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Изоспиновые ограничения для реакций в произвольными спинами

Для реакций с произвольным спином получены как изоспиновые ограничения (равенства и границы), так и теоремы типа Померанчука. Обсуждаются несколько предсказаний для NN -рассеяния и реакций $\pi p \to K\Sigma^*$, $Kp \to \pi\Sigma^*$.

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Isospin Constraints for Reactions with Arbitrary Spins

Isospin constraints (equalities and bounds), as well as the Pomeranchuk-like theorems, for reactions with arbitrary spins are proved. Several predictions for the NN -scattering and $\pi p \to K\Sigma^*$, $Kp \to \pi\Sigma^*$ are discussed.

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In this paper we derive several stringent isospin constraints on the differential observable of the reactions with arbitrary spins related by isospin invariance via two isospin channels. We then prove: i) the Pomeranchuk-like theorems on differential observables of these types of reactions and ii) the constraints on spin polarization parameters when the usual triangular inequalities on differential cross-sections are exactly saturated or degenerated.

Therefore, suppose three reactions

$$a_\ell + b_\ell \to 1_\ell + 2_\ell + \dots + n_\ell , \quad \ell = 1,2,3,$$

are described by N independent amplitudes $f_\ell^{(\alpha)}$ ($\alpha = 1,2,\dots,N$) which are defined so that the differential cross-section is

$$\sigma_\ell = \sum_{\alpha=1}^{N} | f_\ell^{(\alpha)} |^2, \quad \ell = 1,2,3. \tag{1}$$

We shall assume that all the amplitudes $f_\ell^{(\alpha)}$ satisfy the linear relations:

$$\sum_{\ell=1}^{3} c_\ell f_\ell^{(\alpha)} = 0 , \quad (\alpha = 1,2,\dots,N) \tag{2}$$

due to the isospin invariance, where c_ℓ , $\ell = 1,2,3$, is a homogeneous fourth degree polynomial of Clebsh-Gordan coefficients. Then, using the sum rule (2) and Schwartz's inequality, we obtain [1] the usual triangular inequalities which are equivalent to

$$0 \le -\lambda[c_1^2\sigma_1 , c_2^2\sigma_2 , c_3^2\sigma_3] \equiv -\lambda[\sigma], \tag{3}$$

where λ-function is defined in general by

$$\lambda[x_1, x_2, x_3] \equiv x_1^2 + x_2^2 + x_3^2 - 2x_1x_2 - 2x_2x_3 - 2x_3x_1 , \tag{4}$$

Next, let us define σ_β and $\vec{\xi}_\beta \equiv (A_\beta, P_\beta, R_\beta)$ by the relations

$$\sigma_{\beta\ell} = |f_\ell^{(\alpha_p)}|^2 + |f_\ell^{(\alpha_r)}|^2, \tag{5a}$$

$$A_{\beta\ell}\,\sigma_{\beta\ell} = 2\mathrm{Re}[\overset{*}{f}{}_\ell^{(\alpha_p)} f_\ell^{(\alpha_r)}], \tag{5a}$$

$$P_{\beta\ell}\,\sigma_{\beta\ell} = -2\mathrm{Im}[\overset{*}{f}{}_\ell^{(\alpha_p)} f_\ell^{(\alpha_r)}], \tag{5c}$$

$$R_{\beta\ell}\,\sigma_{\beta\ell} = [|f_\ell^{(\alpha_p)}|^2 - |f_\ell^{(\alpha_r)}|^2, \tag{5d}$$

with

$$\vec{\xi}_{\beta\ell}^{\,2} = A^2_{\beta\ell} + P^2_{\beta\ell} + R^2_{\beta\ell} = 1, \tag{5e}$$

for any $\beta \equiv (\alpha_p, \alpha_r), \alpha_p < \alpha_r, \alpha_p \neq \alpha_r = 1,2,3,\dots,N$.
Next, let H_{ij} and $H_{\beta ij}$ be defined by

$$H_{ij} = \sum_{\text{all}\,\beta} H_{\beta ij} \geq 0,\quad H_{\beta ij} = \frac{1}{2}[1 - \vec{\xi}_{\beta i}\cdot\vec{\xi}_{\beta j}]\,\sigma_{\beta i}\,\sigma_{\beta j}. \tag{6}$$

Then, *the sum rules (2) alone imply that the equalities*

$$H \equiv c_1^2 c_2^2 H_{12} = c_2^2 c_3^2 H_{23} = c_3^2 c_1^2 H_{31} \geq 0, \tag{7}$$

and the bounds

$$4H \leq -\lambda[\sigma] \leq 4\min\{c_i^2 c_j^2 \sigma_i \sigma_j\}, \tag{8}$$

are valid for any values of kinematical variables [2] *from the physical domain.*

The proof of eqs. (7) and (8) can be obtained as follows. Start with the bilinear forms

$$Z^{(0)}_{ij} = \sum_{\alpha=1}^{N} \overset{*}{f}{}_i^{(\alpha)} f_j^{(\alpha)},\quad i \neq j = 1,2,3, \tag{9a}$$

$$Y_{\beta ij} = f_i^{(\alpha_p)} f_j^{(\alpha_r)} - f_i^{(\alpha_r)} f_j^{(\alpha_p)},\quad \beta \equiv (\alpha_p, \alpha_r), \alpha_p < \alpha_r, \tag{9b}$$

which have the properties

$$Z^{(0)}_{\ell\ell} = \sigma_\ell,\quad \ell = 1,2,3, \tag{9c}$$

$$|Z^{(0)}_{ij}|^2 = \sigma_i\,\sigma_j - H_{ij},\quad \text{(see definition (6)/,} \tag{9d}$$

$$|Y_{\beta ij}|^2 = \frac{1}{2}[1 - \vec{\xi}_{\beta i}\cdot\vec{\xi}_{\beta j}]\sigma_{\beta i}\sigma_{\beta j} = H_{\beta ij}. \tag{9e}$$

We note that eq. (9d) is a direct consequence of the "Lagrange" identity

$$|Z^{(0)}_{ij}|^2 + \sum_{\text{all}\,\beta} |Y_{\beta ij}|^2 = [\sum_{\alpha=1}^{N} |f_i^{(\alpha)}|^2][\sum_{\alpha=1}^{N} |f_j^{(\alpha)}|^2], \tag{9f}$$

and eqs. (1) and (9e).

Then, using the sum rule (2), we get the relations

$$c_1 c_2 Y_{\beta 12} = c_2 c_3 Y_{\beta 23} = c_3 c_1 Y_{\beta 31}, \tag{10a}$$

$$\mathrm{Re}\,Z^{(0)}_{ij} = (2 c_i c_j)^{-1}[c_k^2\sigma_k - c_i^2\sigma_i - c_j^2\sigma_j], \tag{10b}$$

$$4c_i^2 c_j^2 [\mathrm{Re}\,Z^{(0)}_{ij}]^2 = 4c_i^2 c_j^2 \sigma_i\,\sigma_j + \lambda[\sigma] \geq 0, \tag{10c}$$

$$4c_i^2 c_j^2 [\mathrm{Im}\,Z^{(0)}_{ij}]^2 = -4H - \lambda[\sigma] \geq 0, \tag{10d}$$

which imply the constraints (7) and (8), respectively.

Clearly, the lower bound (8) is more stringent than the usual triangular inequalities (3). Also, it is easy to see that *the constraints (7) and (8) are independent of the spin reference frame,* and that these constraints improve in particular all the bounds derived by Kamei et al. and Dass et al. /see ref. [1] /. Eqs. (7) and (8) require that, *if the bound (3) is exactly saturated then*

$$\vec{\xi}_{\beta 1} = \vec{\xi}_{\beta 2} = \vec{\xi}_{\beta 3}, \tag{11}$$

for all β *and any spin reference frame.*

We note that eqs. (7) and (8) represent a complete extension, to reactions with arbitrary spins, of the corresponding constraints previosly derived [3,4,5] for the $(0^- 1/2^+ \to 0^- 1/2^+)$ and $(0^- 1/2^+ \to 0^- 0^- 1/2^+)$ reactions [6], respectively.

Next, we observe that the lower bound (8) is equivalent to

$$[c_i^2\sigma_i - c_j^2\sigma_j]^2 + 4H \le 2c_k^2\sigma_k[c_i^2\sigma_i + c_j^2\sigma_j - \frac{1}{2}c_k^2\sigma_k], \qquad (12)$$

where $i \neq j \neq k$ are any permutation of channel indices 1,2,3.

Therefore, if

$$\sigma_k[c_i^2\sigma_i + c_j^2\sigma_j] \xrightarrow[s\to+\infty]{} 0, \qquad (13)$$

/$\sqrt{s}$ is the c.m. energy/ *when the other kinematical variables* [2] *are fixed, then the isospin bound (12) implies the Pomeranchuk-like theorems*

$$c_i^2\sigma_i - c_j^2\sigma_j \xrightarrow[s\to+\infty]{} 0, \qquad (14a)$$

$$[A_{\beta i}, P_{\beta i}, R_{\beta i}] - [A_{\beta j}, P_{\beta j}, R_{\beta j}] \xrightarrow[s\to+\infty]{} 0 \qquad (14b)$$

for all β *in any spin reference frame, or, conversely the* σ_k *-differential cross section cannot vanish for* $s \to +\infty$ *if one of the Pomeranchuk-like theorems (14a,b) is violated at high energies.*

Hence, if the indices i, j, k, are chosen such that they correspond to the elastic and change exchange channels, respectively, then the bound (12) makes it possible for us to understand the small elastic differential cross-sections differences [e.g., $\sigma^{el}_{pn} - \sigma^{el}_{pp}$, $\sigma^{el}_{\bar{p}p} - \sigma^{el}_{\bar{p}n}$, $\sigma^{el}_{\pi^+p} - \sigma^{el}_{\pi^-p}$, $\sigma^{el}_{K^+p} - \sigma^{el}_{K^0p}$, $\sigma^{el}_{K^-p} - \sigma^{el}_{\bar{K}^0p}$, $\sigma^{el}_{K^+n} - \sigma^{el}_{K^-n}$, $\sigma^{el}_{K^+p} - \sigma^{el}_{K^-p}$] at high energies and small $|t|$-fixed, in terms of small charge exchange differential cross sections [$\sigma_{pn\to np}$, $\sigma_{\bar{p}p\to\bar{n}n}$, $\sigma_{\pi^-p\to\pi^0n}$, $\sigma_{K^+n\to\bar{K}^0p}$, $\sigma_{K^-p\to\bar{K}^0n}$, $\sigma_{K^0_Lp\to K^0_sp}$, $\sigma_{K^0_Ln\to K^0_sn}$, respectively], and to predict the validity of the Pomeranchuk-like theorems for all the differential observables of the elastic reactions for $s \to +\infty$ and $|t|$-fixed.

For the nucleon-nucleon scattering, since there are five independent amplitudes [7], we are far from having a complete set of measurements for comparison of the isospin constraints (7) and (8) with the experiments. However, by a systematic study of the usual triangular isospin inequalities /see eq. (3)/ one can find the kinematical regions where the bound (3) is exactly saturated. Then, using eqs. (7), (8) we predict the equalities (11), which imply:

$$P^{el}_{pp} = P^{el}_{pn} = P^{CE}_{pn}, \quad D^{el}_{pp} = D^{el}_{pn} = D^{CE}_{pn},$$
$$C^{el}_{pp} = C^{el}_{pn} = C^{CE}_{pn}, \quad K^{el}_{pp} = K^{el}_{pn} = K^{CE}_{pn}, \qquad (15)$$

where P_{NN}, C_{NN}, D_{NN} and K_{NN} denote the "polarization asymmetry", "correlation tensor", "depolarization tensor" and "polarization transfer tensor", for the elastic and charge exchange scattering, respectively, defined in ref. [7]. Therefore, these isospin constraints can be used for a test of the treatments of experimental data since the extraction of $pn \to pn$ data from pd scattering usually employs the Glauber theory and also some further simplifying assumptions. At high energies, one expects get a good bound on elastic proton-neutron differential cross sections because of smallness of charge exchange data. Comparisons of these bounds with the experiments have been carried out by Dass and Froyland [1] for P_{LAB} = 12, 19.2 and 24 *GeV/c*, and they find that the elastic pn data [8] lie consistently above the upper bound. Therefore it would be interesting to have new experimental data for pn-elastic scattering and also the data for the polarization asymmetry for all elastic and charge exchange channels in order to compare the predictions (15) with the experiments.

Next, we remark that the experimental consequences of the isospin constraints (7) and (8) can be easily tested (experimentally) in the quasi-two-body reactions with unpolarized (or polarized) target. Then, using the formalism developed in ref. [9], we obtain a large number of equalities of the form

$$c_1^2 c_2^2 H_{\beta 12} = c_2^2 c_3^2 H_{\beta 23} = c_3^2 c_1^2 H_{\beta 31}, \tag{16}$$

/see eqs. (10a) and (9e)/ in terms of the spin density matrix elements in the transversity quantization system. For example let us consider three $(0^- 1/2^+ \to 0^- 3^+/2)$ reactions related by isospin invariance via two-isospin channels [eq. $\pi p \to K\Sigma^+$, $Kp \to \pi\Sigma^+$, etc.]. Then, for reactions with unpolarized target, we obtain two sets of $[\sigma_{\beta\ell}, \vec{\xi}_{\beta\ell} = (A_{\beta\ell}, P_{\beta\ell}, R_{\beta\ell})]$ observables and $H_{\beta ij}$ which are listed in the table. We note that the exact saturation of the bound (3) on the differential cross sections implies the equalities:

$$\rho^{(1)}_{11} = \rho^{(2)}_{11} = \rho^{(3)}_{11}, \qquad \rho^{(1)}_{-1-1} = \rho^{(2)}_{-1-1} = \rho^{(3)}_{-1-1}, \tag{17a}$$

$$\rho^{(1)}_{33} = \rho^{(2)}_{33} = \rho^{(3)}_{33}, \qquad \rho^{(1)}_{-3-3} = \rho^{(2)}_{-3-3} = \rho^{(3)}_{-3-3}, \tag{17b}$$

$$\rho^{(1)}_{1-3} = \rho^{(2)}_{1-3} = \rho^{(3)}_{1-3}, \qquad \rho^{(1)}_{3-1} = \rho^{(2)}_{3-1} = \rho^{(3)}_{3-1} \quad \text{m, etc.} \tag{17c}$$

These relations are direct consequences of the isospin bound (8). Comparisons of the isospin constraints (16) with the experimental data are of great interest for a detailed test of the isospin invariance in the $(0^-1/2^+ \to 0^-3/2^+)$ reactions of the types $\pi p \to K\Sigma^*$ and $Kp \to \pi\Sigma^*$. They also can be used for a test of the resonance background (or $\Lambda - \Sigma^\circ$) separations.

Finally, we note that *the isospin sum rules (2) impy that each of isospin constraints (7), (8), (12), (13), (14a,b), (15), (16) and (17a,b,c) has an integrated analog.* Proof of this statement can be obtained just as in $(0^-1/2^+ \to 0^-1/2^+)$ scattering case discussed in ref. [4,10].

Table

The observables $\sigma_{\beta\ell}$, $\vec{\xi}_{\beta\ell} = (A_{\beta\ell}, P_{\beta\ell}, R_{\beta\ell})$ and $H_{\beta ij}$ for $(0^-1/2^+ \to 0^-3/2^+)$ reactions with the unpolarized target, in terms of spin density matrix elements.

Observables	(1)	(2)
$\sigma_{\beta\ell}$	$2\sigma_\ell [\rho^{(\ell)}_{11} + \rho^{(\ell)}_{-3-3}]$ *	$2\sigma_\ell [\rho^{(\ell)}_{-1-1} + \rho^{(\ell)}_{33}]$ *
$A_{\beta\ell}$	$2\mathrm{Re}\,\rho^{(\ell)}_{1-3} / [\rho^{(\ell)}_{11} + \rho^{(\ell)}_{-3-3}]$	$2\mathrm{Re}\,\rho^{(\ell)}_{3-1} / [\rho^{(\ell)}_{-1-1} + \rho^{(\ell)}_{33}]$
$P_{\beta\ell}$	$2\mathrm{Im}\,\rho_{1-3} / [\rho^{(\ell)}_{11} + \rho_{-3-3}]$	$2\mathrm{Im}\,\rho^{(\ell)}_{3-1} / [\rho^{(\ell)}_{-1-1} + \rho^{(\ell)}_{33}]$
$P_{\beta\ell}$	$[\rho^{(\ell)}_{11} - \rho^{(\ell)}_{-3-3}] / [\rho^{(\ell)}_{11} + \rho^{(\ell)}_{-3-3}]$	$[\rho^{(\ell)}_{-1-1} - \rho^{(\ell)}_{33}] / [\rho^{(\ell)}_{-1-1} + \rho^{(\ell)}_{33}]$
$H_{\beta ij}$	$4\sigma_i\sigma_j [\rho^{(i)}_{11}\rho^{(j)}_{-3-3} + \rho^{(j)}_{11}\rho^{(i)}_{-3-3} - 2\,\mathrm{Re}(\rho^{*(i)}_{1-3}\rho^{(j)}_{1-3})]$	$4\sigma_i\sigma_j [\rho^{(i)}_{-1-1}\rho^{(j)}_{33} + \rho^{(j)}_{-1-1}\rho^{(i)}_{33} - 2\,\mathrm{Re}(\rho^{*(i)}_{3-1}\rho^{(j)}_{3-1})]$

*The rank constraints: $\rho^{(\ell)}_{11}\rho^{(\ell)}_{-3-3} = |\rho^{(\ell)}_{1-3}|^2$, $\rho^{(\ell)}_{-1-1}\rho^{(\ell)}_{33} = |\rho^{(\ell)}_{3-1}|^2$, $\ell = 1,2,3,$ are equivalently to $\vec{\xi}^{\,2}_{\beta\ell} = 1$.

References

1. *R.J.N.Phyllips. Nuovo Cimento, 26, 103 (1962); O.Kamei and S.Sasaki. Nuovo Cimento, 59A, 535 (1969); G.V.Dass and J.Froyland. Nucl.Phys., B42, 153(1972).*
2. *A.Gheorghe, D.B.Ion and El.Mihul. Ann. Inst. Henri Poincare, 22, 131 (1975).*
3. *M.G.Doncel, L.Michel and P.Minnaert. Phys.Lett., 38B, 42 (1972).*
4. *D.B.Ion. Nucl.Phys., B84, 55 (1975) and JINR preprints E2-7732, E2-7868, Dubna, 1974; E2-8695, Dubna, 1975.*
5. *D.B.Ion. Nucl.Phys., B96, 67 (1975) and JINR preprint E2-8432, Dubna, 1974.*
6. *D.B.Ion. JINR preprints, E2-8865, E2-8866, Dubna, 1975.*
7. *G.L.Kane and U.P.Suchatame. Nucl.Phys., B78, 110 (1974).*
8. *B.G.Gibbard et al. Nucl.Phys., B30, 77 (1971).*
9. *M.G.Doncel, L.Michel and P.Minnaert. CERN preprint /D.Ph.II/ Phys. 74-7, 1974.*
10. *D.B.Ion. JINR preprint E2-9227, Dubna, 1975.*

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