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ON THE SEARCH
FOR LIKE MESON INTERFERENCE

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К поискам интерференции тождественных мезонов

Рассчитано влияние поверхностного испускания мезонов возбужденным объемом на их интерференцию. Обсуждаются способы оценки фона.

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On the Search for Like Meson Interference

It is assumed that in the frame of the statistical model of multiple production only those mesons which are emitted by outer parts of an excited volume reach detectors. The influence of this effect on the interference between like mesons is calculated. The ways to estimate the background are discussed.

The investigation has been performed at the Laboratory of High Energies, JINR.

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Dubna 1975

The existence of interference correlations between momenta of like particles was predicted in 1970[1]*. This effect permits one to measure the lifetime τ and the radius R of the excited volume in which particles are produced. First experimental observations of the effect appeared in 1975 [4-6]. This paper treats two questions arising from these observations.

1. *WHAT TIME DOES ONE MEASURE IN THE INTERFERENCE EXPERIMENT?*

Data of refs. [4-6] are contradictory. The discrepancy can be so far explained by the natural reason that the reactions investigated in [4-6] do not coincide. We shall mainly discuss here some interesting data from ref. [5] according to which $R = 1{,}6 \pm 0{,}2\,f$, $\tau \approx 0{,}64\, f/c$. The value of τ seems to be too small ($\tau < R/c$) and needs to be explained.

As is stated in ref. [7], there are several quantities of time dimension which can influence the interference effect:

a) lifetime of meson sources, τ;

b) dispersion of moments when the sources are switched on, T;

c) time which the meson needs to transverse the excited volume, t^L (see also ref. [8]).

Only some combination of these three times can be measured in first crude experiments [7]. We want to

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* These correlations were considered previously in ref. [2] and some other papers (see, e.g., [3]) from a somewhat different point of view.

imagine a picture when this combination would be small. One can assume, e.g., that

α) no mesons are emitted during the time while the statistical equilibrium is being set in. Only when the process of "termalization" is finished, the meson sources are "switched on". Then nothing hinders us to assume that their lifetime $\tau \ll R/c$ and that $T \ll R/c$*.

β) mesons after generation interact strongly, and really one can see only those which are emitted from an outer surface of the excited volume **.

As we shall see, this effect makes the longitudinal time t^L approximately four times smaller than that for the transparent sphere (see Eq. (17) below).

We want here to notice that the data of ref. /5/ can be explained by combining these two factors, i.e., a small lifetime of the meson sources and not a very large longitudinal time t^L.

The effect of the surface meson sources was calculated already in /9/ for $\tau \gg R/c$. Now we discuss a more general case of arbitrary τ. (For simplicity we take $T = 0$. The effect depends on T and on τ qualitatively similarly. Each statement about τ written below equally concerns T).

Let narrow meson pairs be detected which have been emitted in the direction $\vec{n}$. The difference of their 4-momenta p_1, p_2 is equal to $q = \{q_0, \vec{q}\}$. The meson sources are assumed to be distributed over the forward surface of the hemisphere (its part is shown in *fig. 1*).

Let the Lambert law be fulfilled: the effective density of the sources depends on $\cos\theta$ according to the law

*Otherwise, i.e., when there exists free meson emission during the "termalization process", it is more natural to expect that $\tau > R/c$.

** Like light from the Sun. Probably, this is a way to take into account phenomenologically the final state interaction between mesons.

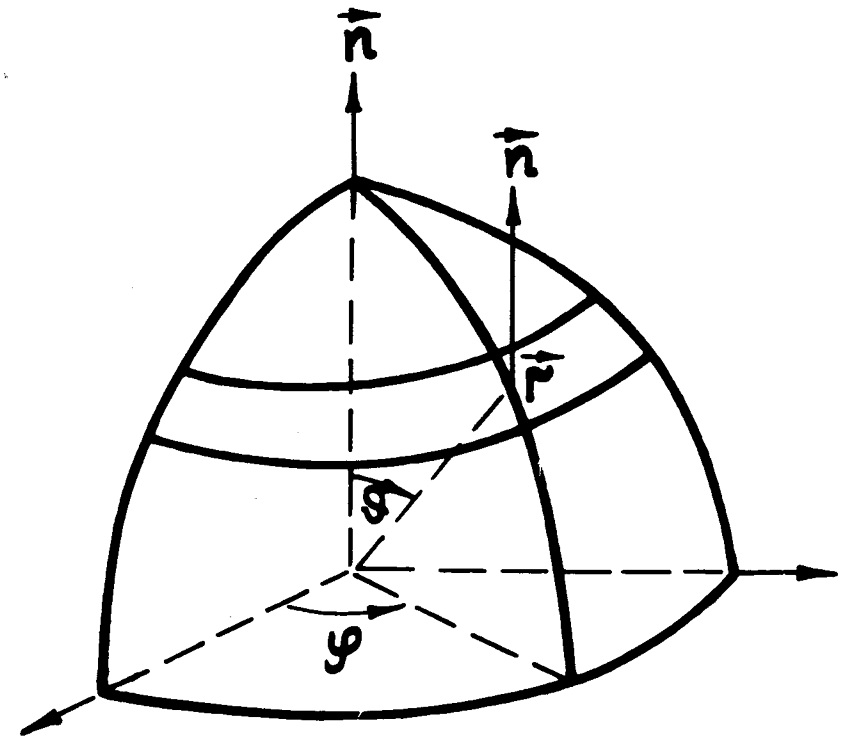

Fig. 1

$$U(\vec r) = \begin{cases} \cos\theta/\pi R^2, & r = R,\ \theta = (\hat{\vec r,\vec n}) < \pi/2, \\ 0 & ,\ r \neq R \ \text{or}\ \theta > \pi/2. \end{cases} \quad (1)$$

The function $U(\vec r)$ is normalized to 1. It is shown in ref. /7/ that the probability of observation of two like pions is given by the expression

$$W \sim \int_S\int_S \left[1 + \frac{\cos\vec q(\vec r_1 - \vec r_2)}{1+(q_0\tau)^2}\right] U(\vec r_1)\, U(\vec r_2)\, d\vec r_1 d\vec r_2 \ . \quad (2)$$

Here $\vec r_1$ and $\vec r_2$ are the coordinates of two sources. The integration is taken over the region where these sources are present. If the both are distributed similarly, we have

$$W \sim [\int_S U(\vec r)\, d\vec r\,]^2 + (1+q_0^2\tau^2)^{-1} \ |\int_S \exp(i\vec q\vec r)\, U(\vec r)\, d\vec r\,|^2 . \quad (3)$$

In our case the region S can be written in the spherical frame

$$\vec r = \{ R\sin\theta\cos\phi,\ R\sin\theta\sin\phi,\ R\cos\theta \},$$
$$d\vec r = R^2 d\cos\theta d\phi, \quad 0 < \theta \leq \pi/2, \quad 0 \leq \phi < 2\pi. \quad (4)$$

As is mentioned above, $\int U(\vec r) d\vec r = 1$. The second term integral is denoted by $f(\vec q)$. Then we have

$$f(\vec q) = \int e^{i\vec q\vec r}\, U(\vec r)\, d\vec r = \frac{1}{\pi}\iint \exp(iq_\perp r_\perp + iq_{||} r_{||})\, \cos\theta\, d\cos\theta\, d\phi.$$

Here we introduce the projection of $\vec q$ along $\vec n$ and transverse to it. After integrating over ϕ, we have *

$$f(\vec q) = 2\int_0^1 \exp(iq_{||}R\cos\theta)\ J_0(q_\perp R\sin\theta)\ \cos\theta\, d\cos\theta. \quad (5)$$

We come to formula

$$W \sim 1 + (1+q_0^2\tau^2)^{-1} |2\int_0^1 \exp(iq_{||}R\cos\theta)\ J_0(q_\perp R\sin\theta)\ \cos\theta\, d\cos\theta|^2. \quad (6)$$

This formula permits the effect of interest to be estimated qualitatively. If $\tau \gg R/c$, only small $q_0 \ll c/R$ and therefore small $q_{||}$ contribute to the interference; one can neglect the exponent in (6). This results in the formula

$$W \sim 1 + (1+q_0^2\tau^2)^{-1}\ [\,2J_1(q_\perp R)/q_\perp R\,]^2 \quad (7)$$

obtained in /9/. The effect of interference appears to coincide with that for a flat disc.

On the contrary, for $\tau \ll R/c$ and for $q_\perp \ll 1/R$ a main contribution is from $q_{||} \sim 1/R$, i.e., from the exponent (see formula (13) below). In this case the convex form of the radiating surface considerably effects the interference.

Let us take an exact calculation now. We expand the exponent in a series and use the formula (6.683.6) from ref. /10/

$$\int_0^{\pi/2} J_\mu(\alpha\sin\theta)(\cos\theta)^{2\rho+1}(\sin\theta)^{\mu+1}\, d\theta = \quad (8)$$
$$= 2^\rho\,\Gamma(\rho+1)\, J_{\rho+\mu+1}(\alpha)\,\alpha^{-(\rho+1)}.$$

Using the notation $q_{||}R = \gamma$, $q_\perp R = \alpha$, we obtain

$$f(\vec q) \equiv f(q_{||}R, q_\perp R) \equiv f(\gamma,\alpha) =$$
$$= 2\sum_0^\infty \frac{(i\gamma\sqrt{2})^n\,\Gamma(\frac{n}{2}+1)}{n!}\ \frac{J_{\frac{n}{2}+1}(\alpha)}{\alpha^{\frac{n}{2}+1}}. \quad (9)$$

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* An additional integration over R within the limits $R-\delta/2$ and $R+\delta/2$ would take into account the effect of depth of an excited volume "photosphere" /5/. For $\delta \ll R$ this effect can be neglected, see the end of this Section.

The effect in eq. (3) is determined by the square of module of $f(\vec{q})$. Separating in (9) the real and imaginary parts, we express $|f(\vec{q})|^2$ as a sum of squares of two rapidly converging series

$$|f(\gamma,a)|^2 = \Big[\sum_0^\infty \frac{2(-\gamma^2)^m}{(2m-1)!!}\,\frac{J_{m+1}(a)}{a^{m+1}}\Big]^2 + \tag{10}$$

$$+\frac{\pi}{2}\gamma^2\Big[\sum_0^\infty \frac{2(-\gamma^2)^m}{(2m)!!}\,\frac{J_{m+\frac{3}{2}}(a)}{a^{m+3/2}}\Big]^2.$$

Now $J_n(a) = O(a^n)$ for small a. Therefore it is convenient to introduce the ratio $J_n(a)/a^n$ as a new function of a. It is designated /11/ as $\Lambda_n(a)$

$$\Lambda_n(a) = \sum_{k=0}^\infty \frac{n!}{k!(n+k)!}\Big(-\frac{a^2}{4}\Big)^k, \quad \Lambda_n(0)=1 .$$

Now

$$|f(\gamma,a)|^2 = \Big[\sum_0^\infty \frac{2(-\gamma^2)^m\,\Lambda_{m+1}(a)}{(2m)!\,(2m+2)}\Big]^2 + \tag{11}$$

$$+\gamma^2\Big[\sum_0^\infty \frac{2(-\gamma^2)^m\,\Lambda_{m+3/2}(a)}{(2m+1)!\,(2m+3)}\Big]^2.$$

The lines of equal values of $|f(\gamma,a)|^2$ are shown in *fig. 2*. Functions $|f(0,a)|^2$ and $|f(\gamma,0)|^2$ are comparatively simple

$$|f(0,a)|^2 = [2J_1(a)/a]^2, \tag{12}$$

$$|f(\gamma,0)|^2 = 4\Big|\frac{\cos\gamma - 1 + \gamma\sin\gamma + i(\sin\gamma - \gamma\cos\gamma)}{\gamma^2}\Big|^2 =$$

$$= \frac{4[\gamma^2 + 2(1-\cos\gamma - \gamma\sin\gamma)]}{\gamma^4}. \tag{13}$$

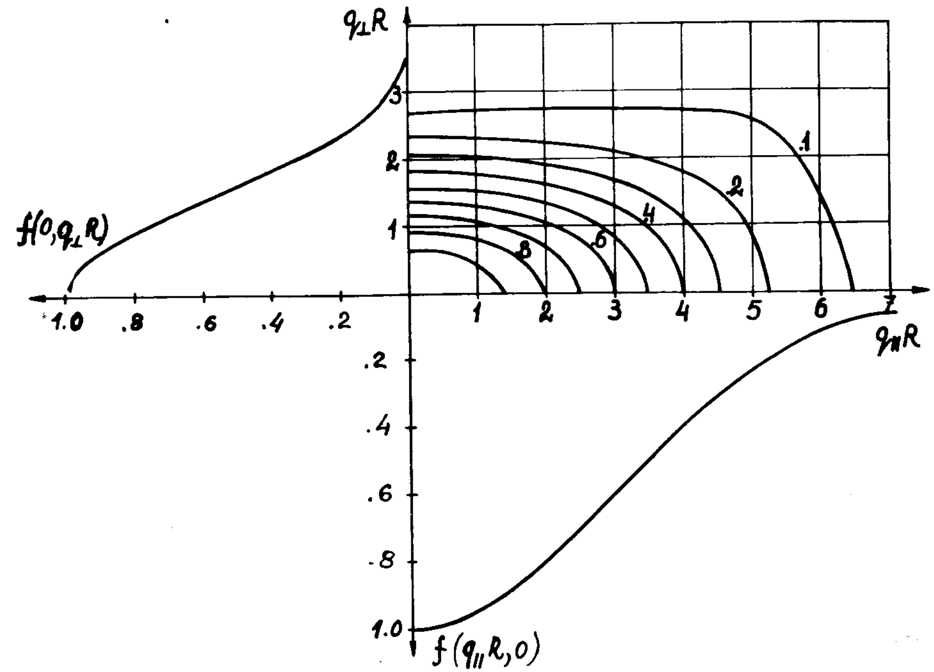

Fig. 2. The lines of equal values of $f(q_\parallel R, q_\perp R)$; *functions* $f(0, q_\perp R)$ *and* $f(q_\parallel R, 0)$.

One can write them for sufficiently small q_0 and $q_\perp$ in the form

$$|f(0,q_\perp R)|^2 \approx \exp\Big(-\frac{1}{4}q_\perp^2 R^2\Big), \quad q_\perp R \ll 1 \tag{14}$$

$$|f(q_0\frac{R}{v},0)|^2 \approx \exp\Big(-\frac{1}{18}q_0^2\frac{R^2}{v^2}\Big), \quad q_0\frac{R}{v} \ll 1. \tag{15}$$

Here $q_0 = q_\parallel v$, where v is the velocity of the pair of particles. Therefore one has for small $q_0, q_\perp$

$$W \sim 1 + \frac{\exp(-\frac{1}{4} q_\perp^2 R^2 - \frac{1}{18} q_0^2 \frac{R^2}{v^2})}{1 + (q_0 \tau)^2} . \qquad (16)$$

If we understand correctly, the experimental data in ref.[5] were fitted to the formula (7).The dependences on $q_\perp$ in (16) and in (7) practically coincide. But if one fits by formula (7) the data which really correspond to formula (16) for $\tau = 0$, one can obtain for τ the value

$$\tau \simeq .236 \frac{R}{v} , \qquad (17)$$

i.e., $\tau < R/c$ as was obtained in[5].

If the discussed model is true, this means that for sufficiently high energies of particles ($v \sim c$) the dimensions of the system determine the widths of spectra of both $q_\perp$ and q_0.

Now it is easy to take into account the effect of "photosphere". Let its depth is δ, its average radius $R \gg \delta$ and a density of sources in it is constant. Integrating over this thin layer we come again to Eq. (16), where the term R^2/v^2 is substituted by $(R^2+\delta^2)/v^2$ (again in approximation $q_\parallel R \ll 1$).

To understand this result qualitatively add W (from (16)) for $R+\delta/2$ and for $R-\delta/2$ and divide by 2. Exact calculation only changes the coefficient in the term δ^2.

2. HOW TO OBTAIN A BACKGROUND OF THE INTERFERENCE EFFECT?

In refs.[5,6] the peak on the $\Delta\Delta$ -plot[12], i.e., on the $(q_\perp^2, q_0)$ plane, for $\pi^\pm\pi^\pm$ pairs is compared with the distribution of $\pi^+\pi^-$-pairs on the same plane. The $\pi^+\pi^-$-pairs are considered as a background of the interference phenomenon; there is no interference between unlike particles. But the pairs $\pi^\pm\pi^\pm$ and $\pi^+\pi^-$ are

distributed differently, as it follows from refs.[5,6], also far from the peak (this can be, for example, due to the production of ρ°-mesons). This hinders to compare the effect with background. Because of this, the phase space distribution of $(q_\perp^2, q_0)$ was used in[6] for background. There is another way to obtain a background. One can take one momentum $\vec{p}_1$ from one event and $\vec{p}_2$ from another[4]. This method seems to give a constant ratio "effect/background" far from the peak. But it has another shortcoming: nonconservation of energy and momentum. Here we propose one more way of obtaining background. It is similar to that in[4] but assures the momentum conservation.

The essence of the method can be seen from such an example. Let one neutron and four mesons $\pi^+\pi^-\pi^+\pi^-$ be obtained in some π^-p event. Their momenta are $\vec{p}_1, \vec{p}_2, \vec{p}_3, \vec{p}_4$ with components $\{x_i, y_i, z_i\}$. The interaction axis is z. We form new momenta $\vec{p}_i{}'$ $(i=1,2,3,4)$ by mixing the components of $\vec{p}_i$:

$$\vec{p}_i{}' = \{x_j, y_k, z_i\}, \quad i \neq j \neq k . \qquad (18)$$

One can do, e.g., such a permutation of the components

$$\vec{p}_1{}' = \{x_3, y_2, z_1\}, \qquad \vec{p}_3{}' = \{x_1, y_4, z_3\},$$
$$\vec{p}_2{}' = \{x_4, y_3, z_2\}, \qquad \vec{p}_4{}' = \{x_2, y_1, z_4\}. \qquad (19)$$

It is easy to see that the total momentum is conserved

$$\sum_i \vec{p}_i{}' = \sum_i \vec{p}_i . \qquad (20)$$

The total energy is also conserved in a nonrelativistic limit

$$\sum_i p_i'^2 / 2m_\pi = \sum_i p_i^2 / 2m_\pi . \qquad (21)$$

But in high energy events, if $|x_j|, |y_k| \ll |z_i|$, it is conserved only approximately

$$\sum_i E_i' \approx \sum_i \{ |z_i| + \frac{m_\pi^2 + x_j^2 + y_k^2}{2|z_i|} \} \approx \sum_i E_i; \qquad (22)$$

The error in the total energy in (22) can be less than the sum of the fractions standing in (22). Let, for example, all $|z_i|$ have the same values, then one has exactly

$$\sum_i |z_i| + \sum_i \frac{m_\pi^2 + x_j^2 + y_k^2}{2|z_i|} = \sum_i |z_i| + \sum_i \frac{m_\pi^2 + x_i^2 + y_i^2}{2|z_i|}. \qquad (23)$$

But the total energy nonconservation can be considerable if all the components x_i, y_i, z_i are of the same order of magnitude, for example, in the CMS of isotropically decaying fireball (as in /5/).

We propose to try as a background the distribution of q_0', $q_\perp'^2$ obtained from the momenta $\vec{p}_i'$. It is clear that this switches off the interference between like mesons.

One can modify the way of "splitting" the momenta components:

1) to mix only the longitudinal components and not to change the transverse components

$$\vec{p}_i' = \{x_i, y_i, z_k\}, \quad k \neq i.$$

The transverse picture of each event remains untouched. This way of "splitting" can be useful if one wants to know whether the azimuthal correlations of like mesons are connected with the $\Delta\Delta$ -plot distributions /12/;

2) one can achieve the conservation of total energy and longitudinal momentum at the expense of the non-conservation of transverse momentum. For this, turn each momentum $\vec{p}_i$ randomly around the z -axis. The polar angles θ_i should not be changed.

Finally we would like to note that the mixing can be done so that the resonances will remain almost unaffected. But we do not describe this matter in detail here.

We are grateful to M.S. Levitsky for his help in calculations.

Note added in proof. After the present work was completed there appeared two preprints /13/ J.Cantor et al., BNL-20516 and /14/ M.Deutschmann et al., CERN-PHYS. 75-44 (a revised version of /5/) which confirm that τ is small. Here we compile existing data on R and τ ($\hbar / m_\pi c = 4.7 \cdot 10^{-24}$ s)

Reference	R, fermi	τ, 10^{-24} s
/6/	.8	3 ÷ 4
/14/	1 ±.2	2.3 ± 1
/13/	1.3 ±.1	1.2 ±.8
/4/	4	6 ± 2

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