mc^2}{\sqrt{1-v^2/c^2}}\sqrt{-G_{00}}=\frac{mc^2}{\sqrt{1-v^2/c^2}}\,\frac{e^{\Delta}}{1-e^{-\Delta}}\Big(1-\frac{\vec{x}^2}{4\pi\alpha'}\cdot\frac{1}{1-v^2/c^2}\Big)^{1/2}. \tag{8}$$

From this we immediately conclude that the quark undergoes a finite motion, the phase diagram of which is defined as

$$\frac{\vec{P}^2}{P^2_{max}}+\frac{\vec{x}^2}{x^2_{max}}\leq 1, \tag{9}$$

where $P_{max}=mc$ and $x_{max}=2\sqrt{\pi\alpha'}$. Assuming $\alpha'=m_\rho^{-2}$ we get $x_{max}=10^{-13}$ cm. Second, motion equation of a particle in the induced gravity takes the form

$$m\frac{d^2x^\alpha}{ds^2}=F^\alpha=\{F^0, \vec{F}\}. \qquad (10)$$

Now we define the force acting in the particle in constant "gravitational" field (7). In the nonrelativistic limit equation (10) takes the form [2]

$$m\frac{d^2\vec{x}}{dt^2}\left(\frac{dt}{ds}\right)^2=\frac{mc^2}{2}\frac{1}{h}\left(-\vec{\nabla}h\right),$$

where $(dt/ds)^2=1/h, \quad h=-G_{00}$.
Thus, according to the formula (9) quark potential is defined as

$$U=\begin{cases}\dfrac{mc^2}{2}\dfrac{e^{4\Delta}}{1-e^{-4\Delta}}\left(1-\dfrac{\vec{x}^2}{R_0^2}\right), & 0\le|\vec{x}|\le R_0\\ 0 & |\vec{x}|>R_0\end{cases} \qquad (11)$$

where $R_0^2=4\pi\alpha'$.

3. Particle Propagator of the Confinement Potential (11)

It is well-known that in the static limit the form of the potential is connected with the form of the interacting particles propagator. For example, in the case of quantum electrodynamics, the Coulomb potential is given by

$$\frac{e}{4\pi r}=\frac{e}{(2\pi)^3}\int d^3p\, e^{i\vec{p}\vec{r}}\,\frac{1}{\vec{p}^2}. \qquad (12)$$

Analogously, the Yukawa potential is responsible for existence of scalar particle-transmitting quanta:

$$\frac{g}{4\pi r}e^{-mr}=\frac{g}{(2\pi)^3}\int d^3p\, e^{i\vec{p}\vec{r}}\,\frac{1}{m^2+\vec{p}^2}. \qquad (13)$$

From (12) and (13) it is easily seen that corresponding particle propagators are photons and mesons, respectively.

Without loss of generality we do not consider tensor structure of the interacting particle propagator and define its form in accordance with the general rules (12) and (13). Thus, in our case we have the following integral equation

$$\frac{\lambda'}{4\pi R_0}\left(1-\frac{\vec{r}^2}{R_0^2}\right)=\frac{\lambda'}{(2\pi)^3}\int d^3p\, e^{i\vec{p}\vec{r}}\,D(\vec{p}^2), \qquad (14)$$

where $\lambda'=(m/M_0)\,2\pi e^{4\Delta}/(1-e^{-4\Delta}), \quad R_0=\frac{1}{M_0}=\sqrt{4\pi\alpha'}$.
To solve this equation we integrate it over angular variables θ,φ. The result reads

$$\frac{\lambda'}{4\pi R_0}\left(1-\frac{\vec{r}^2}{R_0^2}\right)=\frac{\lambda'}{2\pi^2}\frac{1}{r}\int_0^\infty dp\cdot p\sin(pr)D(p^2). \qquad (15)$$

This integral equation has unique solution

$$D(\vec{p}^2)=\sqrt{\frac{2\pi}{R_0}}\;\frac{J_{5/2}(R_0\sqrt{\vec{p}^2})}{(\sqrt{\vec{p}^2})^{5/2}}, \qquad (16)$$

where $J_{5/2}(x)$ is the Bessel function of the order of 5/2.
It is natural to generalize expression (16) for the relativistic case:

$$D(p^2)=\sqrt{\frac{2\pi}{R_0}}\;J_{5/2}(R_0\sqrt{-p^2})\Big/\left(\sqrt{-p^2}\right)^{5/2}, \quad p^2=p_0^2-\vec{p}^2 \qquad (17)$$

and for the particle with mass m

$$D_m(p^2)=\sqrt{\frac{2\pi}{R_0}}\;J_{5/2}(R_0\sqrt{m^2-p^2})\Big/\left(\sqrt{m^2-p^2}\right)^{5/2}. \qquad (18)$$

The latter gives oscillation potential

$$U_m=\frac{\lambda'}{2\pi}\frac{1}{R_0^3}\frac{1}{m}\left(R_0^2-r^2\right)^{1/2}J_1\left(m\sqrt{R_0^2-r^2}\right). \qquad (19)$$

If we assume that force-transmitting quanta giving confinement potentials (14) and (19) are photon and Z -boson like objects (gluons), then expressions (17) and (18) may be written in convenient form (inserting tensor indices):

$$D(p^2) \Rightarrow D_{\mu\nu}(p^2) = V_o(p^2 R_o^2)\, \Delta^{o}_{\mu\nu}(p^2), \qquad (20)$$

$$D(p^2) \Rightarrow D^{m}_{\mu\nu}(p^2) = V_m(p^2 R_o^2)\, \Delta^{m}_{\mu\nu}(p^2), \qquad (21)$$

where

$$V_m(p^2 R_o^2) = \sqrt{2\pi/R_o}\; \mathcal{J}_{5/2}\left(R_o\sqrt{m^2-p^2}\right) \Big/ \left(\sqrt{m^2-p^2}\right)^{1/2}, \qquad (22)$$

$$V_o(p^2 R_o^2) = V_m(p^2 R_o^2)\Big|_{m=0}, \qquad (23)$$

$\Delta^{o}_{\mu\nu}(p)$ and $\Delta^{m}_{\mu\nu}(p)$ are usual local propagators of photon and Z-boson, respectively.

As shown in Table, propagators (20) and (21) possess some interesting properties:

1. These are entire analytic functions of the variable p^2.
2. Their order of growth is, $\rho = 1/2$.
3. They decrease rapidly enough in the **Euclidean** directions $p^2 \to -\infty$
4. They have no poles in the complex plan p^2, so that force-transmitting quanta corresponding to these propagators **are never observable.**
5. Wave functions of these quanta are spread out over space-time

$$A^{nonl}_{\mu}(x) = \int d^4y\, K_o(x-y) A_{\mu}(y) \qquad (24)$$

or

$$Z^{nonl}_{\mu}(x) = \int d^4y\, K_m(x-y) Z_{\mu}(y), \qquad (25)$$

where $K_o(x)$ and $K_m(x)$ are nonlocal generalized functions [3] Fourier transform of which is **defined by** $\left[V_o(p^2 R_o^2)\right]^{1/2}$ and $\left[V_m(p^2 R_o^2)\right]^{1/2}$, respectively. Local wave functions of photons $A_{\mu}(x)$ and Z-bosons $Z_{\mu}(x)$ give usual propagators

$$\langle 0|T\{A_{\mu}(x)A_{\nu}(y)\}|0\rangle = \frac{1}{i}\,\Delta^{o}_{\mu\nu}(x-y),$$

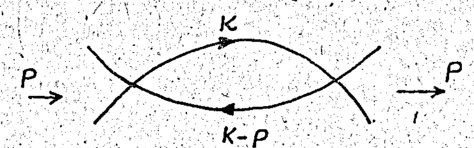
$$\langle 0|T\{Z_{\mu}(x)Z_{\nu}(y)\}|0\rangle = \frac{1}{i}\,\Delta^{m}_{\mu\nu}(x-y)$$

by means of their T-product operators in x-space.

Table 1

Type of interactions	Potential force	Propagator of force-transmitting quanta	Corresponding particle
Electromagnetic	$e/4\pi r$	$g_{\mu\nu}/(-p^2 - i\varepsilon)$	photons
Yukawa	$(g/4\pi r)e^{-mr}$	$1/(m^2 - p^2 - i\varepsilon)$	scalar particles with mass m
Confinement	$\begin{cases} 0, & r > R_o \\ const\left(1-\frac{\vec{r}^{\,2}}{R_o^2}\right), & 0 \le r \le R_o \end{cases}$	$\sqrt{\frac{2\pi}{R_o}}\,\frac{\mathcal{J}_{5/2}(R_o\sqrt{-p^2})}{(\sqrt{-p^2})^{5/2}}$	nonlocal photon-like objects – gluons

Thus, we see that confinement potential due to induced gravitation of strings leads to new type interactions, corresponding force-transmitting quanta become nonlocal and unobservable, and their propagator is entire analytic function. Quantum field theory for such interaction is constructed by means of Efimov method [3,4], in which there are no ultraviolet divergences. For example, in $\phi^4(x)$- interaction matrix element corresponding to simpler loop diagram



has the form

$$\Pi(p) = (\lambda')^2 \frac{i^{-1}}{(2\pi)^4}\int d^4k\, D(k)\, D(k-p),$$

where $D(k)$ is given by (17). After analogous calculations carried out in [4], we have

$$\Pi(p^2)=\lambda'^2\frac{\pi}{16}R_0^4\frac{1}{16\pi^2}\frac{1}{(2i)^2}\int\limits_{-\beta_1+i\infty}^{-\beta_1-i\infty}d\xi\int\limits_{-\beta_2+i\infty}^{-\beta_2-i\infty}d\eta\,\frac{1}{\sin\pi\xi\,\sin\pi\eta}\,\frac{1}{\Gamma(1+\xi)\Gamma(1+\eta)}$$

$$\times\frac{1}{\Gamma(\frac{5}{2}+\xi+1)\Gamma(\frac{5}{2}+\eta+1)}\left(\frac{R_0^2}{4}\right)^{\xi+\eta}\frac{\Gamma(-\eta-\xi-2)}{\Gamma(-\xi)\,\Gamma(-\eta)}$$

$$\times\int\limits_0^1 dx\,(1-x)^{-1-\xi}x^{-1-\eta}\left[m^2-p^2x(1-x)\right]^{2+\eta+\xi}.$$

Assuming $m^2R_0^2\ll 1$, we get a finite result

$$\Pi(p^2)=\lambda'^2\frac{1}{16\pi^2}\sum_{n=0}^{\infty}\frac{1+n}{(\frac{5}{2}+n)(\frac{3}{2}+n)(\frac{1}{2}+n)(\frac{1}{2}-n)}+O(m^2R_0^2).$$

In other words, absence of ultraviolet divergences in our theory is connected with the fact that the Green function of the force-transmitting quanta is finite in x-space. For example, Euclidean Green function corresponding to (17) takes the form:

$$D_E(x)=\frac{1}{(2\pi)^4}\int d^4p\,e^{ipx}\tilde{D}(p^2)=\frac{1}{2\pi^2}\,\frac{1}{R_0^2}\sqrt{1-\frac{x_E^2}{R_0^2}}$$

so that

$$D_E(0)<\infty,$$

as it should.

References

1. Kh.Namsrai, Intern.J.Theor.Phys., 1993, 32 (I), 43.
2. L.D.Landau and I.M.Lifschitz, The Classical Theory of Fields, 3rd Edition, Pergamon Press, Oxford, 1971.
3. G.V.Efimov, Nonlocal Interactions of Quantized Fields, Nauka, Moscow, 1977.
4. Kh.Namsrai, Nonlocal Quantum Field Theory and Stochastic Quantum Mechanics, D.Reidel, Dordrecht, Holland, 1986.

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