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FOUR-QUARK STATES
AND NUCLEON-ANTINUCLEON ANNIHILATION
WITHIN THE QUARK MODEL
WITH QCD VACUUM-INDUCED INTERACTION

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INTRODUCTION

Four-quark mesons were first considered in articles [1], [2] in the MIT version of the bag model. From this consideration it followed in particular that known $a_0(980)$, $f_0(975)$ mesons may be assigned to the lightest $q^2\bar{q}^2$ states. Now there exists some additional experimental indications of multi-quark meson states. Let us numerate the most clear of them. In the experiments on vector meson VV' production in $\gamma\gamma$ scattering there are perhaps observed resonance like signals of four-quark nature [3]. In Serpukhov facility in hadron-hadron scattering the extraordinary C-resonance was discovered in the $\phi\pi$ system with mass ~ 1.5 GeV and quantum numbers ($I=1$; $J^{PC} = 1^{--}$) [4]. Later the GAMS group has reported on the resonance structure in the $\eta\pi$ channel [5] with exotic quantum numbers ($I=1$, $J^{PC} = 1^{-+}$) which is probably a G-partner of the C-resonance.

Reach material for the $q^2\bar{q}^2$ meson spectroscopy can be extracted from $N\bar{N} \rightarrow (q^2\bar{q}^2)(q\bar{q}) \rightarrow \text{mesons}$ reactions. In paper [6] there is analyzed the resonance observed in $\bar{p}n \rightarrow \pi^- X_0(1480) \rightarrow 3\pi^- 2\pi^+$ reaction. The decay of $X_0(1480)$ goes mainly through the $\rho^0\rho^0$ system. This resonance possibly also gives a contribution to $\gamma\gamma \rightarrow \rho^0\rho^0$ reaction and was observed by the TACCO, CELLO and JADE [3] groups. Recently the ASTERIX collaboration presented the data on the annihilation of $p\bar{p}$ into the $\pi^+\pi^-\pi^0$ system [7] where the resonance in the $\pi^+\pi^0$ mass spectrum with a mass ~ 1565 MeV and a width 170 MeV is observed. The Crystal Barrel collaboration in LEAR sees in the $\bar{p}p \rightarrow \pi^0 X_2(1515) \rightarrow \pi^0\pi^0\pi^0$ reaction a tensor meson with a width ~ 120 MeV [8]. In many papers there is widely discussed the E(1420) meson with a width ~ 60 MeV observed for example in $\bar{p}p \rightarrow (K^+K, \pi^+\pi^-)\pi^+\pi^-$ [9], [10]. The decay of the E-meson goes mainly through $a_0(980)\pi$ ($a_0(980) \rightarrow \bar{K}K$). So, if $a_0(980)$ is really a four-quark state then it is natural to suppose that the E-meson is also a four-quark state.

All these experimental data need a theoretical analysis to answer the question: is it really these states may be described as $q^2\bar{q}^2$ states? The calculation of $q^2\bar{q}^2$ meson mass spectrum and their wave functions are necessary to carry out in the quark model where the mixing of hadron states is taken into account. In the first part of this article we describe the quark model with QCD vacuum - induced interaction. In second part we present the mass spectrum of $q^2\bar{q}^2$ mesons, their recouplings and the basis of physical states. Parts 3, 4 and 5 are devoted to applications of the results obtained to the manifestation of four - quark states in $N\bar{N}$ annihilation process. Scalar, vector and tensor mesons are considered and four - quark nature of the $a_0(980)$ - meson state is also discussed.

1 Quark model with QCD vacuum - induced interaction

In [11] the quark bag model has been formulated in which importance of the quark-QCD vacuum field interaction has been stressed. The crucial fact allowing

to construct the realistic model of hadrons is that there is the physical medium surrounding a bag and populated by collective intensive vacuum fluctuations. The interaction of fields inside the bag with external vacuum fields completely changes the structure of the standard bag model. Within this approach the bag surface is determined self-consistently by minimizing the total energy of the hadron as a system of Dirac quarks in an external vacuum field.

Let us consider the hierarchy of fields in the QCD vacuum and the bag-vacuum system. As it is known, the QCD vacuum has quite a complicated structure. Conventionally the nonperturbative fields can be divided in two parts: a short-wave component that provides the helicity sensitive interaction of quarks at small distances and a long-wave component that gives the confinement. In the framework of the instanton liquid model [13, 12] the first part is connected with a finely granular vacuum structure where the one-instanton interaction with effective size $\rho_c \sim 1.5 \div 2 \text{ GeV}^{-1}$ dominates. The second component is related with long-wave collective excitations of the instanton liquid with wave length $\lambda \approx R \approx R_{conf}$, where $R \approx 3\rho_c$ is an average distance between instantons and $R_{conf} \approx 5 \div 6 \text{ GeV}^{-1}$ is a confinement radius.

The main assumption of our model [11] is that the QCD vacuum is almost not destroyed by color fields inside the hadron and the interaction of quarks and gluons localized in the bag with vacuum fields defines the hadron structure. We shall consider the bag and fields localized inside it immersed inside the physical instanton vacuum and suggest that quarks do (almost) not disturb the local properties of this medium. This suggestion is analogously to QCD SR one. In the latter case as a probe, i.e. a nonlocal object selecting the lowest hadron states, the correlator of hadron currents is used like the bag in our quark model. Here, one also suggests that local sources do not perturb the properties of physical vacuum, the quantities of quark and gluon condensates.

There are three different length scales of fluctuations in the bag-vacuum system: small-size instanton fluctuation with characteristic frequency $\epsilon_i \sim 1/\rho_c$, fluctuations of fields localized inside the bag with frequency $\epsilon_q \sim \omega_q/R_{bag}$, and long-wave vacuum fluctuations with $\epsilon_{vac} \sim 1/R_{conf}$. For low-lying excitations of quarks in the bag the factorization of small, intermediate and large distances occurs $1/\rho_c \gg \omega_q/R_{bag} \gg 1/R_{conf}$, and the use of the effective Lagrangian technique is correct [14]. At the scale $r \leq \rho_c$ the main effect is the interaction related with tunneling transition due to instanton and at the scale $r \sim R_{bag}$ it is the confinement of quarks in the bag. With this hierarchy of interactions we can regard that the interaction of quarks in the hadron mainly develops on one instanton in the background of external vacuum medium.

Long-wave vacuum fields $Q(x)$, $A_\mu^a(x)$ satisfy the classical Yang-Mills equations (acting on the state of physical vacuum $|\underline{Q}\rangle$)

$$(i\hat{\nabla} - m_i)Q'(x) | \underline{Q} \rangle = 0, \quad (1.1)$$

$$\nabla_{ab}^\mu G_{\mu\nu}^b(x) | \underline{0} \rangle = g \bar{Q}^i(x) \frac{\lambda^a}{2} \gamma_\nu Q^i(x) | \underline{0} \rangle,$$

$$G_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a + g f^{abc} A_\mu^b A_\nu^c.$$

Their solutions are parametrized by singlet renormalization-invariant averages [15]:

$$\langle \underline{0} | \frac{\alpha_s}{2\pi} : G_{\mu\nu}^a(0) G^{a\mu\nu}(0) : | \underline{0} \rangle \approx 0.012 \text{ GeV}^4, \quad (1.2)$$

$$\langle \underline{0} | \alpha_s^{4/9} : \bar{Q}(0) Q(0) : | \underline{0} \rangle \approx -(250 \text{ MeV})^3, \dots,$$

where the values of condensates are determined phenomenologically within the current algebra and QCD SR. By dots the operators of higher dimensions are denoted and the normal ordering of operators with respect to a perturbative vacuum state $| \underline{0} \rangle$ is implied.

In order to describe the interaction of valence quarks with long-wave vacuum fields let us do the substitution

$$q(x) \Theta_V(x) \rightarrow q(x) \Theta_V(x) + Q(x), \quad A_\mu^a(x) \Theta_V(x) \rightarrow A_\mu^a(x) \Theta_V(x) + \mathcal{A}_\mu^a(x) \quad (1.3)$$

in the bag model Lagrangian

$$\mathcal{L}^{QCD} \Theta_V(x) \rightarrow \mathcal{L}^{QCD} \Theta_V(x) + \Delta \mathcal{L}^{vac}, \quad (1.4)$$

where

$$\Delta \mathcal{L}^{vac} = [\bar{q}(x) \Theta_V(x)] (\frac{i}{2} \overleftrightarrow{\partial}) Q(x) + \bar{Q}(x) (\frac{i}{2} \overleftrightarrow{\partial}) [q(x) \Theta_V(x)] + g \bar{q}(x) \gamma^\mu \frac{\lambda^a}{2} q(x) \mathcal{A}_\mu^a(x) \Theta_V(x), \quad (1.5)$$

Q and \bar{Q} are anticommuting vacuum quark fields, $\mathcal{A}_\mu^a(x)$ is an external gauge field. Localized field components $q(x)$ and $A(x)$ are approximated by the solutions of the bag model equations in the spherical cavity approximation.

The interaction with the external long-wave vacuum field (1.5) results in an additional energy increasing with bag size [11]. As a consequence, the situation occurs when a further growth becomes impossible (that is, large fluctuations of the bag size are strongly suppressed). Thus, within our model the bag stability is due to the interaction of bag fields with vacuum fields.

The interaction of quarks with the short-wave component of vacuum fields, small-size instantons, is approximated by the effective 't Hooft Lagrangian [16, 17] which in the instanton liquid model [12, 21, 14] is:

$$\Delta \mathcal{L}^{inst} = \sum_{i>j}^{i=u,d,s} n_c (-k'^2) \{ \bar{q}_i R q_i L \bar{q}_j R q_j L [1 + \frac{3}{32} \lambda_i^a \cdot \lambda_j^a (1 - \frac{3}{4} \sigma_{\mu\nu}^i \cdot \sigma_{\mu\nu}^j)] + (R \leftrightarrow L) \} \quad (1.6)$$

where the constant

$$k' = \frac{4\pi \rho_c^3}{3} \frac{\pi}{(m_* \rho_c)}$$

characterizes the interaction strength of a quark with an instanton and is proportional to the instanton volume, n_c is the instanton density in the vacuum related to vacuum energy density by $B_{QCD} \approx 2n_c$, ρ_c is an effective size of the instanton in the QCD vacuum, $q_{R,L}(1 \pm \gamma_5)q/2$, $m_* = -\frac{2}{3}\pi^2 \rho_c^2 < \underline{0} | \bar{Q} Q | \underline{0} \rangle$ is the effective mass of the quark with zero current mass in physical vacuum [17], $< \underline{0} | \bar{Q}_i Q_i | \underline{0} \rangle$ is a quark condensate. An effective mass takes into account long-range field correlations in the instanton vacuum. The term $(R \leftrightarrow L)$ in (1.6) corresponds to the interaction through an anti-instanton. In addition, averaging over instanton space positions and orientations in color space is implicitly supposed. The averaging over collective variables provides translation and gauge invariance of instanton interaction. The Lagrangian (1.6) is written for qq -interaction in the $SU_f(2)$ flavor sector of $SU_f(3)$. Selection of $SU_f(2)$ corresponds to the case when one of quarks interacts with a vacuum condensate. For $q\bar{q}$ -system one should change in (1.6)

$$\lambda_q \rightarrow -\lambda_{\bar{q}}^T, \quad \sigma_q \rightarrow -\sigma_{\bar{q}}^T.$$

Let us note that a specific feature of the effective Lagrangian (1.6) is that only amplitudes with quarks being in zero fermionic mode states [16] are nonzero:

$$(\vec{\sigma}_i \oplus \vec{c}_i) | \rangle = 0, \quad (1.7)$$

where $\vec{\sigma}_i$ is the spin, \vec{c}_i is the color (subgroup $SU_c(2)$) of a i -th quark. This yields nontrivial spin-spin forces between zero mode quarks and is especially important in the phenomenology of multi-quark states [18, 19].

The interaction between quarks with long-wave vacuum fields dominates at distances of an order of the confinement radius ($R_c \sim 1 \text{ fm}$). At intermediate distances ($\rho_c \sim 0.3 \text{ fm}$) the vacuum structure is characterized by high-frequency fluctuations approximated by instantons ($\omega_{inst} \sim 1/\rho_c$).

The hadron energy is a sum of the quark kinetic energy and energies of interaction due to one-gluon and one-instanton exchange and of interaction with condensate:

$$E = E_{kin} + \Delta E_{OGE} + \Delta E_{inst} + E_{vac}. \quad (1.8)$$

The constants $\alpha_s, \rho_c, \langle q\bar{q} \rangle$, characterizing various contributions have been chosen as in the QCD sum rules, $\alpha_s = 0.7$, $\langle \bar{q} q \rangle = (-250 \text{ MeV})^3$ and in the instanton liquid model $\rho_c = 2 \text{ GeV}^{-1}$.

Taking into account the interaction with instantons and condensates enables us satisfactorily to describe both the mass spectra of the hadron ground states and to obtain right values of $\pi - \rho$, $N - \Delta$, $\Sigma \rightarrow \Xi$ and $\eta - \eta'$ splittings [14].

2 Mass spectrum of $q^2\bar{q}^2$ mesons in the quark model with QCD vacuum induced interaction

Pair forces in diquark (q^2) and quarkonium ($q\bar{q}$) systems occur due to the interaction via gluon and instanton exchanges. Gluon exchange contribution due to small hyperfine coupling constant is not important as a rule. To calculate the multi-quark hadron spectrum in the quark model we should construct the physical state basis where the energy is diagonal.

The kinetic energy and the energy of interaction with a quark condensate depend only on the quark masses and so these contributions are diagonal in the basis of states with a definite number of s-quarks (magically mixed states or a magic basis).

However the instanton contribution ΔE_{inst} is diagonal in the basis states of the $SU_{csf}(12)$ - group with definite color-spin-flavor quantum numbers.

A situation with constructing the physical state basis for four-quark systems is analogous in many respects to the one for a system of η, η' mesons. First, the physical states of these mesons have no definite number of s-quarks, second, they do not enter into any irreducible representation of the flavor group $SU_f(3)$, but they are the mixture of the singlet - η_1 and octet - η_8 states determined from the diagonalisation of the energy. In the MIT model there is no reason for occurring a basis differing from the magic one. So it is possible there to classify particles with respect to the $SU_f(3)$ multiplets: $9_f, 36_f, 18_f$. As we shall see below, in the general case the instanton interaction mixes irreducible representations of the $SU_f(3)$ group and separation of particles into flavor multiplets becomes meaningless.

As to spin degrees of freedom the ground state of the $q^2\bar{q}^2$ system may be in $J^P = 0^+, 1^+, 2^+$. Let us first consider scalar four - quark states.

2.1 Scalar $q^2\bar{q}^2$ mesons

We shall construct the basis of $q^2\bar{q}^2$ states. A pair of quarks may be in the following color, spin, and flavor representations:

$$3_c \otimes 3_c = 6_c + \bar{3}_c, \quad 2_s \otimes 2_s = 1_s + 3_s, \quad 3_f \otimes 3_f = 6_f + \bar{3}_f. \quad (2.1)$$

Then from the combinations of diquark and anti-diquark which are color singlets and obey Pauli principle we shall construct the basis of $q^2\bar{q}^2$ states [22],[23]:

$$\begin{aligned} |q^2, \bar{q}^2 \rangle &= |(q_c^2, q_s^2, q_f^2), (\bar{q}_c^2, \bar{q}_s^2, \bar{q}_f^2) \rangle, \\ |1 \rangle_{cs} &= |(6_c, 3_s, \bar{3}_f), (\bar{6}_c, 3_s, 3_f) \rangle, \quad |2 \rangle_{cs} = |(\bar{3}_c, 1_s, \bar{3}_f), (3_c, 1_s, 3_f) \rangle, \\ |3 \rangle_{cs} &= |(6_c, 1_s, 6_f), (\bar{6}_c, 3_s, 6_f) \rangle, \quad |4 \rangle_{cs} = |(\bar{3}_c, 3_s, 6_f), (3_c, 1_s, \bar{6}_f) \rangle. \end{aligned} \quad (2.2)$$

Table 1. Masses of $q^2\bar{q}^2 (J^P = 0^+)$ mesons and coefficients of decomposition in the magic basis ($I=1$).

| m, MeV | $C_\pi^s(9)$ | $C_\pi^s(9^*)$ | $C_\pi(36^*)$ | $C_\pi(36)$ | $C_\pi^s(36^*)$ | $C_\pi^s(36)$ |
|--------|--------------|----------------|---------------|-------------|-----------------|---------------|
| 1100 | .814 | -.021 | .025 | .577 | -.004 | .050 |
| 1350 | -.438 | -.034 | -.010 | .556 | .014 | .705 |
| 1700 | .254 | .736 | .084 | -.379 | .017 | -.493 |
| 1700 | -.014 | -.139 | .989 | -.030 | .028 | .022 |
| 1800 | -.282 | .661 | .113 | .462 | .028 | -.506 |
| 2050 | .014 | -.027 | -.032 | -.012 | .999 | -.005 |

Table 2. Masses of $q^2\bar{q}^2 (J^P = 0^+)$ and coefficients of decomposition in the magic basis ($I=1/2$).

| m, MeV | $C_K^s(9)$ | $C_K^s(9^*)$ | $C_K(36^*)$ | $C_K(36)$ | $C_K^s(36^*)$ | $C_K^s(36)$ |
|--------|------------|--------------|-------------|-----------|---------------|-------------|
| 970 | .958 | .000 | .015 | .271 | .008 | .093 |
| 1400 | -.182 | -.133 | .009 | .821 | -.009 | -.524 |
| 1550 | .000 | .985 | -.034 | .060 | -.015 | -.157 |
| 1990 | .000 | .028 | .997 | -.038 | -.040 | .010 |
| 2000 | -.222 | .104 | .058 | .496 | -.014 | .831 |
| 2200 | -.012 | .016 | .040 | .012 | .999 | .002 |

Table 3. Masses of $q^2\bar{q}^2 (J^P = 0^+)$ and coefficients of decomposition in the basis of the MIT model ($I=0$).

| m | C 9 | C 9* | C 36* | C 36 | C* 9 | C* 9* | C* 36* | C* 36 | C** 36* | C** 36 |
|------|-------|-------|-------|-------|-------|-------|--------|-------|---------|--------|
| 800 | .924 | -.003 | -.030 | -.350 | -.045 | .007 | -.040 | .130 | -.002 | .034 |
| 1140 | -.108 | -.107 | -.039 | -.293 | .872 | -.019 | -.020 | .311 | -.022 | -.176 |
| 1350 | .040 | .978 | .084 | .034 | .150 | -.001 | .066 | -.059 | .011 | -.062 |
| 1600 | .123 | .071 | -.550 | .584 | .026 | .101 | -.267 | .485 | -.113 | .074 |
| 1700 | .144 | -.077 | .787 | .443 | .063 | .137 | .013 | .367 | -.000 | -.006 |
| 1700 | .045 | -.010 | -.176 | .095 | .100 | .964 | -.016 | -.134 | .020 | .018 |
| 1950 | .130 | -.080 | -.191 | .240 | .046 | -.052 | .886 | .029 | .197 | -.230 |
| 2100 | -.207 | .096 | -.012 | -.312 | -.144 | .115 | .309 | .487 | -.043 | .694 |
| 2350 | -.036 | .021 | -.025 | -.026 | -.029 | -.005 | -.201 | .120 | .969 | .034 |
| 2600 | .181 | -.075 | .021 | .297 | .419 | -.160 | .028 | -.493 | .073 | .650 |

Taking irreducible flavor representations:

$$\begin{aligned} 3_f \otimes \bar{3}_f &= 9_f = 1_f(9) + 8_f(9), \\ 6_f \otimes \bar{6}_f &= 36_f = 1_f(36) + 8_f(36) + 27_f(36) \end{aligned} \quad (2.3)$$

we obtain for scalar $q^2\bar{q}^2$ mesons ten possible basis states with a definite flavor:

$$\begin{aligned} |1\rangle_{cs} &= 1_f(9), & |2\rangle_{cs} &= 1_f(9), & |3\rangle_{cs} &= 1_f(36), & |4\rangle_{cs} &= 1_f(36), \\ |1\rangle_{cs} &= 8_f(9), & |2\rangle_{cs} &= 8_f(9), & |3\rangle_{cs} &= 8_f(36), & |4\rangle_{cs} &= 8_f(36), \\ & & & & |3\rangle_{cs} &= 27_f, & |4\rangle_{cs} &= 27_f. \end{aligned} \quad (2.4)$$

In the general case the physical states are arbitrary superpositions of these states. Thus, in describing experimental data it is necessary to choose nine mixing angles of these states which are free parameters. So of main interest is the calculation of these parameters within some dynamic mechanism leading to a physical basis. Early we have shown that instanton induced interaction dominates in the unitary multiplets mixing [11]. Thus, we shall suggest this interaction as main dynamical mechanism of construction of the physical basis. The kinetic energy and vacuum energy are easier calculated in the magic basis which is connected with the $SU_f(3)$ basis by known transformations [1], [2]. The connection between the bases allows to rewrite the instanton contribution in the magic basis or the kinetic energy in the $SU_f(3)$ basis. Physical states are found by diagonalisation of the total energy of a hadron. In Tables. 1, 2, 3 there are shown the decompositions of physical states in the magic basis (MIT bag states) obtained in our model.

To analyze the decays of $q^2\bar{q}^2$ mesons it is necessary also to consider the basis composed of mesonic pair $(q\bar{q})(q\bar{q})$ states, for which there are possible the following color - spin states:

$$\begin{aligned} |M\rangle &= |(q\bar{q})(q\bar{q})\rangle = |((q\bar{q})_c, (q\bar{q})_s)((q\bar{q})_c, (q\bar{q})_s)\rangle \\ |1\rangle_{cs} &= |(1_c, 1_s)(1_c, 1_s)\rangle, & |2\rangle &= |(1_c, 3_s)(1_c, 3_s)\rangle, \\ |3\rangle_{cs} &= |(8_c, 1_s)(8_c, 1_s)\rangle, & |4\rangle &= |(8_c, 3_s)(8_c, 3_s)\rangle. \end{aligned} \quad (2.5)$$

For exotic mesons ($I > 1$) in the basis (2.5) the values of masses and the recoupling coefficients practically coincide with the ones presented in [1]. For the remaining nonexotic $q^2\bar{q}^2$ mesons the basis (2.5) has to be supplemented by the flavor recoupling:

$$\begin{aligned} (K\bar{K})^{I=1}, & & (\pi\eta_s)^{I=1}, & & (\pi\eta_0)^{I=1}, \\ (K\pi)^{I=1/2}, & & (K\eta_0)^{I=1/2}, & & (K\eta_s)^{I=1/2}, \\ (\pi\pi)^{I=0}, & & (\eta_0\eta_0), & & (K\bar{K})^{I=0}, \\ (\eta_0\eta_s), & & (\eta_s\eta_s), & & \\ \eta_s = s\bar{s}, & & \eta_0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d}). & & \end{aligned} \quad (2.6)$$

Table 4. Recoupling coefficients for $q^2\bar{q}^2 (J^P = 0^+)$ mesons into the pair of $(q\bar{q})(q\bar{q})$ mesons ($I=1$).

| m, MeV | f | $ 1\rangle_{cs}$ | $ 2\rangle_{cs}$ | $ 3\rangle_{cs}$ | $ 4\rangle_{cs}$ |
|--------|---------------|------------------|------------------|------------------|------------------|
| 1100 | K^+K^- | -.288 | .026 | -.087 | -.274 |
| | K^0K^0 | .288 | -.026 | -.087 | .274 |
| | $\pi^0\eta_s$ | .453 | -.029 | -.090 | .345 |
| | $\pi^0\eta_0$ | .373 | .121 | .219 | -.364 |
| 1350 | K^+K^- | .387 | -.070 | .113 | -.072 |
| | K^0K^0 | -.387 | -.070 | -.113 | .072 |
| | $\pi^0\eta_s$ | .096 | .093 | .234 | -.522 |
| | $\pi^0\eta_0$ | .358 | .091 | .233 | -.345 |
| 1700 | K^+K^- | .130 | -.182 | -.113 | -.387 |
| | K^0K^0 | -.130 | .182 | .113 | .387 |
| | $\pi^0\eta_s$ | .267 | .398 | .428 | .109 |
| | $\pi^0\eta_0$ | -.241 | -.001 | -.208 | .222 |
| 1700 | K^+K^- | .001 | .057 | .037 | .024 |
| | K^0K^0 | -.001 | -.057 | -.037 | -.024 |
| | $\pi^0\eta_s$ | .021 | -.045 | -.066 | -.059 |
| | $\pi^0\eta_0$ | .021 | .730 | -.651 | -.149 |
| 1800 | K^+K^- | .001 | -.253 | -.342 | .112 |
| | K^0K^0 | -.001 | -.253 | .342 | -.112 |
| | $\pi^0\eta_0$ | -.461 | .261 | .167 | .281 |
| | $\pi^0\eta_s$ | .302 | .166 | .115 | -.307 |
| 2050 | K^+K^- | .012 | .380 | -.314 | -.082 |
| | K^0K^0 | -.012 | -.380 | .314 | .082 |
| | $\pi^0\eta_s$ | .037 | .512 | -.471 | -.119 |
| | $\pi^0\eta_0$ | -.009 | -.026 | .016 | .013 |

Table 5. Recoupling coefficients for $q^2\bar{q}^2(J^P = 0^+)$ mesons into the pair of $(q\bar{q})(q\bar{q})$ mesons ($I=1/2$).

| m, MeV | f | 1 > c_s | 2 > c_s | 3 > c_s | 4 > c_s |
|--------|---------------|-----------|-----------|-----------|-----------|
| 970 | $K^+\pi^-$ | .575 | -.004 | -.074 | .367 |
| | $K^0\pi^0$ | -.407 | .003 | .052 | -.260 |
| | $\pi^0\eta_0$ | -.204 | .071 | .168 | -.458 |
| | $\pi^0\eta_s$ | .050 | .022 | .033 | -.059 |
| 1400 | $K^+\pi^-$ | .137 | .007 | .097 | -.331 |
| | $K^0\pi^0$ | -.097 | -.005 | -.069 | .234 |
| | $\pi^0\eta_0$ | .514 | .170 | .311 | -.359 |
| | $\pi^0\eta_s$ | -.338 | -.099 | -.207 | .328 |
| 1550 | $K^+\pi^-$ | -.108 | .443 | .453 | .271 |
| | $K^0\pi^0$ | .076 | -.313 | -.320 | -.191 |
| | $\pi^0\eta_0$ | .119 | -.330 | -.267 | -.228 |
| | $\pi^0\eta_s$ | -.102 | -.039 | -.054 | .100 |
| 1900 | $K^+\pi^-$ | .003 | .312 | -.257 | -.051 |
| | $K^0\pi^0$ | -.002 | -.221 | .182 | .036 |
| | $\pi^0\eta_0$ | .016 | .626 | -.580 | -.131 |
| | $\pi^0\eta_s$ | -.034 | -.039 | .006 | .038 |
| 2000 | $K^+\pi^-$ | .002 | .107 | .140 | -.202 |
| | $K^0\pi^0$ | -.001 | -.076 | -.099 | .143 |
| | $\pi^0\eta_0$ | .371 | .075 | .092 | -.226 |
| | $\pi^0\eta_s$ | .535 | .136 | .347 | -.515 |
| 2200 | $K^+\pi^-$ | -.005 | .021 | .000 | -.007 |
| | $K^0\pi^0$ | .003 | -.015 | .000 | .005 |
| | $\pi^0\eta_0$ | .014 | .022 | -.025 | -.012 |
| | $\pi^0\eta_s$ | .042 | .743 | -.645 | -.170 |

Table 6. Recoupling coefficients for $q^2\bar{q}^2(J^P = 0^+)$ mesons into the pair of $(q\bar{q})(q\bar{q})$ mesons ($I=0$).

| m, MeV | f | 1 > c_s | 2 > c_s | 3 > c_s | 4 > c_s |
|--------------|----------------|--------------|-----------|-----------|-----------|
| 800 | $\pi^+\pi^-$ | .394 | -.063 | -.162 | .513 |
| | $\pi^0\pi^0$ | .278 | -.044 | -.115 | .363 |
| | K^+K^- | .024 | .000 | .045 | -.050 |
| | K^0K^0 | .024 | .000 | .045 | -.050 |
| | $\eta_0\eta_0$ | -.540 | -.053 | -.028 | -.105 |
| | $\eta_0\eta_s$ | .083 | -.009 | .047 | -.034 |
| | $\eta_s\eta_s$ | .022 | .004 | .016 | -.021 |
| | 1140 | $\pi^+\pi^-$ | -.121 | -.078 | -.072 |
| $\pi^0\pi^0$ | | -.085 | -.055 | -.051 | -.002 |
| K^+K^- | | .425 | -.004 | -.010 | .183 |
| K^0K^0 | | .425 | -.004 | -.010 | .183 |
| | $\eta_0\eta_0$ | -.134 | -.038 | -.057 | .220 |
| | $\eta_0\eta_s$ | -.320 | .062 | .211 | -.528 |
| | $\eta_s\eta_s$ | -.114 | .047 | -.058 | .113 |
| | 1350 | $\pi^+\pi^-$ | -.091 | .472 | .409 |
| $\pi^0\pi^0$ | | -.064 | .334 | .290 | .202 |
| K^+K^- | | .038 | .016 | -.047 | .061 |
| K^0K^0 | | .038 | .016 | -.047 | .061 |
| | $\eta_0\eta_0$ | .094 | -.255 | -.336 | -.242 |
| | $\eta_0\eta_s$ | -.104 | .032 | -.029 | -.050 |
| | $\eta_s\eta_s$ | -.040 | -.003 | -.032 | .037 |
| | 1600 | $\pi^+\pi^-$ | .200 | -.096 | .259 |
| $\pi^0\pi^0$ | | .142 | -.068 | .183 | -.024 |
| K^+K^- | | .169 | -.090 | .151 | -.141 |
| K^0K^0 | | .169 | -.090 | .151 | -.141 |
| | $\eta_0\eta_0$ | .267 | -.285 | .502 | -.289 |
| | $\eta_0\eta_s$ | .187 | -.033 | .309 | -.165 |
| | $\eta_s\eta_s$ | .043 | -.071 | .104 | -.027 |
| | 1700 | $\pi^+\pi^-$ | .215 | .231 | -.185 |
| $\pi^0\pi^0$ | | .152 | .164 | -.131 | -.087 |
| K^+K^- | | .130 | .080 | .108 | -.067 |
| K^0K^0 | | .130 | .080 | .108 | -.067 |
| | $\eta_0\eta_0$ | .215 | .602 | -.248 | -.385 |
| | $\eta_0\eta_s$ | .152 | -.008 | .047 | -.232 |
| | $\eta_s\eta_s$ | -.004 | -.001 | -.002 | .004 |

| m, MeV | f | 1 >_{cs} | 2 >_{cs} | 3 >_{cs} | 4 >_{cs} |
|----------------|----------------|----------|----------|----------|----------|
| 1700 | $\pi^+\pi^-$ | .047 | -.053 | .052 | .006 |
| | $\pi^0\pi^0$ | .033 | -.037 | .037 | .004 |
| | K^+K^- | -.092 | .291 | .270 | .272 |
| | K^0K^0 | -.092 | .291 | .270 | .272 |
| | $\eta_0\eta_0$ | .029 | -.094 | .139 | -.038 |
| | $\eta_0\eta_s$ | .006 | -.461 | -.444 | -.262 |
| | $\eta_s\eta_s$ | .013 | .010 | -.006 | -.015 |
| 1950 | $\pi^+\pi^-$ | .138 | -.081 | .040 | -.011 |
| | $\pi^0\pi^0$ | .098 | -.057 | .028 | -.008 |
| | K^+K^- | .049 | .314 | -.301 | -.079 |
| | K^0K^0 | .049 | .314 | -.301 | -.079 |
| | $\eta_0\eta_0$ | .072 | -.058 | .227 | -.127 |
| | $\eta_0\eta_s$ | .008 | .494 | -.368 | -.125 |
| | $\eta_s\eta_s$ | -.140 | .106 | -.221 | .110 |
| 2100 | $\pi^+\pi^-$ | -.203 | .023 | .018 | .013 |
| | $\pi^0\pi^0$ | -.144 | .017 | .013 | .009 |
| | K^+K^- | .099 | .198 | .047 | -.201 |
| | K^0K^0 | .099 | .198 | .047 | -.201 |
| | $\eta_0\eta_0$ | -.089 | -.091 | -.151 | .218 |
| | $\eta_0\eta_s$ | .321 | .167 | -.069 | -.218 |
| 2350 | $\eta_s\eta_s$ | .445 | .091 | .310 | -.425 |
| | $\pi^+\pi^-$ | -.029 | .001 | .016 | -.002 |
| | $\pi^0\pi^0$ | -.021 | .001 | .011 | -.001 |
| | K^+K^- | .024 | -.065 | .090 | -.031 |
| | K^0K^0 | .024 | -.065 | .090 | -.031 |
| | $\eta_0\eta_0$ | -.000 | -.028 | -.005 | .025 |
| | $\eta_0\eta_s$ | .054 | -.089 | .125 | -.011 |
| 2600 | $\eta_s\eta_s$ | .061 | .726 | -.613 | -.185 |
| | $\pi^+\pi^-$ | .183 | -.012 | -.011 | -.016 |
| | $\pi^0\pi^0$ | .130 | -.008 | -.008 | -.011 |
| | K^+K^- | .012 | -.093 | -.195 | .254 |
| | K^0K^0 | .012 | -.093 | -.195 | .254 |
| | $\eta_0\eta_0$ | .093 | .087 | .132 | -.207 |
| $\eta_0\eta_s$ | -.454 | .038 | -.034 | .068 | |
| $\eta_s\eta_s$ | .422 | .169 | .217 | -.417 | |

Table 7. Recoupling coefficients for $q^2\bar{q}^2$ mesons into pair of vector color-singlet VV' and color-octet VV' states.

| | VV' | VV' |
|--------|--------------|---------------|
| 9_f | $\sqrt{2/3}$ | $-\sqrt{1/3}$ |
| 36_f | $\sqrt{1/3}$ | $\sqrt{2/3}$ |

Table 8. Recoupling coefficients in flavor for tensor mesons into vector mesons.

| flavor st. | $\rho^+\rho^-$ | $\rho^0\rho^0$ | $K^{*+}K^{*-}$ | $K^{*0}K^{*0}$ | $\omega\omega$ | $\omega\phi$ | $\rho^0\phi$ | $\rho^0\omega$ | $\phi\phi$ |
|-----------------|----------------|----------------|----------------|----------------|----------------|---------------|--------------|----------------|------------|
| $C^0(9_f)$ | $\sqrt{1/2}$ | 1/2 | 0 | 0 | -1/2 | 0 | 0 | 0 | 0 |
| $C^*(9_f)$ | 0 | 0 | 1/2 | 1/2 | 0 | $-\sqrt{1/2}$ | 0 | 0 | 0 |
| $C_\pi^*(9_f)$ | 0 | 0 | -1/2 | 1/2 | 0 | 0 | $\sqrt{1/2}$ | 0 | 0 |
| $E(36_f)$ | $-\sqrt{1/3}$ | $\sqrt{2/3}$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| $C_\pi^0(36_f)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 | 0 |
| $C^0(36_f)$ | $\sqrt{1/6}$ | $\sqrt{1/12}$ | 0 | 0 | $\sqrt{3/4}$ | 0 | 0 | 0 | 0 |
| $C_\pi^*(36_f)$ | 0 | 0 | 1/2 | -1/2 | 0 | 0 | $\sqrt{1/2}$ | 0 | 0 |
| $C^*(36_f)$ | 0 | 0 | 1/2 | 1/2 | 0 | $\sqrt{1/2}$ | 0 | 0 | 0 |
| $C^{**}(36_f)$ | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |

Table 9. The spin recoupling coefficients $\langle N\bar{N} | (q^2\bar{q}^2)(q\bar{q}) \rangle_s$.

| J_{total}^{pc} | $(q^2\bar{q}^2, q\bar{q})$ | TV | SV | VS | SS | VV |
|---------------------|------------------------------------|-------------------------------|------------------------|----------------------|----------------------|----------------------|
| | | 2+1- | 0+1- | 1+0- | 0+0- | 1+1- |
| 1-(3 _s) | (9 _s , 4 _s) | $\frac{\sqrt{20}}{3\sqrt{3}}$ | $-\frac{1}{3\sqrt{3}}$ | $\frac{\sqrt{2}}{3}$ | 0 | 0 |
| 0-(1 _s) | (9 _s , 4 _s) | 0 | 0 | 0 | $\frac{1}{\sqrt{3}}$ | $\sqrt{\frac{2}{3}}$ |
| 1-(3 _s) | (3 _s , 4 _s) | 0 | 0 | $\frac{1}{\sqrt{3}}$ | 0 | $\sqrt{\frac{2}{3}}$ |
| 0-(1 _s) | (3 _s , 4 _s) | 0 | 0 | 0 | 0 | 1 |
| 1-(3 _s) | (1 _s , 4 _s) | 0 | 1 | 0 | 0 | 0 |
| 0-(1 _s) | (1 _s , 4 _s) | 0 | 0 | 0 | 1 | 0 |

Table 10. The ratios between production channels of the $a_0(980)$ meson in $N\bar{N}$ reaction, $\cos^2\theta \sim 2/3$.

| | $p\bar{p} \rightarrow a_0\rho$ | $p\bar{p} \rightarrow a_0\eta$ | $p\bar{p} \rightarrow a_0\omega$ | $\bar{p}n \rightarrow a_0^-\eta$ | $\bar{p}n \rightarrow a_0^-\omega$ |
|---|--------------------------------|--------------------------------|----------------------------------|----------------------------------|------------------------------------|
| $\frac{\sigma(N\bar{N} \rightarrow a_0\pi)}{\sigma(p\bar{p} \rightarrow a_0\pi)}$ | 1/3 | $\cos^2\theta$ | 1/3 | $2\cos^2\theta$ | 2/3 |

Table 11. The ratios between production channels of the tensor mesons in $N\bar{N}$ reaction.

| | $\frac{\sigma(N\bar{N} \rightarrow TV)}{\sigma(\bar{p}n \rightarrow E^-\rho^+)}$ | | $\frac{\sigma(N\bar{N} \rightarrow TV)}{\sigma(\bar{p}n \rightarrow E^-\rho^+)}$ |
|--------------------------------------|--|--------------------------------------|--|
| $\bar{p}n \rightarrow E^-\rho^0$ | 1/2 | $\bar{p}n \rightarrow C_\pi^-\omega$ | 1/2 |
| $\bar{p}p \rightarrow E^-\rho^+$ | 1/4 | $\bar{p}p \rightarrow C_\pi^-\rho^+$ | 1/4 |
| $\bar{p}p \rightarrow E^+\rho^-$ | 1/4 | $\bar{p}p \rightarrow C_\pi^+\rho^-$ | 1/4 |
| $\bar{p}p \rightarrow C_\pi^0\rho^0$ | 1/4 | $\bar{p}p \rightarrow C_\pi^0\omega$ | 1/4 |

The states from (2.5), (2.6) form together the basis of states with definite color-spin-flavor content. In Tables 4, 5, 6 the recoupling coefficients of physical $q^2\bar{q}^2$ states in the mesonic pair $|(q\bar{q})(q\bar{q})\rangle$ states are presented.

The $q^2\bar{q}^2$ mesons decay into a pair of $q\bar{q}$ mesons through the OZI - superallowed diagram (Fig. 1). The width of the decay is determined by:

$$\Gamma_{C_i \rightarrow mm'}(s) = \frac{|\langle C_i | mm' \rangle|^2}{16\pi M_i} F_{mm'}(s),$$

$$\langle C_i | mm' \rangle = g_0 \langle C_i | mm' \rangle_c \langle C_i | mm' \rangle_s \langle C_i | mm' \rangle_f \quad (2.7)$$

where $\langle C_i | mm' \rangle_c, \langle C_i | mm' \rangle_s, \langle C_i | mm' \rangle_f$ are the recoupling coefficients for a four-quark C_i state with a pair of mesons MM' for color, spin and flavor, respectively, and are collected in Tables 4, 5, 6. In [25], [26] in order to describe the signals of resonance type in the $\gamma\gamma \rightarrow VV'$ reaction the four-quark states were used.

2.2 Vector and tensor mesons

As it was shown in details in [22] the spectrum of $J^P = 1^+ q^2\bar{q}^2$ mesons has the following features. The main effect of the instanton interaction is that unlike the one gluon exchange interaction it mixes multiplets: 18_f with $\bar{1}8_f$ and $1S_b$ with 36_f . Further, the states with different isotopic spins which are irreducible in masses in the MIT model are splitted by the instanton mechanism ($\sim 50-200$ Mev).

At the same time for tensor mesons the instanton interaction practically do not change the results of the MIT model. In Tables 7, 8 the color-spin and flavor recoupling coefficients for $q^2\bar{q}^2$ mesons into the pair of vector mesons VV' are presented.

So, we obtain (Table 1-8) the basis of physical states of scalar, vector and tensor four - quark mesons and their mass spectrum within the quark model with QCD vacuum induced quark interaction.

3 Four-quark states in $N\bar{N}$ annihilation

The production of $q^2\bar{q}^2$ mesons may go according to the OZI - allowed diagram of Fig. 2a. In this process the annihilating quark - antiquark pair has vacuum quantum numbers: ${}^3P_0, J^{PC} = 0^{++}$. Then it follows that the P-wave of $N\bar{N}$ process will be dominate in this diagram. Due to this fact this reaction may be used as a good filter for $q^2\bar{q}^2$ states.

There exists also alternative way of $q^2\bar{q}^2$ meson production. In Fig. 2b it is shown the OZI - superallowed diagram which is enhanced as compared with the diagram of Fig. 2a. Moreover, the processes in Fig. 2b may go in the S-wave with $q\bar{q}$ meson being in the $J^{PC} = 0^{-+}$ or 1^{-+} state and $q^2\bar{q}^2$ meson being in $0^{++}, 1^{++}$ or

2^{++} state. From Fig. 2b it is seen that the diquark from a nucleon with the anti-diquark from an antinucleon produce the $q^2\bar{q}^2$ state which may be easily presented as a superposition of known basis states: The remaining quark from the nucleon and antiquark from the antinucleon may be also easily combined into usual meson states. Then, the dynamics of the interaction reduces to the vertex of Fig. 2c. The formula for the cross-section of the process of $N\bar{N}$ annihilation into mesons is the following:

$$d\sigma(N\bar{N} \rightarrow mm') \sim \left| \sum_i \frac{\langle N\bar{N} | C_i m'' \rangle \langle C_i m'' | mm' m'' \rangle}{m_i^2 - s - i\sqrt{s}\Gamma_i(s)} \right|^2, \quad (3.1)$$

where C_i is four - quark state, m is meson state, m_i and $\Gamma_i(s)$ are mass and width of C_i , $\langle C_i m'' | mm' m'' \rangle \sim \langle C_i | mm' \rangle$. To calculate it we should know the matrix elements $\langle N\bar{N} | C_i m'' \rangle$. First of all, let us consider the wave function of a nucleon as a q^2q system :

$$|N\rangle = |q^2q\rangle = |(q_c^2 q_s^2 q_f)(q_c q_s q_f)\rangle,$$

$$|N\rangle = \frac{1}{\sqrt{2}} (|(\bar{3}_c 3_s 6_f)(3_c 2_s 3_f)\rangle + |(\bar{3}_c 1_s 3_f)(3_c 2_s 3_f)\rangle), \quad (3.2)$$

$$|\bar{N}\rangle = \frac{1}{\sqrt{2}} (|(\bar{3}_c \bar{3}_s \bar{6}_f)(\bar{3}_c \bar{2}_s \bar{3}_f)\rangle + |(\bar{3}_c 1_s 3_f)(\bar{3}_c 2_s \bar{3}_f)\rangle).$$

From the group - theoretical point of view the wave function of the $N\bar{N}$ system may be decomposed in a complete set of $|q^2\bar{q}^2, q\bar{q}\rangle$ states. From (3.2) one can easily obtain expanded: ¹

$$|N\bar{N}\rangle = \sum_i \langle N\bar{N} | (q^2\bar{q}^2)(q\bar{q}) \rangle_i | (q^2\bar{q}^2)_f, (q\bar{q})_f \rangle_i | (q^2\bar{q}^2)_s, (q\bar{q})_s \rangle_i \quad (3.3)$$

$$= \frac{1}{2} (|36_f, 9_f\rangle |9_s, 4_s\rangle + \sqrt{2}|18_f, 9_f\rangle |3_s, 4_s\rangle + |9_f, 9_f\rangle |1_s, 4_s\rangle).$$

The total spin of the $|(q^2\bar{q}^2)_s, (q\bar{q})_s\rangle$ system is defined by the total spin of $N\bar{N}$ which may be $J^P = 0^-, 1^-(1_s, 3_s)$. In Table 9 we present the spin recoupling coefficients $\langle N\bar{N} | (q^2\bar{q}^2)(q\bar{q}) \rangle_s$.

The flavor recoupling of the $p\bar{p}$ system into the $|(q^2\bar{q}^2)_f, (q\bar{q})_f\rangle$ states has the following form:

$$|36_f, 9_f\rangle = \frac{1}{3}|E_{\pi\pi}^+ \pi^- \rangle + \frac{1}{3}|C_{\pi\pi}^+ \pi^- \rangle - \frac{2}{3\sqrt{3}}|E_{\pi\pi}^0 \pi^0 \rangle - \frac{1}{3}|C_{\pi\pi}^0 \pi^0 \rangle - \frac{1}{3\sqrt{6}}|C_{\pi^0} \rangle$$

$$+ \frac{1}{3}|E_{\pi\pi}^- \pi^+ \rangle + \frac{1}{3}|C_{\pi\pi}^- \pi^+ \rangle + \frac{1}{3}|C_{\pi\eta} \rangle + \frac{1}{\sqrt{6}}|C_{\eta} \rangle, \quad (3.4)$$

¹The color decomposition is trivial, so we do not present it here.

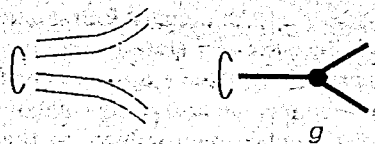


Figure 1. The diagrams of $q^2 \bar{q}^2$ meson decay

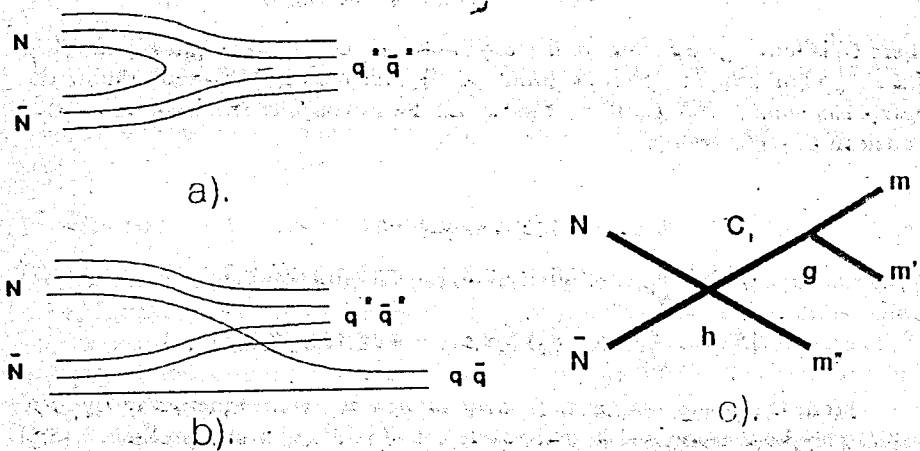


Figure 2. The diagrams of $q^2 \bar{q}^2$ meson production in $N\bar{N}$ annihilation a) the OZI - allowed diagram, b) the OZI - superaligned diagram; c) the vertex corresponding to b).

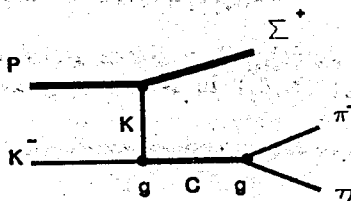


Figure 3. The diagram of $a_0(980)$ -meson production in pK^- scattering.

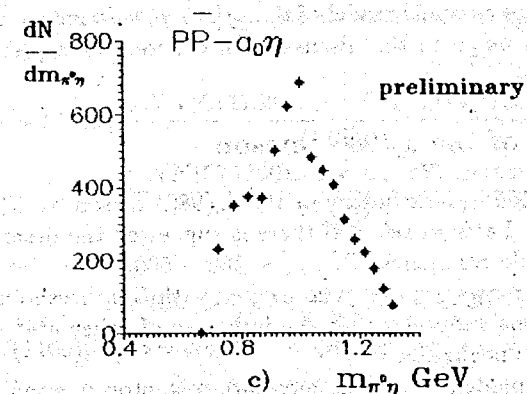
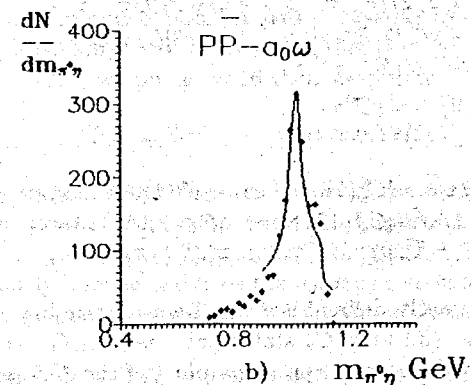
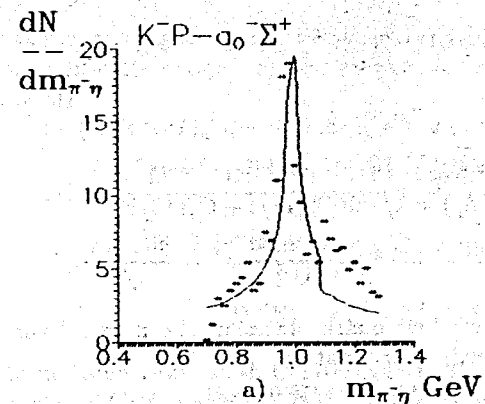


Figure 4. The mass spectrum of the $\pi\eta$ system in reactions a) $K^- p \rightarrow (C(1100) + C(1350))\Sigma^+ \rightarrow (\pi^-\eta)\Sigma^+$ and b) $p\bar{p} \rightarrow a_0\omega$, c) $p\bar{p} \rightarrow a_0\eta$.

$$|9_f, 9_f \rangle = 1/\sqrt{2}(-|C^0 \eta \rangle + |C^0 \pi^0 \rangle), \quad (3.5)$$

$$|18_f, 9_f \rangle = 1/\sqrt{6}(|\bar{C}_\pi^0(18_f)\eta \rangle - |\bar{C}_\pi^0(18_f)\pi^0 \rangle) - \quad (3.6)$$

$$-1/\sqrt{3}(|C_\pi^+(18_f)\pi^- \rangle + |C_\pi^-(18_f)\pi^+ \rangle),$$

$$\bar{C}_\pi^0(18_f) = 1/\sqrt{2}(C_\pi^0(18_f) + C_\pi^0(\bar{1}8_f)),$$

$$\eta = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}, \pi^0 = \frac{d\bar{d} - u\bar{u}}{\sqrt{2}}.$$

Here $E_{\pi\pi}, C_\pi, C$ are four-quark states in the magic basis, and sign of $E_{\pi\pi}^\pm, C_\pi^\pm, \pi^\pm$ mean the charge of the state.

For the $\bar{p}n$ system we have the expansions:

$$|36_f, 9_f \rangle = \sqrt{2/27}|E_{\pi\pi}^0 \pi^- \rangle + 1/\sqrt{27}|C_\pi^- \rangle - \quad (3.7)$$

$$-\sqrt{2/3}|E_{\pi\pi}^- \pi^0 \rangle + \sqrt{2/3}|C_\pi^- \eta \rangle +$$

$$+2/3|E_{\pi\pi}^- \pi^+ \rangle,$$

$$|9_f, 9_f \rangle = |C^0 \pi^- \rangle, \quad (3.8)$$

$$|18_f, 9_f \rangle = 1/\sqrt{6}(-|C_\pi^0(18_f)\pi^- \rangle + |C_\pi^0(\bar{1}8_f)\pi^- \rangle) + \quad (3.9)$$

$$+1/\sqrt{3}(|C_\pi^-(18_f)\eta \rangle - |C_\pi^-(\bar{1}8_f)\eta \rangle) -$$

$$-1/\sqrt{3}(|C_\pi^-(18_f)\pi^0 \rangle + |C_\pi^-(\bar{1}8_f)\pi^0 \rangle).$$

The $\bar{p}n$ system has exactly defined isospin quantum numbers $I=1, I_z = -1$ and thus in the expressions (3.9) the C_π states have definite G-parity.

So we obtained color, flavor, and spin recoupling of the $N\bar{N}$ wave functions into the $(q^2\bar{q}^2)(q\bar{q})$ wave functions based only on group - theoretical considerations. It allows us to formulate a simple model of the $(q^2\bar{q}^2)(q\bar{q})$ meson production in the $N\bar{N}$ annihilation. Let us go to the discussion of concrete examples from meson spectroscopy.

4 The problem of the $a_0(980)$ meson

The indication of four-quark nature of the $a_0(980)$ meson as $C_\pi^+(9_f)$ state is yet given in article [1]. Later in ref. [24] there is suggested the description of the $a_0(980)$ meson as a wide resonance ($\Gamma_{a_0 \rightarrow \pi\eta} \sim 300 - 500 MeV$). In this case the narrow peak in the $\pi\eta$ mas spectrum is connected with the threshold influence of the $K\bar{K}$ channel. It was supposed that the influence of other $q^2\bar{q}^2$ states of the MIT-model spectrum is negligible.

However in our model, as a consequence of instanton mechanism, in order to describe the $a_0(980)$ meson there are essential two states with masses $m_{theor} = 1100 MeV$ and $m_{theor} = 1350 MeV$ (Table 1). In Fig. 4a there is presented the

mass spectrum of the channel $\pi\eta$ in the reaction $K^-p \rightarrow (C(1100)+C(1350))\Sigma^+ \rightarrow (\pi^-\eta)\Sigma^+$ which is practically defined by the amplitude of $\langle K^-K^0|C(1100)+C(1350)|\pi^-\eta \rangle$ (Fig. 3):

$$\frac{dN}{dm_{\pi\eta}} \sim 1/16\pi \left| \frac{\langle K^-K^0|C(1100) \rangle \langle C(1100)|\pi\eta \rangle}{D_c(E)} + \frac{\langle K^-K^0|C(1350) \rangle \langle C(1350)|\pi\eta \rangle}{D_c(E)} \right|^2, \quad (4.1)$$

$$D_c(E) = m_c^2 - E^2 - iE\Gamma_c(E).$$

It follows from Table 1 that both $C(1100)$ and $C(1350)$ have a considerable coupling with the states $C_\pi^+(9_f)$ and $C_\pi(36_f)$. The coupling of 36_f with vector mesons as compared with 9_f is 20 times stronger. The use of simple considerations of the vector dominance model makes it possible to obtain the experimental value of the width $a_0(980) \rightarrow (C(1100)+C(1350)) \rightarrow \gamma\gamma$ [28] without additional parameters. If in (2.7) we put $g_0 = 16$ then we obtain

$$\Gamma_{a_0 \rightarrow \gamma\gamma} Br(a_0 \rightarrow \pi^0\eta) \approx 0.19 K eV. \quad (4.2)$$

In the experiment [30] there was observed the $a_0(980)$ meson in the $p\bar{p} \rightarrow a_0\pi, a_0\omega, a_0\eta$ reactions. In this connection it is important to note that in the $N\bar{N}$ recoupling there is absent nonet 9_f state with isospin $I = 1$ ab the MIT model- C_π^+ which was previously interpretative as the $a_0(980)$ meson in ref. [1], [24]. It means that $a_0(980)$ can not be observed in $N\bar{N}$ annihilation within suggestions of ref. [1], [24].

In our model the $a_0(980)$ meson is strongly coupled with the $C_\pi(36_f)$ state, which is present in the $N\bar{N}$ decomposition (3.4), (3.7). In Figs. 4b, 4c there are shown the mass spectrum of the $\pi\eta$ system in the reactions $p\bar{p} \rightarrow a_0\omega, p\bar{p} \rightarrow a_0\eta$ which is defined by:

$$\frac{dN}{dm_{\pi\eta\omega}} \sim 1/16\pi \left| \frac{\langle N\bar{N}|C(1100)\omega \rangle \langle C(1100)\omega|\pi\eta\omega \rangle}{D_c(E)} + \frac{\langle N\bar{N}|C(1350)\omega \rangle \langle C(1350)\omega|\pi\eta\omega \rangle}{D_c(E)} \right|^2, \quad (4.3)$$

$$\langle N\bar{N}|C\omega \rangle \sim 1/2\left(\frac{1}{2.3}\right), 1/3 \langle C|36_f \rangle h_0, \quad (4.4)$$

$$\langle C(1100)|36_f \rangle = 0.577, \quad \langle C(1350)|36_f \rangle = 0.556, \quad (4.5)$$

$$\langle C\omega|\pi\eta\omega \rangle \approx \langle C|\pi\eta \rangle,$$

$$\langle C(1100)|\pi\eta \rangle = 0.043g_0, \quad \langle C(1350)|\pi\eta \rangle = 0.237g_0, \quad (4.6)$$

where the coefficients in (4.5) are taken from Table 1 and (4.6) are from Table 4. In (4.4) the unpolarized initial nucleons are used:

$$|N\bar{N}\rangle = \frac{1}{2}(\sqrt{3} + 3_s + 1_s). \quad (4.7)$$

Using only the recoupling (3.4), (3.7) we can predict $a_0(980)$ meson production cross-section ratios for different reaction channels (Table 10).

So in the framework of our approach a satisfactory description of the existing experimental data for production $a_0(980)$ and width $a_0(980) \rightarrow \gamma\gamma$ is achieved and the predictions for the cross-sections in $N\bar{N}$ annihilation is obtained. The properties of this meson are unusual due to its two state $C(1100)$, $C(1350)$ description and strong coupling with $C_\pi^*(9_f)$ and $C_\pi(36_f)$.

5 Tensor $q^2\bar{q}^2$ mesons in $N\bar{N}$ scattering

The production of $q^2\bar{q}^2$ tensor mesons is possible only in an accompaniment with the vector meson ($N\bar{N} \rightarrow TV$). The most convincing manifestation of tensor mesons must become observation of the exotic meson $E_{\pi\pi}(36_f)$ in the reaction

$$\bar{p}n \rightarrow E_{\pi\pi}^{--}\rho^+ \rightarrow \rho^-\rho^-\rho^+ \rightarrow \pi^-\pi^0\pi^-\pi^0\pi^+\pi^0. \quad (5.1)$$

with a vertex:

$$\langle \bar{p}n | E^{--}\rho^+ \rangle = 1/2 \frac{\sqrt{20}}{3\sqrt{3}} 2/3 h_0. \quad (5.2)$$

with coupling h_0 . Let us numerate other interesting channels of tensor meson production:

$$\begin{aligned} \bar{p}n &\rightarrow E_{\pi\pi}^-\rho^0 \rightarrow (\rho^-\rho^0)\rho^0 \rightarrow (\pi^-\pi^0\pi^+\pi^-)(\pi^+\pi^-), \\ \bar{p}p &\rightarrow E_{\pi\pi}^-\rho^+ \rightarrow (\rho^-\rho^0)\rho^+, \end{aligned} \quad (5.3)$$

$$\begin{aligned} \bar{p}p &\rightarrow E_{\pi\pi}^+\rho^- \rightarrow (\rho^+\rho^0)\rho^-, \\ \bar{p}n &\rightarrow C_\pi^-\omega \rightarrow (\rho^-\omega)\omega, \\ \bar{p}p &\rightarrow C_\pi^-\rho^+ \rightarrow (\rho^-\omega)\rho^+, \end{aligned} \quad (5.4)$$

$$\begin{aligned} \bar{p}p &\rightarrow C_\pi^+\rho^- \rightarrow (\rho^+\omega)\rho^-, \\ \bar{p}p &\rightarrow C_\pi^0\omega \rightarrow (\rho^0\omega)\omega, \\ \bar{p}p &\rightarrow C_\pi^0\rho^0 \rightarrow (\rho^0\omega)\rho^0. \end{aligned}$$

The ratios between different $q^2\bar{q}^2$ production channels are presented in Table 11.

From (4.4), (5.2) and (4.7) one can estimate the relative yield of the E^{--} mesons:

$$\frac{\sigma(\bar{p}p \rightarrow a_0\pi)}{\sigma(\bar{p}n \rightarrow E^{--}\rho^+)} \leq \frac{3}{80}. \quad (5.5)$$

The mass spectrum of the $\rho\rho$ system for the reaction like (5.3) has the form:

$$\frac{dN}{dm_{\rho\rho'}} \sim F_{\rho\rho'}/\pi \left| \frac{\langle N\bar{N} | E\rho \rangle \langle E | VV' \rangle}{m_E^2 - s - i(\sqrt{s}\Gamma(s) + a/2)} \right|^2, \quad (5.6)$$

$$F_{\rho\rho'} = 1/\pi^2 \int_{4m_\pi^2}^{(\sqrt{s}-2m_\pi)^2} dm^2 \frac{m\Gamma_\rho(m)}{|D_\rho(m)|^2} \int_{4m_\pi^2}^{(\sqrt{s}-2m_\pi)^2} dm'^2 \frac{m'\Gamma_\rho(m')}{|D_\rho(m')|^2} \rho(s, m, m'),$$

$$m\Gamma_\rho(m) = m_\rho\Gamma_\rho m_\rho/m \left(\frac{q(m)}{q(m_\rho)} \right)^3 \frac{1 + (Rq(m_\rho)^2)}{1 + (Rq(m))^2},$$

$$q(m) = 1/2\sqrt{m^2 - 4m_\pi^2}, D_\rho(m) = m_\rho^2 - m^2 - im\Gamma_\rho(m),$$

$$s = m_{\rho\rho}^2, R = 2GeV^{-1}, m_\rho = 0.321, \Gamma_\rho = 0.154GeV,$$

$$\Gamma_{E \rightarrow VV'}(s) = \frac{\langle E | VV' \rangle^2}{16\pi\sqrt{s}} F_{VV'}(s), \Gamma_E(s) = \sum_{VV'} \Gamma_{EVV'}, \quad (5.7)$$

$$\langle E | \rho\rho \rangle = g_0 \frac{1}{\sqrt{3}}.$$

and corresponding formula for the $\rho\omega$ mass spectrum of the system for the reactions (5.4) has the form:

$$\frac{dN}{dm_{\rho\omega}} \sim F_\rho/\pi \left| \frac{\langle N\bar{N} | C_\pi V \rangle \langle C_\pi | VV' \rangle}{m_{C_\pi}^2 - s - i(\sqrt{s}\Gamma(s) + a/2)} \right|^2, \quad (5.8)$$

$$F_\rho(s) = 1/\pi^2 \int_{4m_\pi^2}^{(\sqrt{s}-2m_\omega)^2} dm^2 \frac{m\Gamma_\rho(m)}{|D_\rho(m)|^2} \rho(s, m_\omega, m),$$

$$\langle C_\pi | \rho\omega \rangle = g_0 \frac{1}{\sqrt{3}},$$

$$\Gamma_{C_\pi \rightarrow \rho\omega}(s) = \frac{\langle C_\pi | \rho\omega \rangle^2}{16\pi\sqrt{s}} F_\rho(s).$$

The parameters m_E, m_{C_π}, g_0, a may be extracted from the data of the reaction $\gamma\gamma \rightarrow q^2\bar{q}^2 \rightarrow \rho\rho, \rho\omega$. In article [25] it was shown that $m_E \approx 1.44GeV, m_{C_\pi} =$

1.4GeV, $g_0^2/(4\pi) \approx 16.4, a \approx 0.65$. However, a partial wave analysis of $\gamma\gamma \rightarrow \rho\rho$ reaction can change these results.

As to scalar charged $q^2\bar{q}^2$ mesons then their production is suppressed as compared to that of tensor mesons by 20 times. It is enhanced only in an accompaniment with scalar meson (for example in the reaction $N\bar{N} \rightarrow E\pi \rightarrow \rho\rho\pi^1$).

The situation for neutral mesons is more difficult. The $N\bar{N}$ system is strongly coupled with scalar mesons from the nonet $C^0(9_f)$ and with tensor mesons from 36_f (E^0, C). We obtain for neutral $q^2\bar{q}^2$ mesons²:

$$\bar{p}n \rightarrow (E_{\pi\pi}^0 + C(36_f) + C(9_f))\rho^- \rightarrow \begin{cases} (\rho^+\rho^-)\rho^- \\ (\rho^0\rho^0)\rho^- \\ (\omega\omega)\rho^- \end{cases}, \quad (5.9)$$

$$\bar{p}p \rightarrow (E_{\pi\pi}^0 + C(36_f) + C(9_f))\rho^0 \rightarrow \begin{cases} (\rho^+\rho^-)\rho^0 \\ (\rho^0\rho^0)\rho^0 \\ (\omega\omega)\rho^0 \end{cases}, \quad (5.10)$$

$$\bar{p}p \rightarrow (C(36_f) + C(9_f))\omega \rightarrow \begin{cases} (\rho^+\rho^-)\omega \\ (\rho^0\rho^0)\omega \\ (\omega\omega)\omega \end{cases} \quad (5.11)$$

The mass spectrum of the $\rho\rho$ system in the reactions (5.9), (5.10) has the form:

$$\begin{aligned} \frac{dN}{dm_{\rho\rho}} &\sim F_{\rho\rho}/\pi \left| \frac{\langle N\bar{N}|E_{\pi\pi}V^n \rangle \langle E|\rho\rho \rangle}{D_E} + \right. \\ &+ \frac{\langle N\bar{N}|C(36_f)V^n \rangle \langle C(36_f)|\rho\rho \rangle}{D_{C(36_f)}} + \\ &+ \left. \frac{\langle N\bar{N}|C(9_f)V^n \rangle \langle C(9_f)|\rho\rho \rangle}{D_{C(9_f)}} \right|^2, \end{aligned} \quad (5.12)$$

where

$$\langle E|\rho\rho \rangle = g_0 \frac{1}{\sqrt{3}} \begin{cases} -1/\sqrt{3}, & \text{for } \rho^+\rho^- \\ \sqrt{2/3}, & \text{for } \rho^0\rho^0, \end{cases} \quad (5.13)$$

$$\langle C(36_f)|\rho\rho \rangle = g_0 \frac{1}{\sqrt{3}} \begin{cases} 1/\sqrt{6}, & \text{for } \rho^+\rho^- \\ 1/\sqrt{12}, & \text{for } \rho^0\rho^0 \\ \sqrt{\frac{3}{4}}, & \text{for } \omega\omega, \end{cases} \quad (5.14)$$

¹some results for these reaction was presented in ref. [31], [32]

²The scalar neutral meson $C(36_f)$ is neglected due to its weak spin coupling.

$$\langle C(1350)|\rho\rho \rangle = g_0 \begin{cases} 0.472, & \text{for } \rho^+\rho^- \\ 0.334, & \text{for } \rho^0\rho^0 \\ -0.255, & \text{for } \omega\omega, \end{cases} \quad (5.15)$$

$$\langle p\bar{p}|TV \rangle = h_0 \frac{1}{2} \frac{\sqrt{20}}{3\sqrt{3}} \begin{cases} -\frac{2}{3\sqrt{3}}, & \text{for } E^0(36_f)\rho^0 \\ -\frac{1}{3\sqrt{6}}, & \text{for } C(36_f)\rho^0 \\ +\frac{1}{3\sqrt{6}}, & \text{for } C(36_f)\omega, \end{cases} \quad (5.16)$$

$$\langle p\bar{p}|SV \rangle = h_0 \frac{1}{2} \langle C(9_f)|C(1350) \rangle \begin{cases} \frac{1}{\sqrt{2}}, & \text{for } C(9_f)\rho^0 \\ -\frac{1}{\sqrt{2}}, & \text{for } C(9_f)\omega, \end{cases} \quad (5.17)$$

$$\langle \bar{p}n|TV \rangle = h_0 \frac{1}{2} \frac{\sqrt{20}}{3\sqrt{3}} \begin{cases} \frac{2}{\sqrt{27}}, & \text{for } E^0(36_f)\rho^- \\ \frac{1}{\sqrt{27}}, & \text{for } C(36_f)\rho^-, \end{cases} \quad (5.18)$$

$$\langle p\bar{p}|SV \rangle = h_0 \frac{1}{2} \langle C(9_f)|C(1350) \rangle \cdot 1, \quad \text{for } C(9_f)\rho^-. \quad (5.19)$$

In the above expressions all numerical coefficients are taken from Tables 8,9 and Eqs. (3.3)- (3.8). In (5.15), (5.17), (5.19) we have the $C(1350)$ as an example of scalar meson. These formula provide a set of predictions on the observation of the tensor mesons in the $N\bar{N}$ annihilation. Thus, in $N\bar{N}$ annihilation we have a good filter for $0^+, 2^+ q^2\bar{q}^2$ mesons.

CONCLUSION

The spectrum of $q^2\bar{q}^2$ states in the model, which takes into account the interaction between quarks through the nonperturbative vacuum of QCD, has been calculated. Nonperturbative vacuum of QCD is determined by two parts. The first one takes into account the long wave vacuum components of quark and gluon fields (the quark and gluon condensates). The second one is constructed with the effective Lagrangian t'Hooft, describing quark interaction through the instantons of small sizes. The account of these two aspects of the vacuum interaction allows to formulate and study a bag model, which gives a good description of hadron spectrum. The spectrum of multi-quark states is of a particular interest. Among a great number of the $q^2\bar{q}^2$ states one can find some states having the similar quantum numbers which in general case must be mixed. The presented approach

allows to calculate the mixing angles. These calculations give concrete predictions for the cross sections of production and the widths of decay of the $q^2\bar{q}^2$ mesons.

A searching of the multiquark states frequently demands a formulation of features, which allow to distinguish these states from the hybrid, glueball and other ones. The analysis of nucleon-antinucleon annihilation to mesons allows, from our viewpoint, very easy to proceed the identification in the most of four-quark states. In particular, one has obtained a satisfactory description of a scalar $a_0(980)$ meson ($N\bar{N} \rightarrow a_0\omega$ data). The nature of the $a_0(980)$ meson can be clarified by measuring the cross section of this meson production in the different annihilation channels $N\bar{N} \rightarrow a_0\pi, a_0\omega, a_0\rho, a_0p$.

We are waiting also for a big number of resonances with the four-quark structure in the different annihilation channels to three vector mesons $N\bar{N} \rightarrow VV'V''$ in the system of mass of two vector mesons. The cross sections of four-quark meson production must satisfy to exact ratios. This point allows easy to extract $q^2\bar{q}^2$ mesons from a big number of the observable particles.

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